Mihailo Petrović

ALAS

Life
Work
Times

Serbian Academy of Sciences and Arts

MIHAILO PETROVIĆ ALAS
150th birth anniversary

MIHAILO
PETROVIĆ
150th ALAS
birth anniversary

SERBIAN ACADEMY OF SCIENCES AND ARTS

MIHAILO PETROVIĆ ALAS: LIFE, WORK, TIMES
ON THE OCCASION OF THE 150th ANNIVERSARY OF HIS BIRTH

Publisher
Serbian Academy of Sciences and Arts
Knez Mihailova 35, Belgrade

Acting publisher
Academician Vladimir S. Kostić

Editor-in-chief
Academician Marko Anđelković

Editors of publication
Academician Stevan Pilipović
Academician Gradimir V. Milovanović
Professor Dr Žarko Mijajlović

Cover design
Dragana Lacmanović-Lekić

Prepress
Dosije Studio, Belgrade

Selection of artworks
Maja Novaković

English translation
Tatjana Ćosović, Natalija Stepanović
Tanja Ružin Ivanović, Žarko Radovanov, Dora Seleši

Proofreading and editing
Jelena Mitrić

Printing
Planeta print, Belgrade

Print run: 500 copies

ISBN 978-86-7025-818-1

The publication was financially supported by the Ministry of Education, Science and Technological Development of the Republic of Serbia and Telekom Srbija.

MIHAILO PETROVIĆ ALAS
LIFE, WORK, TIMES

ON THE OCCASION OF THE 150th ANNIVERSARY OF HIS BIRTH

SERBIAN ACADEMY OF SCIENCES AND ARTS

Exclusive editions, such as this monograph, call for the engagement, enthusiasm and cooperation of a number of individuals and institutions. We would like to use this opportunity and extend our gratitude to everyone who has taken part or in any way contributed to, or supported the creation and publication of this monograph.

First of all, we would like to express our gratitude to the authors of papers for their effort taken to provide expert and high level insights into some main points of Mihailo Petrović Alas' life and work, at the same time preserving an important aspect of being easy to read and appealing to a broader readership. In addition, we would like to thank to Ms. Snežana Krstić-Bukarica and Ms. Nevena Đurđević from SASA Publishing Section for performing a thorough proofread of the papers, thus making the writing even more articulate.

The monograph features a number of photographs and the copies of documents that have been obtained owing to the kindness of the SASA Archive, SASA Library, SASA Mathematical Institute, Archive of Serbia, Mr. Viktor Lazić from the "Adligat" Society, Mr. Jovan Hans Ivanović and his "Mihailo Petrović Alas" Foundation, "Mihailo Petrović Alas" Primary School, "Svetozar Marković" University Library, Belgrade City Museum, Zavod za udžbenike (Institute for Textbook Publishing) in Belgrade, Virtual Library of Faculty of Mathematics in Belgrade and Digital Legacy of Mihailo Petrović Alas.

The publication of the monograph was financially supported by JP Srbijagas, the Ministry of Education, Science and Technological Development, primarily through scientific projects in which the majority of the authors of the papers takes part, and Telekom Srbija. We would like to express our deep gratitude for their support.

Finally, we would like to express our gratitude to Mr. Mirko Milićević from the publishing house "Dosije Studio" for excellent prepress preparation of the monograph.

S. Pilipović, G. Milovanović, Ž. Mijajlović

CONTENTS

EDITOR'S FOREWORD

As soon as one first encounters the work of Mihailo Petrović, it becomes evident that he was a person that according to its numerous traits was a polymath. Above all, the academician Petrović was a gifted mathematician and a renowned professor at the University of Belgrade, but also a fisherman, writer, philosopher, musician, world traveler and a travel writer. He earned a degree in mathematics at the Belgrade Grand School and a licentiate degree in mathematics, physics and chemistry at the Sorbonne. At the age of 26, only a year after he had completed his studies, he defended his PhD degree in mathematics at the same university, as a student of the famous French mathematicians Henri Poincaré, Charles Hermite and Charles Émile Picard. In the same year (1894) he was elected to the position of professor at the Grand School to which he brought the spirit of the French mathematical school. It was at that point that his long and prolific journey through science began, whereas, owing to him, Belgrade achieved parity with other major European centers in mathematical sciences. He became an initiator and a leader of the Serbian mathematics and strongly contributed to the spirit of the modern European science in Serbia.

Petrović's expertize spanned several mathematical areas in which he achieved scientific results of world-class relevance: differential equations, numerical analysis, theory of functions of a complex variable and geometry of polynomials. He was also interested in natural sciences, chemistry, physics and biology, and he published scientific papers in these fields, too. In his scientific endeavor he managed to meet the most rigorous standards of the most developed European countries. In a brilliant rise, in a few years' time, up to the early 20th century, he wrote around thirty papers that he published in the leading European mathematical journals. It was due to this fact that he was elected a member of the Serbian Royal Academy as early as at the age of 30, and soon after he became a member of a number of foreign academies and prominent expert societies. He won the greatest respect of the global mathematical community: he was among few mathematicians (13) who delivered at least five plenary lectures or lectures as a visiting lecturer at the International Congress of Mathematicians (ICM). He delivered five such lectures (1908, 1912, 1924, 1928 and 1932). One such invitation has been considered by the mathematical community as an equivalent of an induction to a hall of fame. In addition, it has been considered that Petrović was a founder of new scientific disciplines, namely mathematical phenomenology and spectral theory. He invented several analogue computing machines, possessed technical patents and was the main cryptographer of the Serbian and Yugoslav Army.

Up to the Second World War he was the mentor of all doctoral thesis in mathematics defended at the University of Belgrade. Aforementioned is related to one of professor Petrović's greatest and most important achievements – he was a founder of the Serbian mathematical school that has produced a great number of renowned and successful mathematicians not only in Serbia but also around the world.

In 2018, the Serbian Academy of Sciences and Arts and mathematicians in Serbia celebrate the 150th anniversary of the birth of Mihailo Petrović Alas. Throughout this year, the Academy has organized a large exhibition dedicated to Petrović, alongside a solemn gathering and a conference. This monograph commemorates this important jubilee of the Serbian mathematics. Given the fact that a lot of articles on Petrović have already been written, and that his collected works were published at the end of the last century, the editors and authors of the papers in this monograph were faced with a daunting task of finding some new details from professor Petrović's life and career. Even more so given that his body of work is immense, spanning different scientific areas and encompassing topics that at first glance one finds difficult to combine. As Dragan Trifunović, Petrović's biographer and a man who most thoroughly studied his life and work, noted on one occasion that almost an institute was necessary that would encompass professor's entire body of work. Therefore, we set a relatively modest goal to ourselves to shed light upon some main points of Petrović's life and work, times and circumstances he lived in, as well as to elaborate on the present developments in relation to the Serbian mathematical school, through a selection of papers. The authors of the papers steered clear of technical details and excessive use of mathematical language. Hence, the monograph is intended for a broader readership, in particular to those readers who are interested in the history of Serbian science and its evolvement at the turn of the 20th century, but also to those who want to gain a deeper insight into the life of a brilliant mathematician and a polymath, and, we can quite freely say, an unusual personality.

Ž. Mijajlović, S. Pilipović, G. Milovanović

MIHAILO PETROVIĆ ALAS: LIFE AND WORK

MIHAILO PETROVIĆ ALAS AND HIS AGE

Žarko MIJAJLOVIĆ
University of Belgrade, Faculty of Mathematics

Mihailo Petrović's appearance in Serbian science and culture coincided with quite specific circumstances in Serbia and Europe. The Obrenović and Karađorđević dynasties alternately ruled Serbia. In the last four decades of the 19th century, the Obrenovićs ruled Serbia until the coup d'état in 1903, when the Karađorđevićs assumed power. Not so long before, in 1878, after the wars with Turkey and the Congress of Berlin, Serbia gained full independence and defined its borders. Until then it was considered a part of the Ottoman Empire. The bulk of the Serbian people lived in the diaspora, while science and culture were only emerging. Funds that the Serbian government could spend on science were very modest. Still, Petrović's biography suggests that these events did not stand in the way of his education and science. This can be ascribed to several facts. Petrović was born into a family that cherished the tradition of solid education of its younger generations. Besides, the family was wealthy enough to enable such education. Moreover, as the Serbian authorities recognised the importance of education, they constantly invested in it and enhanced the educational system in accordance with European standards. Gymnasiums and the Great School provided their pupils and students with knowledge that enabled them to continue their education and defend their doctoral theses at European universities.

In the 19th century, particularly its second half, science gained momentum, heralding its present-day development. The foundations of contemporary mathematics were laid. Universally recognised as the most influential mathematicians of the time, Henri Poincaré and David Hilbert introduced new mathematical concepts and a new style of abstract mathematical thinking. New mathematical theories with multiple applications in technical engineering and physics were emerging, and old theories were receiving novel grounding. It is not possible to elaborate on this in merely a few words as this deserves separate examination. Still, let us give at least two examples. Somewhat earlier, with his ε-δ definition of limit, Karl Weierstrass formally defined the analysis, while his student Georg Cantor created the set theory, establishing the framework and universal language of contemporary mathematics. Mihailo Petrović, Poincaré's direct student, began his academic career equipped with such knowledge and the understanding of science and culture that existed in Europe at the time. As a young man, he was already a well-formed mathematician and highly prolific academically. Fond of analysis, he was well familiar with the works of mathematicians of the French school in this area, examining the most topical issues in his works on differential equations and the theory of functions.

Mihailo Petrović was born on 23 April 1868 (Julian calendar) in Belgrade to a reputable family, of mother Milica and father Nikodim. Nikodim Petrović (1843–1875) earned a doctorate in theology in Kiev and taught theology at the Belgrade Faculty of Theology. As Nikodim died young, Mihailo barely remembered his father. Mihailo's maternal grandfather, parish priest Novica Lazarević (1821–1902) took care about his grandson and his education[1]. Mihailo was very close to this grandfather, until his death, as indicated by the intensive correspondence between the two of them, which has been preserved in Petrović's legacy[2]. Mihailo completed the First Belgrade Gymnasium in 1878–85. His school peers were Milorad Mitrović, Jovan Cvijić, Pavle Popović and others, who later became important figures of Serbian culture and science. He already then showed interest in mathematics, winning awards for his term papers and attracting his professors' attention with his talent. He enrolled in the Scientific-Mathematical Department of the Belgrade Faculty of Philosophy. He graduated in 1889 and soon went to Paris for specialisation and further studies of mathematics.

Petrović was gifted at mathematics and other sciences, writing his first mathematical work already as a first-year student[3]. He gained solid knowledge during his studies of mathematics at the Belgrade Faculty of Philosophy from professor Dimitrije Nešić. After arriving in Paris, he underwent one-year preparations for the entrance exam for the prestigious L'Ecole Normale Supérieure. Other candidates often prepared for this exam for several years, which attests to Petrović's success and talent. Petrović passed the exam with distinction, gaining the privilege to study at Collège de Sorbonne, the best European school of mathematics at the time. Petrović availed of this exceptional circumstance and gained extraordinary mathematical education. At Sorbonne, he first graduated from chemical sciences in 1891, from mathematics in 1892 and then from physics in 1893. As the best student in his generation, he was received by the President of


Advanced Pedagogical College, (l'École normale supérieure), Paris, 1885

the French Republic both in 1893 and 1894. He enrolled in doctoral studies at the same university. He received a Serbian scholarship, but was obliged to complete the studies in 1895. In 1894, one year ahead of schedule, he defended his doctoral thesis in the field of differential equations[4] and was commended by the examination committee, which consisted of reputable professors and leading French mathematicians of the time Charles Hermite, Émile Charles Picard and Paul Painlevé. According to the Mathematics Genealogy Project[5], the former two were co-mentors of the dissertation, though Petrović's results were close to Painlevé's area of expertise. Mihailo's grandfather Novica financed the first two years of his stay in Paris. The Ministry of Education of Serbia later recognised the talent and success of this Serbian student and granted him a state scholarship. Serbian envoy to France Milutin Garašanin attended the defence of his doctorate.

When Petrović returned from Paris to Belgrade in 1894, his erstwhile professor Dimitrije Nešić retired. Petrović applied for the vacancy of a professor at the Great School, together with Petar Vukićević, his somewhat older colleague from the Belgrade Faculty of Philosophy. Vukićević earned his doctoral degree in Vienna in 1894, also in the field of differential equations and was a good mathematician. Immediately after the defence, the results from his thesis were cited in the famous Schlesinger's monograph about differential equations[6]. Petrović was chosen for the post, after getting one vote more than Vukićević. Vukićević subsequently became a gymnasium professor and, probably under the influence of his defeat at the competition, no longer dealt with science or pursued an academic career. At the time, the Great School consistently applied the *numerus clausus* principle, limiting the number of teaching posts. Regarding his election for the post, Petrović once said: “Had I not obtained that additional vote at the competition for a Great School professor, I would have never dealt with mathematics. I would have lived on Serbian rivers, not on a boat, but in a dinghy”.

At the time, there were several mathematicians in Serbia who engaged in scientific work: Ljubomir Klerić, Dimitrije Nešić, Petar Živković, Dimitrije Danić, Mijalko Ćirić and Bogdan Gavrilović. Most of them attended German and Austro-Hungarian schools and all of them, apart from Petar Živković, lived in Belgrade. As a state scholar, Klerić completed high schools in Germany and Switzerland, becoming a mining engineer. Upon returning to Serbia, in addition to his mining jobs, he taught mechanics at the Great School. He was elected a member of the Serbian Learned Society and the Academy upon its foundation, and held several ministerial posts in the Serbian Government. He remained remembered by the construction of the tractoriograph, second-line curve drawing device, mechanical construction of transcendental numbers π and *e*, and other measurement mechanical devices, some of which were used for military purposes. In Serbia, Klerić was Mihailo Petrović's precursor in the construction of computing and other mechanical devices.

Dimitrije Nešić (1836–1904), Petrović's professor

Dimitrije Nešić studied at the Lyceum and high technical schools in Vienna and Karlsruhe. He was a professor at the Great School from its foundation in 1863 until his retirement in 1894. He significantly improved the teaching of mathematics. He was publishing in the Academy's *Glas (Voice)* and wrote a voluminous textbook *Algebraic analysis* on advanced mathematics. He was a rector of the Great School and president of the Academy. Živković first taught in a gymnasium in Belgrade, and as of 1889 in Užice. He wrote around twenty papers, most of them published in the Academy's *Glas*, and was elected a corresponding member of the Academy. As far as we know, Živković was the only member in the Academy's history who spent his entire career as a gymnasium professor. Danić, the first Serbian doctor of mathematics, was a professor at the Military Academy and author of solid university textbooks. Gavrilović, educated in Novi Sad, completed mathematical sciences in Pest, where he defended his doctoral thesis in 1887. He came to Belgrade in the same year, where he was elected a mathematics professor at the Great School. He spent some time in Germany, where he attended Weierstrass's lectures. He stayed in Belgrade over the course of his entire academic career, as

Dimitrije Danić (1862–1932), first Serbian doctor of mathematics

Bogdan Gavrilović (1864–1947), friend and colleague of Mihailo Petrović

Charles Hermite (1822–1901)

Mihailo Petrović's friend and closest colleague. His merits for the establishment of Belgrade University are significant. He authored excellent university textbooks on linear algebra and analytic geometry, which Radivoj Kašanin assessed as follows: "Both of these textbooks, particularly the latter, would do the honour to any nation; many nations, larger and happier than us at the time, did not have such works". Klerić and Nešić were professors at the Great School and members of the Academy, while Gavrilović became that somewhat later[7]. Although he attended Hermite's lectures in Paris just like Petrović, Mijalko Ćirić, a professor at the Great School, did not leave a visible trace in Serbian science. From this group, Nešić, Danić, Živković and Gavrilović were pure mathematicians, judging by their published papers and academic focus, while Klerić and Ćirić dealt with applied mathematics, as mechanics was often called at the time. Two doctors of mathematics, Đorđe Petković and Petar Vukićević also lived in Belgrade at approximately the same time. They also spent their careers as gymnasium professors.

There were also several other distinguished intellectuals in Belgrade who specialised in sciences akin to mathematics. For instance, Stevan Bošković, a general of the Serbian Army and full member of the Academy, is the most renowned Serbian geodesist, topographer and cartographer. He was educated at high military schools in St Petersburg and the famous Pulkovo Observatory. He carried out the first geodetic survey of Serbia, applying state-of-the-art numerical, mathematical and astronomic methods. This is also attested by the books of his professor in Russia Nicholai Zinger, a well-known Russian geodesist and astronomer from the second half of the 19th century, which he translated. Kosta Stojanović was a professor of theoretical mechanics and the first Serbian author of an advanced book on mathematical economics. This book is probably one of the most important and best works on economic sciences written by a Serbian author, which still attracts interest. Finally, let us also mention Milan Andonović, an honourable member of the Serbian Royal Academy, a geodesist, engineer and author of astronomy books. Andonović brought to Serbia the first knowledge in the field of statistics and Gaussian law of error.[8]

In the late 19th century, though Belgrade barely had 70,000 inhabitants, it had a pleiad of illustrious scientists, such as Jovan Cvijić, Sima Lozanić, Stojan Novaković and others. Given the size of Belgrade and the number of its inhabitants, we may conclude that at the time it had a significant number of learned men and a good school producing new generations of educated young people. New technological inventions began to apply in Belgrade. Electric lighting was introduced in 1893 and the first electric tram in 1894. Given all this, we may say that the time of Petrović's return to Serbia overlapped with the zeitgeist of new-century Europe which was already felt in Belgrade. However, in other Serbian towns, science and education were reduced, in the best case, to what was taught in gymnasiums, and real science and high education in Serbia were concentrated in the capital. This continued into the second half of the last century, when the universities in Skopje (1949), Novi Sad (1960), Niš (1965), Priština (1970), Podgorica (Titograd, 1974) and Kragujevac (1976) sprang up from Belgrade University. The first studies of mathematics outside Belgrade were established in Novi Sad in 1954 at the Mathematics Department of the Faculty of Philosophy.

In the late 19th century, the bulk of the Serbian people were illiterate and the government, through scholarships and by sending students to study abroad, was more inclined to supporting and developing practical sciences such as construction, mining, legal and technical sciences, rather than fundamental sciences. The aim was to strengthen the young state in economic and military terms as soon as possible. Regardless of this, in his scientific work Mihailo Petrović met the highest standards of the most advanced European states. During his brilliant academic rise, within four years only, i.e. until the start of the 20th century, he wrote around thirty papers and published them in leading European mathematical journals. This success brought to Petrović high reputation, as well as widespread recognition. Already in 1897, about to turn thirty, Petrović was elected a corresponding member of the Serbian Royal Academy and in 1899 he became its full member. At the start of the new century, Serbia got its king of mathematics. Petrović's name and mathematical results soon surpassed the borders of Serbia, and later of Yugoslavia, after its creation. He was elected an honorary member of several foreign academies – in Bucharest, Prague, Warsaw and Krakow. He was elected a corresponding member of the Yugoslav Academy of Sciences in Zagreb, and became a member of numerous European scientific societies.

In terms of his scientific work, Mihailo Petrović belongs to a specific time. In the late 19th century, mathematics became a complex and high edifice with many floors. New mathematical disciplines emerged, with others practically dying away. In the 17th and 18th centuries, natural sciences, particularly mechanics and astronomy, were in full swing, directly impacting the development of mathematics for their own purposes. The scientists of the time dealt with their own sciences and mathematics to an almost equal extent. They introduced new mathematical concepts and developed methods with the foremost aim to describe and resolve topical problems in their sciences. In the second half of the 19th century, this need was not so acute as the mathematical apparatus was sufficiently developed to satisfy the majority of requirements of other sciences. Moreover, due to the abundance of mathematical knowledge, it was difficult, if not impossible, for an individual to be well familiar with the entire field of mathematics. Specialisation also gained momentum as a toll of scientific productivity. The time of universal mathematicians and scientists was practically gone. Given the scope of his scientific work in the fields of mathematics, mechanics and philosophy, Henri Poincaré was certainly one of the last *homo universalis* of science. And it was Poincaré who was one of Mihailo Petrović's professors. Petrović, as a young student from Serbia, attended Poincaré's lectures and passed two exams, one which was mathematical physics. Judging by Petrović's later mathematical work, we can conclude that he was imbued with the spirit of universalism of his professor. He was equally familiar with and achieved first-class results in several mathematical fields: differential equations, numerical analysis, theory of functions of a complex variable and geometry of polynomials. He was also interested in natural sciences – chemistry, physics and biology, and he published papers in these fields. In addition, Petrović is considered to have founded new scientific disciplines – mathematical phenomenology and the spectral theory.

In hindsight, the influence of Mihailo Petrović on the development of mathematics in Serbia was enormous. He was the *spiritus movens* of Serbian mathematics and gave a strong contribution to the spirit of contemporary European science in Serbia. Moreover, he would gather and motivate people. This was not only the opinion of the Serbian mathematical public, but world reference publications testify to it as well. For instance, a third of the article *The Balkan Trilogy: Mathematics in the Balkans before World War I* by Snezana Lawrence and six pages in *The Oxford Handbook of the History of Mathematics*[9] are dedicated to the biography of Mihailo Petrović and his contributions to mathematics. In the former article, he is by far most widely cited author, with eleven works. In the article, Petrović is described as the greatest and most famous Serbian mathematician. It is stated that Petrović, as the most renowned Serbian mathematician of the time, defined the directions of development of the Serbian mathematical school on the foundations of French mathematics, although all Serbian doctors of mathematics of the time, apart from Petrović, were educated at Austro-Hungarian and German universities[10].

Henri Poincaré, around 1910

Milutin Milanković, around 1928
(Photo Archive of LSASA, F 240)

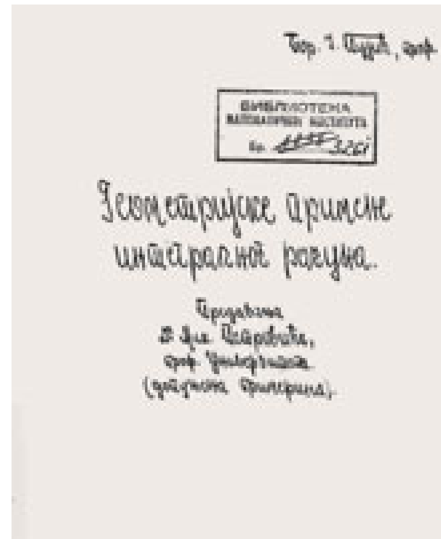
During his scientific career, Petrović published around 400 papers, of which around 300 on mathematics. He published 12 books. There are 14 casebooks from his lectures, prepared by his students or himself. As a member of the Serbian Royal Academy, the highest Serbian scientific institution, Petrović was highly active in its Department for Natural Sciences and Mathematics. Together with his friends and colleagues Bogdan Gavrilović and Milutin Milanković, he contributed to its high reputation. For instance, together with Gavrilović, Petrović wrote the Statute of the National Committee of Mathematicians for the Kingdom of the Serbs, Croats and Slovenes[11], which was adopted by the Academy's Presidency.

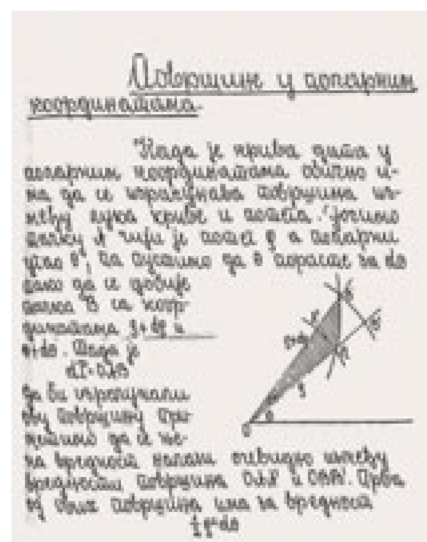
Mihailo Petrović's academic career was related primarily to the Great School, until it grew into University in 1905, and to Belgrade University, until the end of his working age. As he himself said once, as a pupil, student and professor he spent total 55 years in Captain Miša's edifice, where his gymnasium and the Great School were located. We shall therefore present several basic facts about these leading and, in addition to the Military Academy, the only higher educational institutions in Serbia at the turn of the 20th century. We shall also give a separate overview of the Mathematics Department of the Faculty of Philosophy, given that the Department was the main place of Petrović's scientific and pedagogical work.

The Great School, pursuant to the Law on its organisation from 1863 was "a scientific institution for high and professional education"[12]. After the reform of the Lyceum in the same year, it was divided into the Faculties of Philosophy, Technical Sciences and Law. The School was located in the building known today as Captain Miša's edifice. It was a large building at the time, bestowed upon the Serbian people by captain Miša Anastasijević for the needs of the Great School. In 1894, when Petrović was elected a professor at the Faculty of Philosophy, there were two departments: Historical-Philosophical and Natural Sciences – Mathematics. Until 1873, mathematics and natural sciences were taught at the Faculty of Technical Sciences only. That year, the studies of mathematics were introduced into the Faculty of Philosophy, and the Mathematics Department was set up at the same time. Education lasted for three years. The first student, Mihailo Banić, graduated from the Mathematics Department already in 1875. From the establishment of the Department until the outbreak of World War I in 1914, only 35 students graduated from mathematics[13]. In addition to Petrović, there were also other notable persons in the fields of science, culture and politics. Petar Vukićević, one of six Serbian doctors of mathematics in the 19th century and Petrović's opponent at the competition for a professor at the Great School, graduated in 1886. For instance, it is little known that Stanislav Binički, a renowned Serbian conductor, composer and music pedagogue, graduated from mathematics at the Great School in 1894. It was only after he completed these studies that Binički dedicated himself fully to music. Mladen Berić, the first doctoral student whom Mihailo Petrović tutored and the first mathematician who defended his doctoral thesis at Belgrade University, graduated in 1909. Sima Marković, the second doctoral student under Petrović's mentorship and, together with Filip Filipović, the first secretary of the Communist Party of Yugoslavia, graduated in 1911. Let us also mention Borivoj Pujić, a student who graduated in 1914. In the 1960s, Pujić, or someone of his successors, granted to the Mathematical Institute of the Serbian Academy of Sciences and Arts 24 manuscripts – lectures of mathematics professors at the Great School and Belgrade University Dimitrije Danić, Kosta Stojanović, Mihailo Petrović and Milutin Milanković. Of these, 14 manuscripts are Mihailo Petrović's lectures. The majority of these manuscripts were written by Borivoj Pujić in a nice handwriting. These books are an important testimony to the teaching of mathematical sciences at higher educational institutions in Belgrade at the turn of the 20th century[14].

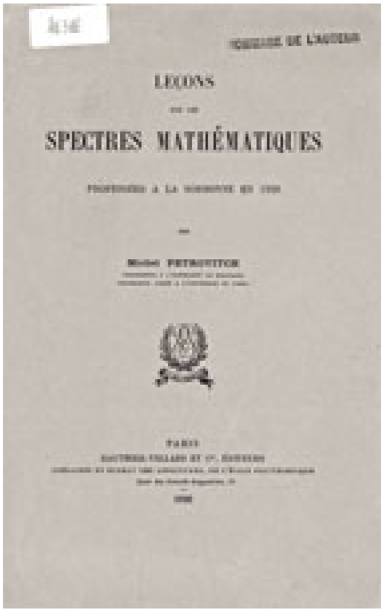
Before Petrović was elected a professor, Dimitrije Nešić was the only professor at the Mathematics Department until the arrival of Bogdan Gavrilović in 1887. After Gavrilović's arrival, mathematics was taught at two departments of the Great School – higher and lower mathematics. Dimitrije Nešić led the Higher Mathematics Department, and Bogdan Gavrilović took the Lower Mathematics Department. At the time, teachers could have the title of professors, junior lecturers and lecturers. There was also the title of an honorary professor, who could be appointed by the minister independently of the Great School Council.

During his professorship, Petrović taught various subjects. In one period, from the establishment of Belgrade University in 1905 until the appointment of Milutin Milanković as a professor of applied mathematics in 1909, Petrović was the only mathematics professor at the Faculty of Philosophy. It would happen that in a single academic year he practically taught all subjects, as seen in the number of preserved manuscripts, i.e. notes from his lectures from Pujić's





Notes from professor Petrović's lectures, taken by student Borivoje Pujić 1910–1914 (Mathematical Institute of SASA)



Lectures from theory of specters
at Sorbonne, 1928
(SASA Library, 46316)

collection. Those were subjects in linear algebra, analytic geometry in plane and space, differential calculus and its application, primarily in solving geometry tasks, including ordinary and partial equations, function theory[15] and algebraic equations. The last manuscript, standing out from others in terms its interesting content, was the basis for the textbook *Theory of Algebraic Equations,* co-authored and published by Mihailo Petrović and Nikola Saltikov in 1927. As this book contained a great number of solved problems on polynomials, mostly of the 3rd and 4th degrees, students and advanced secondary school pupils used it for a long time, even after World War II. By inspecting the content of these manuscripts, we can conclude that the structure and distribution of topics were to an extent similar to what was taught at the first courses on linear algebra, analysis and differential equations at the Faculty of Science and Mathematics, until the first several decades after World War II. This is entirely understandable given that professors at the time were either Petrović's doctoral students[16] or students of the first generation of Petrović's successors[17]. Judging by the manuscripts, Petrović's lectures were of algorithmic type, i.e. were not structured in the form of strict definitions, theorems and proofs, but had a continuous flow, with many examples and gradual introduction of concepts and procedures for solving concrete mathematical problems. Manuscripts were not hard to read and it seems they contained exactly the material that students had to learn. At the time, already available were the excellent books *Theory of Determinants* and *Analytic Geometry*, which were written by Bogdan Gavrilović[18] in the late 19th century and are interesting even for the contemporary reader. These textbooks could also fit into the courses held by Petrović, but were too broad and ambitious for an average student. For instance, the second book, although concerning only plane analytic geometry, had over 900 pages. This is why professor Petrović held lectures by his choice, with the aim to have students master the main techniques of higher mathematics and to successfully pass the exam. He published three university textbooks: *Computing with Number Intervals*, 1932; *Elliptic Functions*, 1937 and *Integration of Differential Equations by Use of Series*, 1938. He also published the textbook *Leçons sur les spectres mathématiques*, Paris, 1928, according to which he held lectures at Sorbonne in Paris in 1927–1928. Of these, particularly important is the book *Elliptic Functions*, which is a monograph-type publication and can be interesting for the contemporary reader as well.

Petrović lectured for 44 years at the Great School, and later at Belgrade University, together with Bogdan Gavrilović, another great name of Serbian mathematics, until his retirement in 1938. In 1894, the same year when Petrović returned from France to his homeland, mathematics for students at the Faculty of Technical Sciences became a separate course, led by Bogdan Gavrilović. Mihailo Petrović stayed at the Faculty of Philosophy. Over 15 years, the two of them were the only professors of pure mathematics at the Great School, i.e. Belgrade University, from 1894 until 1909 when Milutin Milanković became a professor of applied mathematics.

As regards Petrović's work at the University, with the help of his friend and somewhat older colleague Bogdan Gavrilović, Petrović raised Serbian mathematics to the European level. In his necrology devoted to Gavrilović, Milutin Milanković said that the two of them laid the foundations of Serbian mathematics. Petrović did so in scientific and Gavrilović in organisational terms, by significantly contributing to the transformation of the Great School into Belgrade University. The work of these two scientists, who can be considered the creators of contemporary Serbian mathematics, deserves particular attention and analysis. Without carrying out an in-depth analysis, we shall point out several details. As Poincaré's student, Petrović was an eminent representative of the French school of mathematics from the late 19th century, while Gavrilović, as Weierstrass's student, was under the main influence of German and British mathematicians who were at the time developing abstract algebra and were applying algebra in geometry. Gavrilović and Petrović were rather complementary in terms of their mathematical interests. While the focus of Petrović's work was on analytic methods, Gavrilović dealt more with linear algebra and geometry. Petrović mostly published scientific papers, while Gavrilović wrote valuable textbooks on algebra and geometry of monographic character. Unlike Petrović's, Gavrilović's works have not left a deep imprint in the Belgrade mathematical milieu, perhaps undeservedly so, let alone in the international environment as all of them were published in Serbian, although Gavrilović was a polyglot. This was certainly due to the rule that papers published in the Academy's *Glas*, in which Gavrilović was publishing, had to be in Serbian. On the other hand, more than a half of Petrović's papers were published in French, in the leading European journals. Let us emphasise once again that over entire 15 years, the two of them were the only professors of the Great School, later Belgrade University, at the height of their academic careers, owing, in fact, to the then cap on the number of university professors. In any case, both Petrović and Gavrilović, each in his own way, contributed to the development of mathematics in our country and the creation of a special atmosphere, owing to which Belgrade was transformed from a provincial town into a scientific centre.

The complementarity between Petrović and Gavrilović was not exhausted in education and science only, but was seen in their everyday lives as well. Petrović was a passionate fisherman, Gavrilović cultivated peaches. Petrović was a world traveller, while Gavrilović usually spent his free time on his estate in Grocka. Gavrilović had a family and many children, while Petrović never got married and left no direct descendants. Gavrilović was close to the Court, while Petrović was not, mainly due to his friendship with Prince Đorđe Karađorđević, who was

Petrović (violinist with a hat) leading his music troupe "Suz" at a tavern celebration (SASA Archive, 14197/II-1)

in King's disfavour. Gavrilović was the rector of Belgrade University and president of the Academy (1931–1937). Such proposals coming from the academic milieu for Petrović, in 1927 and 1931, were not accepted or approved by the authorities, which the majority of authors ascribes to King's animosity towards him. However, it should be noted that Petrović was not hampered in his scientific work or other activities. On the contrary, by inspecting the daily press and archival documents of the time, we can see that he received from the Ministry funds for his frequent travels and was highly respected as a great scientist and renowned expert both by the public and the authorities. He was engaged in important state affairs. For instance, he was the main cryptographer of the Serbian, and later the Yugoslav army, representing his homeland in international committees and delegations concerning education and fishery[19]. Petrović's unconventional life may have contributed to his not becoming the rector and president of the Academy. As the mythological deity Janus, Petrović had two faces. One was turned to mathematics, philosophy and the spiritual world, while the other looked at distant travels, fishing adventures and tavern parties. It is possible that respectable gentlemen and a part of the authorities could not imagine this other Petrović's face – the image of a rector who almost daily wades in his fisherman's boots through Danube backwaters and catches fish, and then plays the violin to entertain folks in a tavern[20]. However, those were, though high, only administrative duties, and Petrović did not complain a lot about not having them. It is possible that he himself did not want to undertake such tasks as they would have only impeded the life that he led and loved[21].

Regardless of the said differences, Petrović and Gavrilović shared the same love towards science, students and the university. They were colleagues and the cornerstone of the Mathematical Club[22] between the two world wars. They were friends, who socialised not only in the Mathematical Club, but also in taverns, and engaged in various fishing adventures. When Milutin Milanković became a professor at Belgrade University, he immediately joined the two of them, both in scientific and social terms. These three scientists were outstanding personalities and pillars of mathematical sciences in Serbia until World War II and the arrival of a new scientific generation. Although highly individualistic in their work – for instance, none of them wrote their scientific papers with a co-author or a visible associate – they were connected through their work at the University and their warm friendship. Milanković wrote about this with a lot of sympathy in a nice biographical novel *Mika Alas – Notes about the Life of Great Mathematician Mihailo Petrović*. The atmosphere they created at Belgrade University is described by Radivoj Kašanin, Petrović's doctoral student, Gavrilović's assistant and successor at the Mathematics Department of the Faculty of Technical Sciences: "In addition to their exceptional educational background and original scientific works, all three of them featured something that I value the most and consider the highest human value – love towards young generations, understanding of young people, unselfishness and sincere help to young, talented people in their advancement. They knew how to rejoice and enjoy when young people advanced. *I was lucky to develop myself and work next to them – those great authorities of science and morality. To take pride in their friendship. I do not believe there was anywhere such an environment as was created by Gavrilović, Petrović and Milanković*".

Already in 1894, the library of the Mathematical Seminar was set up, offering to generations of mathematicians of Belgrade University broad possibilities for scientific work. Until World War I, Bogdan Gavrilović and Mihailo Petrović were in charge of the library, only to be later followed by other mathematicians. The library had a relatively rich collection of books, sets of journals, monographs and other mathematical literature. When the Mathematics Department moved in 1938 to the new building constructed next to the old building of the Faculty of Philosophy in Captain Miša's edifice, the library was moved there as well. Unfortunately, only two days before the liberation of Belgrade, on 18 October 1944, the enemy army, during their retreat, set fire to the library and destroyed it. Only a few books borrowed by individuals survived out of the entire library. As the first book of the inventory until 1907 was preserved, we found out that Bogdan Gavrilović entered by hand books until number 110, and Mihailo Petrović from this number until 301.

The story about the life path of Mihailo Petrović is inseparable from the development of Belgrade University and mathematical sciences in Serbia. In the late 19th and particularly early 20th century, higher education in Serbia was rapidly developing. In 1896, the Great School obtained some autonomy, and the mathematics curriculum came close to European standards.

The first eight professors of the University of Belgrade in 1905. Sitting from the left: Jovan Žujović, Sima Lozanić, Jovan Cvijić and Mihailo Petrović. Standing from the left: Andra Stevanović, Dragoljub Pavlović, Milić Radovanović and Ljubomir Jovanović. (SASA Archive, 14197/II-18)

The regulations of 1900 raised further the level of teaching and scientific work. Faculties became independent entities of the Great School and obtained the organisational framework preserved after World War II. In 1900, the Seminar for Mathematics, Mechanics and Theoretical Physics was created, involving the professors of mathematics, mechanics and astronomy from the Faculties of Philosophy and Technical Sciences. Mihailo Petrović and Bogdan Gavrilović played the main role in the work of the Seminar. After years-long preparations and delays, in 1905 the Great School was transformed into the university as "the highest self-managing body for higher professional education and pursuit of science". The university consisted of four faculties – of philosophy, law, technical sciences and theology. Eight full professors were appointed at the Faculty of Philosophy: Jovan Žujović, Sima Lozanić, Jovan Cvijić, Mihailo Petrović Alas, Andra Stevanović, Dragoljub Pavlović, Milić Radovanović and Ljubomir Jovanović. On their proposal, several days after the appointment, the Ministry of Education issued a decree on the appointment of professors of the Faculty of Technical Sciences. Bogdan Gavrilović, Mihailo's colleague and friend, was among them. He was immediately elected a full professor of the Faculty of Technical Sciences[23]. Petrović and Gavrilović thus gained the leading role in the organisation of scientific work and mathematics courses at the newly established university.

Until 1909, lectures in theoretical mathematics at the Faculty of Philosophy were held by Mihailo Petrović and occasionally by Bogdan Gavrilović, as an honorary professor. That year, on their proposal, Belgrade University invited from Vienna Milutin Milanković to teach applied mathematics. This voluminous subject, taught by Milanković for years, was in fact mainly a course in mechanics and astronomy. At the time, in 1908/9, Petrović was the dean of the Faculty of Philosophy.

In the meantime, science at Belgrade University advanced to such an extent that the first doctorate in mathematical sciences was defended, under the mentorship of professor Petrović. This doctorate in the field of differential equations was defended in 1912 by Mladen Berić, a junior lecturer at the First Belgrade Gymnasium and assistant to professor Petrović. Already the following year, Sima Marković defended his doctoral dissertation on the Riccati differential equation, also under Petrović's tutorship. Those were the beginnings of the Belgrade School of Mathematics. Unfortunately, the work of Belgrade University was often halted due to wars. In the academic 1912/13 year, the University did not work due to the Balkan Wars. During the academic 1913/14 year, the University was re-opened, but World War I suddenly interrupted the work and further development of this institution. Its students and professors went to war. In August 1914, just after the war started, a part of Captain Miša's edifice was destroyed during the bombing. The enemy troops plundered the deserted and destroyed building. Mihailo Petrović took part in the war as a reserve officer. At the start of the war, in 1914, he led a group of soldiers at Skeljanska ada on the Sava river and captured around thirty Hungarian soldiers. Skeljanska ada is located opposite Obedska bara and the Kupinovo village, with the remains of the old royal town of Kupinik, the seat of despots Stefan Lazarević and Đurađ Branković.

Immediately after the war, in 1919, the University began to work, although in difficult circumstances. The curricula and university regulations were adjusted to the needs to restore and develop the country as several generations of students and experts perished, and the University cadre had already been decimated in earlier wars. In the early 1920s, the University experienced accelerated development within a short period – the number of professors increased. Among others, mathematicians Nikola Saltikov and Anton Bilimović came from Russia, significantly reinforcing the teaching cadre of the Mathematics Department of the Faculty of Philosophy. Although Mladen Berić and Sima Marković were appointed lecturers at the Mathematics Department, already in the early 1920s they left the University. Berić was forced to do so due to a personal adverse situation, while in the case of Marković, the Ministry of Education did not confirm his election for an associate professor due to his political engagement as a communist. Although professor Petrović laid a lot of hope in his first students, he was not particularly lucky with them.

In the mid-1920s, a new generation of mathematicians matured: Tadija Pejović, Radivoj Kašanin, Jovan Karamata and Miloš Radojčić. They were all Mihailo Petrović's graduate and

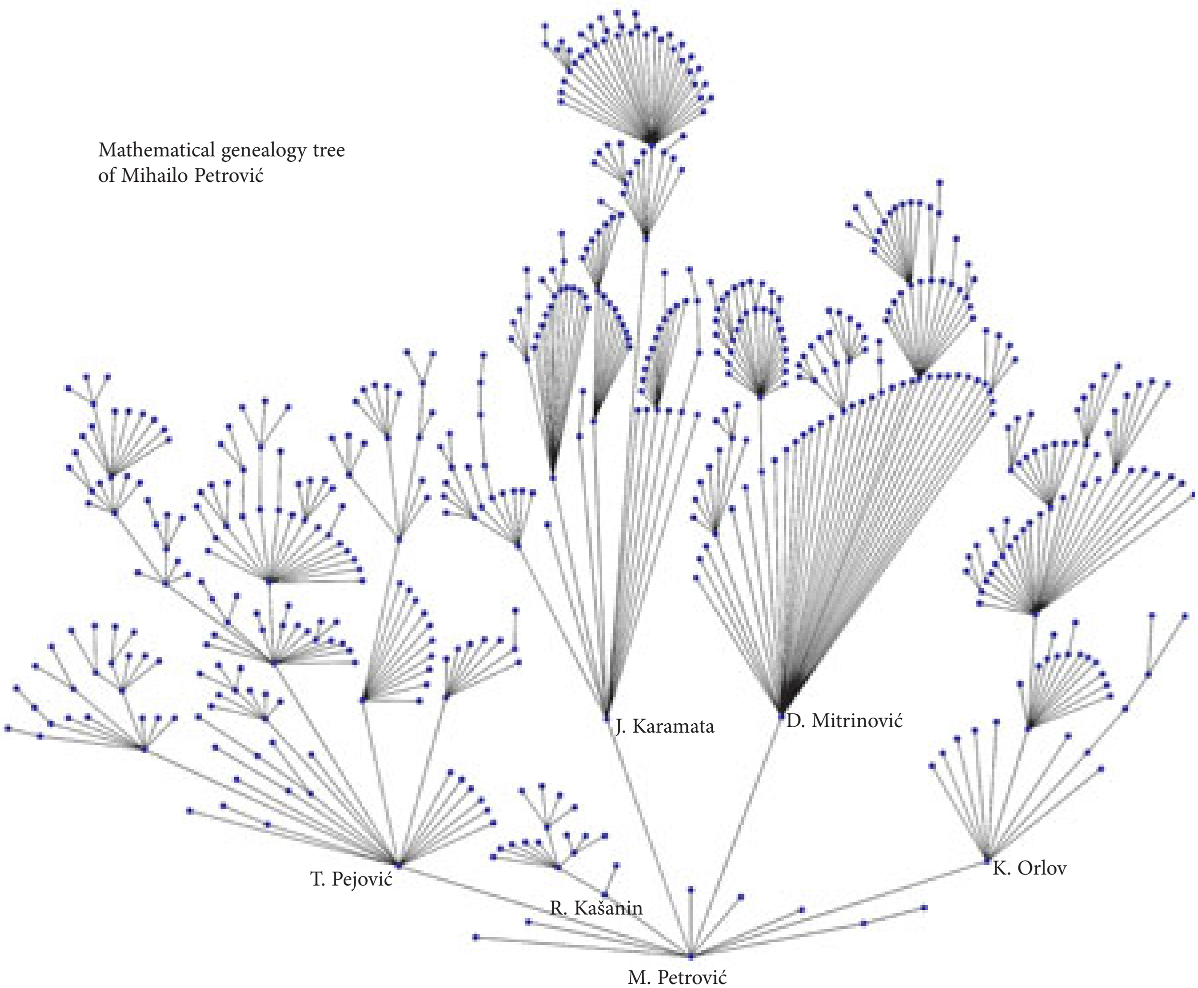
Historical shot: Belgrade school of mathematics, 1926. Miloš Radojčić, Tadija Pejović, Vjačeslav Žardecki, Anton Bilimović, Petar Zajončkovski, (Jelenko Mihailović, seismologist) Radivoj Kašanin, Jovan Karamata (standing). Nikola Saltikov, Mihailo Petrović, (Pavle Popović, rector), Bogdan Gavrilović, (K. Petković, dean of Faculty of Philosophy), Milutin Milanković (sitting)
(SASA Archive, 14197/II-16)

doctoral students. In the 1930s, Dragoslav Mitrinović, Danilo Mihnjević, Konstantin Orlov, Petar Muzen and Dragoljub Marković also defended their doctoral theses under the tutorship of professor Petrović[24]. We shall give the names of all mathematicians at Belgrade University in 1926. The Department of Theoretical Mathematics of the Faculty of Philosophy consisted of: full professors Mihailo Petrović and Nikola Saltikov, lecturer Tadija Pejović and administrative assistants Jovan Karamata and Miloš Radojčić. The Department of Applied Mathematics consisted of: full professors Milutin Milanković and Anton Bilimović, associate professor Vojislav Mišković and lecturer Vjačeslav Žardecki. The Mathematics Department of the Faculty of Technical Sciences consisted of: full professors Bogdan Gavrilović and Petar Zajončkovski, lecturer Radivoj Kašanin; and the Department of Applied Mathematics consisted of: Ivan Arnovljević and Jakov Hlitčijev. All professors and assistants of theoretical and applied mathematics from the University made up the Club of Mathematicians of Belgrade University. This seminar was in fact the mathematics school of Belgrade University and the main point of gathering of Belgrade mathematicians. It is possible to say that this was the golden age of Serbian mathematics. The Club did not have any special rules, except for monthly meetings, when works of Club members were presented and academic discussions held.

Expedition of Belgrade astronomers and mathematicians to Fruška Gora, with the aim of determining a place for building the new observatory. From the left: R. Kašanin, J. Mihailović, M. Petrović, P. Popović, A. Bilimović, M. Milanković, V. V. Mišković, G. Gračanin and the guide (SASA Archive, 14188/7-2)

The mathematical successors of professor Petrović, connected through tutorship in the preparation of doctoral dissertations, make up the mathematical genealogical tree consisting of around 800 nodes – mathematicians[25]. Of this number, somewhat over 500 are Serbian mathematicians and the rest are foreign. Successors are placed in eight circles – generations, the several last of which include the majority of contemporary Serbian mathematicians. Looking at Petrović's mathematical tree, we can see four large clusters, with Tadija Pejović, Jovan Karamata, Dragoslav Mitrinović and Konstantin Orlov representing their roots. Each of them introduced new areas into Serbian mathematics or set up their own schools of mathematics. The successors of professor Pejović usually dealt or deal with logics or algebra, although Pejović's main focus were differential equations. The mathematicians gathered around professor Karamata usually engaged or engage in analysis. The students of professor Mitrinović work in discrete and numerical mathematics, while the mathematical successors of professor Orlov deal with differential equations and numerical analysis. These professors or their direct students created new mathematical centres in Serbia. Professor Bogoljub Stanković opened his own mathematical school in Novi Sad, while professor Mitrinović developed scientific work in the field of mathematics in Niš and at the Belgrade Faculty of Electrical Engineering. Of course, these divisions are rather rough, but give some sort of picture about mathematics in Serbia today.

Mathematical genealogy tree
of Mihailo Petrović



Members of the Belgrade mathematical seminar were publishing their works mainly in the Academy's *Glas* and Yugoslav Academy of Sciences' *Rad (Work)* in Zagreb. Unfortunately, the Academy's rule was to publish papers in Serbian, which is why they remained unknown to the wider global public. There was an acute need for a new journal where papers would be published in world languages. Owing to the endowment of Luka Ćelović – Trebinjac in 1932 and the initiative of Anton Bilimović, and with the support of Mihailo Petrović and Milutin Milanković, a new journal was launched – *Publications de l'Institut Mathématique Université de Belgrade*[26]. Papers were published in Russian, English, French and German. In the same year, at the International Congress of Mathematicians in Zurich, Mihailo Petrović and Anton Bilimović presented *Publications* to the world public. The works of Belgrade mathematicians became known to the global mathematical community. Until World War II, seven volumes of *Publications* were published. In each of these issues, Petrović published at least one paper.

Petrović's last paper was published after his death in the first post-war issue of *Publications* in 1947. In addition to Serbian mathematicians, mathematicians from other Yugoslav centres, Zagreb and Ljubljana, were also publishing in this journal. For instance, Đuro Kurepa, who was educated in Paris and was a professor in Zagreb at the time, published in 1935, in the fourth issue of *Publications* his entire doctoral thesis *Ensembles ordonnés et ramifiés*. This dissertation contains some of the key contributions to the contemporary set theory. After World War II, professor Kurepa had a very strong influence on the development of mathematics in Yugoslavia, including Serbia. The double issue VI–VII was dedicated to professor Petrović. The last pre-war volume VIII was printed on the eve of World War II, and was lost in the enemy bombing of Belgrade in 1941. In addition to mathematicians from Belgrade University, world-renowned mathematicians were publishing their papers in *Publications* both at the time and later, such as, for instance: Elie Cartan, Wacław Sierpinski, Paul Montel, Josip Plemelj, Đuro Kurepa, Paul Erdös and Saharon Shelah.

In his younger age, Petrović was interested primarily, if not exclusively, in abstract or the so-called pure mathematics. Whatever he did – differential equations, analysis or distribution of zeros of polynomials in complex plane – Petrović would detect a problem, formulate a theorem and give proofs. His scientific writings did not go further than this nor was he interested in wider application of the results obtained. Already at the start of the 20th century, he demonstrated particular interest in the practical aspect of mathematics. He pondered over how mathematics appeared in natural sciences, i.e. how it could apply to the exploration of natural phenomena. Just like Leibniz tried to arrive at a *characteristica universalis*, a universal and formal language to express all mathematical, scientific and metaphysical concepts, Petrović tried to invent a universal method to solve problems of other sciences. Analogies had a fundamental place in his examinations. He sought and gave examples of entirely disparate phenomena that were described by the same differential equations. These efforts resulted in his original work – *Mathematical Phenomenology*. He published three books, two in Serbian and one in French, in which he presented his theory[27].

Petrović was a highly prolific and versatile mathematician. He published several hundreds of papers, mainly in the leading foreign journals. He also put forward new and original ideas, and made significant breakthroughs in world science. This fact must be particularly appreciated given the circumstances in Serbia in which Mihailo Petrović worked. His algebraic results are a good example of his contributions to mathematics and his influence on the work of other mathematicians. The results in this field are closely related to the function theory and were recognised, cited and further developed by the leading mathematicians such as: Hermite, Landau, Polya, Fejér, Hardy, Montel and others. Around thirty Petrović's papers, four in algebra and number theory, are presented in the German reference mathematical journal *Zentralblatt* *für Mathematik und ihre Grenzgebiete*. It should be borne in mind that those are only papers published after 1930, when this journal was launched. Petrović's theorems and works on the geometry of polynomials are contained in the most famous monograph in this field – *Geometry of Polynomials* by Morris Marden. This issue of the American Mathematical Society (the book was published twice, in 1949 and 1966) quotes four Petrović's papers[28]. This monograph also contains references to several other Serbian mathematicians: J. Karamata, M. Tomić, B. Rašajski, D. Marković and Š. Raljević. Dragoljub Marković, the founder of the Algebra Department at the Faculty of Science and Mathematics in Belgrade, is cited the most (six papers), and can be considered the true successor of Petrović's work in the field of the geometry of polynomials in our country. In the 1970s, our renowned algebraist Slaviša Prešić made significant contributions in this field. It is therefore reasonable to accept the opinion of academician Miodrag Tomić that the geometry of polynomials, together with the function theory (which can be hardly separated from the former), is perhaps the most important Petrović's field, in which he made his greatest achievements. Moreover, Petrović brought this area to our milieu. Owing to his influence, several important Serbian mathematicians were engaged in this field, making recognisable and valuable contributions.

Mihailo Petrović retired in 1938, receiving the highest recognition from his students and colleagues. The following year, he received an honorary doctoral degree of Belgrade University and the order of St Sava of the first rank. The proposal for the honorary doctorate submitted to the Council of the Faculty of Philosophy clearly highlights Petrović's merits for the creation of the mathematical school in this region: "M. Petrović created the Mathematical School, the first in Yugoslavia, raising mathematical courses at Belgrade University to the level of modern world schools. Our Faculty, University and this entire state must give the highest recognition to Mihailo Petrović". At the same time, the members of the Mathematical Seminar paid respects to their teacher and colleague. They proposed that the unit for theoretical mathematics be separated from the Seminar and called the *Institute for Theoretical Mathematics of Mihailo Petrović*. The explanation read as follows: "Our Mathematical Seminar owes to Petrović eternal gratitude as it was he who founded it, worked there and was developing it for entire 44 years. He gathered around him a large number of young people and prepared them for scientific work".

Petrović held a high rank of a reserve officer, was a reserve engineering lieutenant colonel. When the Germans attacked Serbia in April 1941, he was drafted, although he was 73 at the time. He was soon after captured and spent several months in captivity. According to some sources, he was released after the intervention of his friend, Prince Đorđe Karađorđević, while according to other sources, he was released due to his age and illness. He soon began to contract illnesses, stopped going out and, as Milanković wrote, would sit all the time in his room and write.

Mihailo Petrović had a rich, interesting and unconventional life. It is hard to enumerate in one place, let alone describe in detail everything that he engaged in. Besides tackling various mathematical issues, Petrović was present at many other, often unexpected places. He was the author of laws and proposals of intergovernmental agreements[29] and the inventor and owner of successful and implemented patents. Many consider Petrović one of our most important philosophers and the creator of the original theory in natural philosophy – mathematical phenomenology. He wrote in a nice and interesting way, and some of his novels became a part of obligatory school reading and belonged, as they do today, to favourite youth literature. In addition to novels, he wrote essays and travelogues, and was an associate of daily papers. He wrote scientific papers and was also interested in other natural sciences, primarily astronomy, relativity theory and chemistry. He created the cryptographic system and was the main cryptographer of the Serbian and Yugoslav armies. He played the violin and led the music company "Suz", which had one of the leading roles in Belgrade's bohemian life until the start of World War II. He also collected folk poems and folklore elements. Finally, he was a passionate fisherman and a great world traveller and seafarer on northern and southern seas. This great mathematician and great traveller passed away silently, dreaming about a new and great oceanic travel. Mihailo Petrović died in Belgrade on 8 June 1943, in his home on Kosančićev venac 22.

Many Serbian and several foreign authors have written about the work and life of Mihailo Petrović. The author of this text had a difficult task not to fall into the dangerous trap of repetition and mere enumeration of the already well-known facts. We have therefore focused on the newly revealed archival records, for instance from *Adligat*, and on details and personalities surrounding Petrović. Some sources have not been explicitly mentioned here, while some less known have been stated. We have not given the list of authors who have written about Petrović as we would have thus wronged others whom we might have left out. However, I must mention Dragan Trifunović, Petrović's biographer, historian of mathematics and professor at Belgrade University. He wrote and edited several books about Petrović and his age. He not only shed light on Petrović's name in the past half a century, giving Petrović the place he deserves in Serbian mathematics, but also revealed numerous, less known details from Petrović's life. If we failed to specify some sources, they probably derive from the above works by Dragan Trifunović. In addition

Mihailo with his mother Milica in Bern in 1918 (SASA Archive, 14188/25) to all Petrović's works, *The Collected Works of Mihailo Petrović*, published by the Institute for Textbook Publishing and Teaching Aids in 1997–99, contain complete and valuable scientific analyses and contributions of renowned Serbian mathematicians. Owing to the Academy's engagement and by curtesy of this publishing house, *The Collected Works* have been digitised and are available to the interested public in the Virtual Library of the Belgrade Faculty of Mathematics and its digital legacy devoted to Mihailo Petrović. Owing to authors' contributions in this publication, a reader can learn about other details of Petrović's work and interesting episodes from his life[30].

ACADEMICIAN MIHAILO PETROVIĆ –
HIS CONTRIBUTIONS TO SCIENCE AND EDUCATION
– 150 years later –

Stevan PILIPOVIĆ
Serbian Academy of Sciences and Arts
University of Novi Sad, Faculty of Sciences

Academician Mihailo Petrović passed away 75 years ago, aged 75. This year we mark the 150^{th} anniversary of his birth. We shall attempt to answer several questions in this paper, most notably why professor Petrović, who became an academician aged 31, is so important for our mathematics and science in general? Why is Mihailo Petrović so loved and esteemed among our people?

A lot has been written about academician Mihailo Petrović, mainly about his mathematical results, travelogues and philosophical treaties and essays. There is ample literature about his friendships with fishermen, numerous fishing adventures, "Suz" music orchestra and his bohemian life. Almost all his students wrote about him, particularly professors Dragoslav Mitrinović, Tadija Pejović and Radivoj Kašanin. Within commemorative programmes organised to mark his birth anniversary, our numerous mathematicians analysed almost all his works. One of our greatest scientists, academician Milutin Milanković and our famous seismologist Jelenko Mihailović, who was Petrović's peer and friend, depicted Belgrade in their authentic notes[31], in the years between the two world wars and Mihailo Petrović as the icon of Belgrade's scientific and social life in the late 19^{th} and the first half of the 20^{th} century.

The work of our eminent historian of mathematics Dragan Trifunović also contains plenty of information. In 1998, the Institute for

Textbook Publishing and Teaching Aids, in cooperation with the Faculty of Natural Sciences in Belgrade and the Society of Mathematicians of Serbia, published an outstanding book – *The Collected Works of Mihailo Petrović*, in 15 volumes[32]. Trifunović headed the editorial board of this publishing project, which sheds light on the personality and work of Mihailo Petrović. Our most eminent academicians participated in the preparation of *The Collected Works*. Another important work, a forerunner of *The Collected Works*, is *The Chronicle of the Life and Work of Mihailo Petrović, 24 April 1868 – 8 June 1943*, by the same author. Trifunović collected available archival data, classified by year Petrović's biography and arranged his letters, scientific works, descriptions of his travels and other events from his life. Particularly interesting are the letters reflecting Petrović's personality and his line of reasoning. Original notes fill the reader with excitement and anticipation as one can feel the spirit of the time in which Mihailo Petrović lived and worked, and gain a genuine picture about him as a modest man of great capabilities. We have today all of his works, owing primarily to professor Žarko Mijajlović and his associates who digitalised all the books concerning Mihailo Petrović.

Mihailo Petrović Alas, or our Mika, as his contemporaries called him, was a fisherman and seafarer, musician and thinker, writer and bohemian, but, first and foremost, a great mathematician. He was highly enduring and physically strong, which enabled him to participate in numerous adventures and travels. For instance, he survived malaria at his last transoceanic travel, during an expedition in the southern part of the Indian Ocean. He was already 65 at time. According to him, he survived by drinking only water and mild wines. From this distance, I have the impression I am writing about an exceptionally interesting man of a great creative talent, a genius, who wove events from ordinary and everyday life into uninterrupted creative thought, which beset him wherever he was and whatever he did. I believe his activities outside mathematics were the periods of relaxation, during which he still ruminated about mathematics and philosophy, or practical solutions of problems, such as patents. Exceptional creativity and universality are the main features of Mihailo Petrović's scientific work. By carefully examining Petrović's activities, including fishing, one can discern his outstanding creative spirit. Our enthusiasm with academician Petrović is based on our understanding of his simplicity and modesty, as well as, given the level of mathematics at the time he lived in, his achievements in mathematics. Petrović was one of the most educated mathematicians Serbia has ever had.

This paper focuses on Petrović's scientific and teaching activities. In this light, we also examine today's achievements in the field of analysis within the so-called Novi Sad School of Analysis. Other mathematical fields that Petrović dealt with will be presented by our eminent colleagues. The literary, historical, travelogue, ethnographic and professional works on fishing and music will be described in other sections of this monograph.

Mihailo Petrović Alas (“Mihailo Petrović Alas” Primary School, Belgrade)

MATHEMATICS IN SERBIA IN THE SECOND HALF OF THE 19th CENTURY

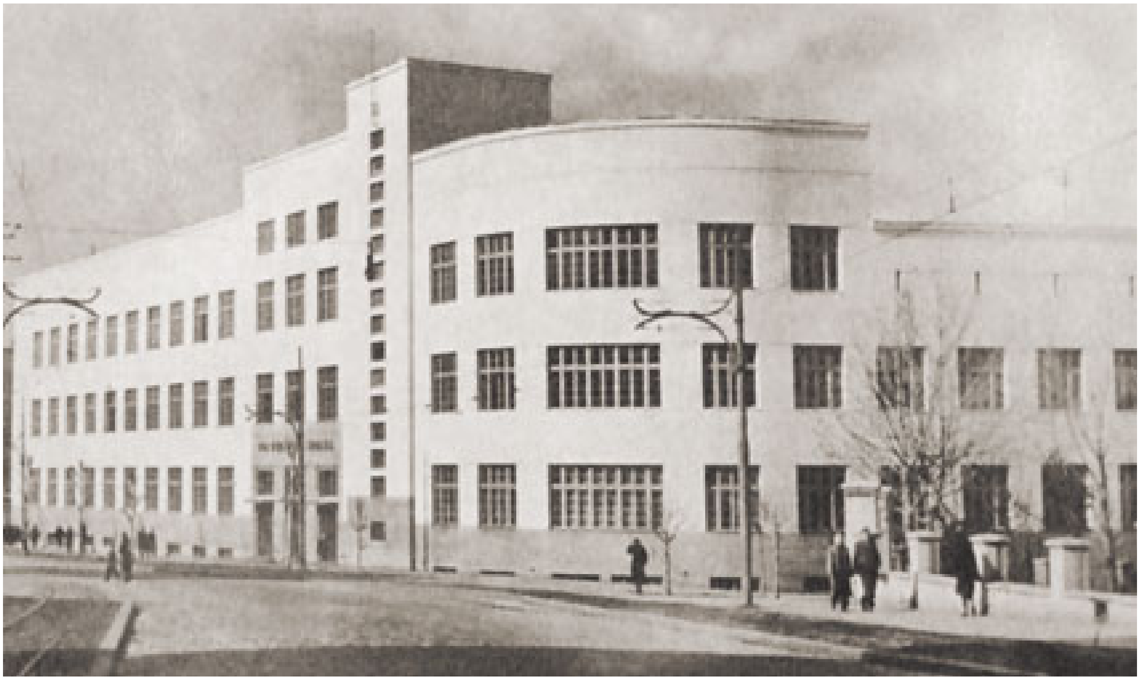
In the 19th century, from the establishment of the *Society of Serbian Letters*, the Serbian language was the dominant theme of the most educated people in Serbia. Natural sciences and mathematics were in the background given the lack of educated mathematicians. However, in addition to members of the *Serbian Learned Society* and professors of the Great School Dimitrije Nešić and Bogdan Gavrilović, there are several other professors of the Great School and authors of the first university mathematics textbooks in our country that are worth mentioning: Atanasije Nikolić, the founder of the Society of Serbian Letters (together with Jovan Sterija Popović), Emilijan Josimović, also a member of the Society of Serbian Letters, Dimitrije Stojanović and Petar Živković, members of the Serbian Learned Society. Professor Nešić, later Member of the Serbian Royal Academy, began his career with an article on squaring the circle in 1878. Nešić's ideas were of much higher quality than the results were not possible to be proved, as earlier established by Hermite and Lindeman. Nešić also wrote several interesting works, printed in Serbian in *Glasnik* of the Serbian Royal Academy.

Some of the world's most brilliant mathematicians were active at the time – for instance, Bernhard Riemann, Sophus Lie and Henri Poincaré. Is it necessary or even possible to compare the situation in Serbia with that in France, Russia, Germany, Italy, Holland, Austria, Switzerland or Hungary? Nonetheless, in the last decade of the 19th century, some great scientific minds started their work in Serbia, such as Jovan Cvijić, Ivan Đaja, Jovan Žujović, Sima Lozanić, Kosta Stojanović, Đorđe Stanojević, Slobodan Jovanović, Branislav Petronijević and mathematician Mihailo Petrović. This is why such a comparison in the late 19th century is, after all, possible.

Around ten years before these great names of our science appeared in Serbia, we had two extraordinary figures in America – one of the greatest innovators in world history, Nikola Tesla, 12 years older than Mihailo Petrović, and Mihajlo Pupin[33], a great scientist and professor, 14 years older than Petrović. The two of them already had exceptional careers, demonstrating supreme talent that we take pride in and that inspires us with the awareness that we can create great deeds.

Let us return to the description of the development of mathematics in Serbia at the time. Above all, let us highlight the merits of our professors: Dimitrije Nešić, Petrović's mathematics professor at the Great School before he left for France, and Petrović's colleague and friend Bogdan Gavrilović, a member of the Serbian Royal Academy and its later president. Academician Miodrag Tomić gives us plenty of selected information about Dimitrije Nešić and his merits for mathematics in his article *Contributions about Mathematical Sciences among the Serbs* (Mathematical Institute, 1992). Professor Gavrilović (also Rector of the University of Belgrade) was extremely important for the development of higher education in Serbia, though in scientific terms he remained in Petrović's shadow. A distinguished pedagogue, writer of several solid and studious mathematical higher education textbooks, Bogdan Gavrilović occupies an important place in our history, i.e. the most important after Mihailo Petrović, in the late 19th and early 20th century.

Historical figures in all fields of human activity are formed in certain time periods and favourable social and economic circumstances. The geniuses of science, art and invention live in any time and in all parts of the world. However, historically the most important personalities in any creative field, in addition to being ingenious, must be lucky to be born at the right moment. Namely, I wish to highlight an almost concurrent appearance of several historical figures in our scientific and cultural milieu by the late 19th century. We are proud of all of them, although with the enormous development of science their achievements are today merely smaller elements of highly developed and advanced theories. It is in this context that we observe academician Mihailo Petrović Alas, one of the most important scientists in Serbia and the founder of the mathematical educational and scientific system in Serbia.



First Belgrade Gymnasium, XIX century

FRAMEWORK OF SCIENTIFIC WORK OF MIHAILO PETROVIĆ

The scientific and pedagogical activity of academician Mihailo Petrović marked the path in the development of mathematics and university teaching of mathematics in Serbia. His biography is an important contribution in understanding historical events in Serbia in the late 19th and the first half of the 20th century.

After completing the Department of Natural Sciences of the High School in Belgrade, in 1985–1989 Mihailo Petrović, owing to favourable circumstances and his enormous talent that must have been evident, became a student in Paris at the most opportune time. He gained diplomas at Sorbonne in Paris, first in mathematics, and later in physics. A diploma on completed studies of chemistry was also recently found in his legacy. He was one of the first foreign doctoral students in mathematics at l'École normale supérieure. This school was and still is the most famous educational mathematical centre, whose lecturers were some of the greatest mathematical minds from the late 19th and the first half of the 20th century: Jules Henri Poincaré (1854–1912), Jean-Gaston Darboux (1842–1917), Paul Appell (1855–1930), Paul Tannery

(1843–1904), Charles Hermite (1822–1901), Paul Painlevé (1863–1933). Painlevé served twice as the Prime Minister of the Republic of France – during and after World War I. Petrović's friends were his peer Félix Édouard Justin Émile Borel (1871–1956) and around ten years younger Paul Antoine Aristide Montel (1876–1975), both of them exquisite personalities of world science. In the company of the greatest mathematicians whose originality of scientific work was their main feature, Mihailo Petrović demonstrated all his qualities. By defending his doctoral dissertation in 1894 before the Dissertation Committee consisting of Hermite, Picard and Painlevé, and having published a paper in the French Academy's journal, the famous *Compte Rendus*, Mihailo Petrović entered the world of great mathematicians of his time and wrote a number of important papers. He dedicated his doctoral dissertation to Tannery and Painlevé.

According to the information analysed by Trifunović, academician Mihailo Petrović published 393 works, of which 328 are mathematical manuscripts from twelve different areas, according to Trifunović's categorisation, in the leading world journals both then and today. He published 30 papers in *Compte Rendus*, where reports of French academicians on mathematical manuscripts considered important were printed. Even more important are the following journals where Petrović was publishing: *Acta Mathematica*, *Mathematische Annalen* – two papers, *Bulletin de la Société Mathématique de France* – fourteen papers, and *American Journal of Mathematics* – three papers. He also published in numerous other journals, primarily in Switzerland, Germany, the Czech Republic and Poland. Information about the number of papers is not reliable given that Petrović published some papers both in Serbian and French. In any case, Petrović was one of our most prolific mathematicians, whose papers were published in the journals still highly ranked on the lists of mathematical journals.

To better understand the time in which Mihailo Petrović worked on his doctoral dissertation, we should underline that in the 1880s and 1890s the greatest minds of French and world mathematics, Picard, Painlevé and Fucs studied nonlinear second-order equations with immovable branch points. The concept of the Painlevé transcendent, which is still highly topical, derives from the analysis of special functions arising as solutions of specific classes of nonlinear second-order differential equations in the complex plane. Among them are elliptic functions (doubly periodic functions), which are one of the most important classes of special functions. Painlevé transcendents are defined by nonlinear second-order differential equations with solutions whose singularities have the Painlevé characteristic: the only movable singularities are poles. Painlevé, followed by Fucs and Bertrand Gambier (1910) described the equations of this type – the so-called Painlevé equations. For the equation

$$y'' = F(x,y,y')$$

where F is the quotient of two polynomials under y and y' with coefficients which represent holomorphic functions, Painlevé found fifty general forms with immovable branch points and reduced them to six essentially new equations known as the Painlevé equations. After Fucs and Gambier supplemented his results, the solutions of these equations are known as the Painlevé transcendents. These six generic cases have a great importance today and are applied in statistical mechanics, physics of plasmas, theory of nonlinear waves, quantum gravity theory, quantum field theory, relativity theory and nonlinear optics. They are non-reducible in the class of classical special functions and have a number of exceptional characteristics. Integrable systems are reduced to these types of equations. For instance, the solutions of the soliton type of nonlinear differential equations are reduced to the Painlevé equations by the method of inverse scattering. Let us mention that this problem is being examined, by applying the methods of algebraic geometry, a modern mathematical field, by our colleagues in the group of professor Vladimir Dragović. Several their papers dedicated to the Painlevé equations have been recently printed or are about to be printed in the leading world journals. The theory related to the Painlevé equations is today unusually rich and modern. In 2017, the American Institute of Mathematics in San Jose organised a high-level conference devoted to the application of the Painlevé equations in the study of random matrices and the number theory. I highlight all this because I believe that the works of Mihailo Petrović, and probably those of his students, contain the forgotten original roots of modern mathematical research.

Painlevé published his first results in 1887 and 1895. Besides him, Poincaré and Picard worked on this issue. The problem of three bodies, linked to the movement of planets, was (almost) solved by Poincaré. This is worth emphasising because, I reiterate, Mihailo Petrović defended his doctoral dissertation before the Dissertation Committee consisting of Picard and Painlevé. Let us mention that Poincaré participated in developing the theme of the doctoral dissertation. Even Poincaré's biography contains information that Mihailo Petrović was his student. The third member of the Committee, Hermite, as the oldest and highly respected professor, a mentor to Poincaré, Tannery, Stieltjes, considerably contributed to the reputation of this Committee.


THÈSES
A LA FACULTÉ DES SCIENCES DE PARIS
PARIS.


PREMIÈRE PARTIE.
CHAPITRE I.

Book covers and the first page from Petrović's doctoral dissertation ("Svetozar Marković" University Library)

DOCTORAL DISSERTATION

Mihailo Petrović's doctoral dissertation deals with the immovability of zeros, poles and essential singularities of solutions of particular classes of algebraic first-order and second-order differential equations. Let us explain in short the movability of zeros and singularities of the general solution of a differential equation which, in a general case, depends on the constants directly calculated from initial conditions. First of all, let us remind that singularities are poles, essential singularities, finite order and logarithmic branch points. Movability means that zeros and singularities change incessantly with a change in initial conditions, i.e. it is a "well-posed" problem in contrast to "ill-posed" problem which often cannot be solved if initial conditions change a little. The adjectives "ill" and "well" do not have a formal meaning in mathematics, and in this particular case they only testify to the constant dependence of solutions on initial conditions. The problems studied by Mihailo Petrović are those in respect of which the solutions of equations have singularities independent of a change in initial conditions.

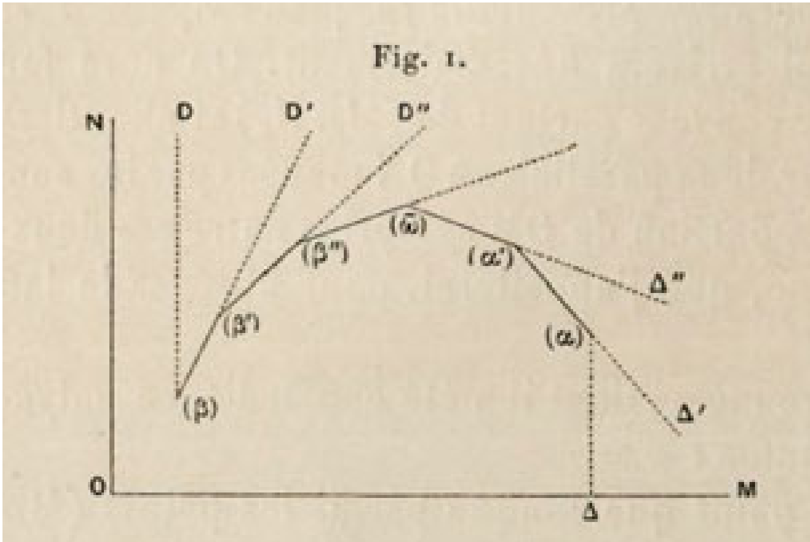
In the first part of his doctoral dissertation, Petrović analyses first-order nonlinear equations with products of powers of derivative of an unknown function y', function y, and holomorphic function $\varphi_i(x)$ under the independent variable x. Their shape is the following:

$$F(x, y, y') = \sum_{i=1}^{s} \varphi_i(x) y^{m_i} y'^{n_i} = 0.$$

In this analysis, Petrović constructs the polygon added to the following equation with vertices

$$(M_i, N_i),\ N_i = n_i,\ M_i = m_i + n_i, \qquad i = 1, \dots, s.$$

He arranges them according to specific rules, with (M_α, N_α) and (M_β, N_β) signifying the most distant/proximate vertices to the axis ON, while λ designates the coefficients of the direction of



Polygon R, a drawing from Petrović's dissertation ("Svetozar Marković" University Library)

the sides of the polygon. In the first chapter of the first part of the thesis, Petrović gives a complete answer to the defined problem, by analysing the given convex polygon. For the infinities (as Petrović calls poles and essential singularities) of the general solution (integral) of the first-order equation $F(x,y,y) = 0$ not to change with the integration constant, it is necessary and sufficient that the polygon joined to F does not have any vertex to the right from the most highly elevated vertex of the polygon. The existence of the movable zero of order λ is necessary and sufficient for the polygon to have a side with the slope λ while for it to have a movable pole of order λ it is necessary and sufficient for the polygon to have a side with a slope $-\lambda$. Based on these two out of six assertions from the first chapter, we can clearly see the simplicity of Petrović's formulations.

Such geometric interpretations are more easily acceptable (let us remember Newton's polygons in the case of partial differential equations) and essentially give a new geometric method for the analysis of the solution of the equation. The conditions for essential singularities are also given. There may be a finite number of poles and essential singularities in the case of their immovability. In the second chapter, Petrović explains the application of theorems from the first chapter, connecting them to the results of Painlevé and Fucs which arise from immovability of branch points. Let us mention an assertion concerning the rational function R and equation $y' = R(x,y)$: this equation may not have more than three different uniform (single-valued) solutions, which I will hereinafter call only solutions. If there are three of them, it is Riccati's. If there are two of them, it is linear or Riccati's or has a solution of a particular shape. If it has one solution, it is reduced to one of the earlier forms or a special form. We do not mention special forms.

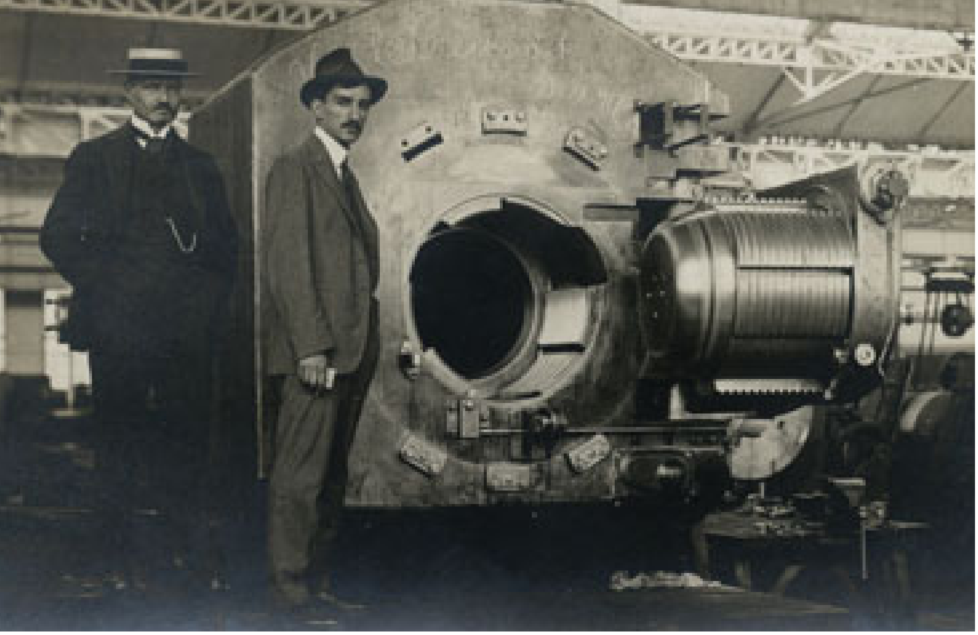
For precise formulation of these assertions, see the paper by professor V. Dragović in this monograph. Furthermore, in the second part of the first chapter, he examines various forms of the right side of the equation, function $R(x,y)$, and gives general forms of the solutions of equations.

In the second part of the thesis, Mihailo Petrović deals with nonlinear ordinary differential equations of the second order, with similar questions as in the first part. He deals with the movability of the singularity of the equation and gives sufficient conditions for the immovability of poles, essential singularities and zeros, under the assumption that branch points are immovable. Starting from the results of Poincaré and Fucs and by drawing polygons under the same principle as in the first chapter, though in a much more complex form, Petrović shows his results rather than illustrations, as sufficient conditions, and less as necessary and sufficient conditions in the form of a theorem. The necessary conditions in the mathematical sense were achieved only in special cases. In the second chapter of the second part of his doctoral dissertation, Mihailo Petrović gives examples of solutions through holomorphic solutions of adjoint equations. Under appropriate conditions, starting again from the results of Painlevé and Fucs about the immovability of branch points, he finds functional links, the so-called first integrals that the solutions must meet and specifies the number of solutions.

Academician Bogoljub Stanković translated Petrović's dissertation into Serbian. This translation is part of the first volume of *The Collected Works*, prepared by academician Stanković.

DIFFERENTIAL EQUATIONS

The turn of the 19th into the 20th century brought great discoveries in almost all sciences, particularly mathematics. New mathematical theories and new methods emerged. Though at the time it might have looked different, but the events that happened have their repercussions still today. Mihailo Petrović belonged to the generation of the greatest minds in the field of the analytic theory of differential equations, which at the time reached its pinnacle, while on the other hand, new mathematical disciplines emerged, which were theoretically highly demanding, including the theory of partial differential equations. Mihailo Petrović was at the height of this mathematical field, the most developed at the time, and it is in this light that we should observe the exceptionally high scientific level of his research at the time when he was writing his doctoral dissertation, including the destiny of these results as well. This is worth emphasising because generations of our mathematicians who dealt with differential equations did not go much further than the results of Mihailo Petrović from the period when he defended his doctoral thesis, or from the works he wrote in the first twenty years of the 20th century. The majority of his doctoral students explored in their theses the topics relating to ordinary nonlinear differential equations, mainly of Riccati type, or with a qualitative analysis of some classes of equations. It is possible that the basic problem was the fact that Mihailo Petrović's successors, and later his students in the fields of differential equations, perhaps less followed the development of mathematical physics or other branches of mathematics, where the results of Painlevé and Picard remained dominant. Besides, insufficiently followed is the theory of partial differential equations, which at the time was developing exceptionally fast both in theoretical terms and in terms of application in almost all natural and technical sciences. Petrović himself did not follow new directions of development of the theory of partial differential equations with entirely new methods arising from the then modern areas – set and algebraic topology, geometry and algebra, particularly functional analysis. The results of Hilbert, Lebesgue, Dirac, Banach, Sobolev and many others brought entirely new insights into mathematical research and its link with other scientific areas. The progress and development of mathematics was particularly intensive after World War I. One should bear in mind that Mihailo Petrović was aged around 50 at the time, and was increasingly outside scientific developments due to complicated circumstances in Serbia in the post-war period. Some of our colleagues mathematicians unjustifiably wrote that Petrović, with his exceptional energy in the areas that he dealt with, hindered the development of other, also important mathematical fields. It is hard to provide arguments, particularly from this perspective, outside the time during which Mihailo Petrović lived and worked. He did not encourage the areas he was not familiar with, nor did he impose upon anyone doctoral dissertations relying on the research that he personally engaged in.



Mihailo Petrović with Prince Đorđe Karađorđević (Foundation "Mihailo Petrović Alas"/ Primary School "Mihailo Petrović Alas", a gift from Jovan Hans Ivanović)

SCIENTIFIC WORKS – DIFFERENTIAL EQUATIONS

The first work of Mihailo Petrović signalled the problems explored in his doctoral dissertation, while the works that followed the dissertation relied on its results. The independence of singularities, zeros, extremes or other features of a general solution with respect to constants, dominates in all these works as one of the determinants of Petrović's scientific work. This is the essential, constitutive feature of a model related to an equation describing the model. When analyzing the residue of the function and exploring the so-called binomial first-order equations, i.e. the asymptotic solution, or when writing about a "class of second-order differential equations" or the nature of the solution, he aims to examine the internal structural relationship between the dependent and independent variable given by equations. This can also be seen in some of his works reprinted in the first volume of *The Collected Works*. A great creative potential in his scientific papers initiated by his doctoral dissertation is also seen in the works written and published during World War I, when his papers in *Compte Rendus* were presented by Picard, Appel and Hadamard. During the war, he was dispatched to Switzerland, in service of Prince Đorđe Karađorđević, to be a cryptographer at the Serbian Military Command. Even there did he have sufficient strength to tackle serious mathematical problems. It is interesting to note that his paper in which he described his own most important results in the period until 1922 was published in France. This analysis and overview of Petrović's results, written by academician Bogoljub

Stanković, clearly attest to the quality and value of results presented in the doctoral dissertation. An excellent overview of Petrović's scientific and teaching opus in the article by academician Miodrag Tomić, in the first volume of *The Collected Works*, clearly confirms all already mentioned qualities of Mihailo Petrović as a scientist and a renaissance personality in our science.

According to Dragan Trifunović's statistics, in addition to his doctoral dissertation, Mihailo Petrović published 86 papers on differential equations. In the first volume of *The Collected Works*, 14 papers were reprinted and translated into Serbian, primarily in the field of the analytic theory of differential equations. The second volume contains 23 papers and the monograph *The First Definite Integrals*. Those were mainly works in the field of the qualitative analysis of general linear and nonlinear equations, and typical solution of some classes of equations. Papers are presented chronologically so that a careful reader can follow the development of ideas and the quality of publications compared to already printed manuscripts.

In the works published in the second volume of *The Collected Works*, Petrović studied the qualitative features of solutions of differential equations. He upgraded his new ideas based on those from his doctoral dissertation. In this context, he particularly examined various forms of the Riccati equation $z'(t) = a(t)z(t) + b(t) + c(t)z(t)^2$ and generalisations which can be solved with integrations and known special functions. He based the classification of differential equations on appropriate transformations of independent and dependent variables. Petrović used his exceptional knowledge of the theory of analytic functions. He thus found canonical forms of various classes of equations and solved them. The analysis of equations through the formulation of sufficient conditions for equation coefficients, along with appropriate initial conditions, is the domain of the qualitative analysis of differential equations to which the majority of Petrović's papers are devoted in the second volume of *The Collected Works*. The theorems about the comparison of solutions of equations relative to the comparison of coefficients or right-hand sides of equations, the so-called Sturm-type theorems, were the inspiration to his students in their doctoral dissertations. He studied Champling-type equations independently of Champling, and before him, although he did not go into the detailed analysis which was later formulated. Applying subtle mathematical insight and having clear motivation, he obtained results of general character for various classes of linear and nonlinear equations. The solution asymptotics also has an important role in the analysis of equations – those were again mainly nonlinear, Riccati-type equations. When reading the papers from the second volume of *The Collected Works* which Mihailo Petrović published aged 50 and 60, one can see a decline in scientific enthusiasm, although these papers still contained new ideas of qualitative generalisations and classification of equations.

Unlike today's practice, Mihailo Petrović wrote his works by explaining a theory starting from special to general mathematical conclusions. As if in a story, easy conclusions are given and generalised; then, fully explained, they are turned into assertions that are only at the end formulated. This enables the reader to easily follow the narrative, with clear objectives that are already defined at the beginning. We can add another analysis of Mihailo Petrović's works, written by himself, to our assessment of the quality of papers from the second volume of *The Collected Works*. It is worth noting that professor Ljubomir Protić wrote a systematic article in the same volume related to Petrović's work. Together with professor Milorad Bertolino, he was one of the important followers of Mihailo Petrović in the theory of ordinary nonlinear differential equations.

APPLICATION OF EQUATIONS – PHENOMENOLOGY

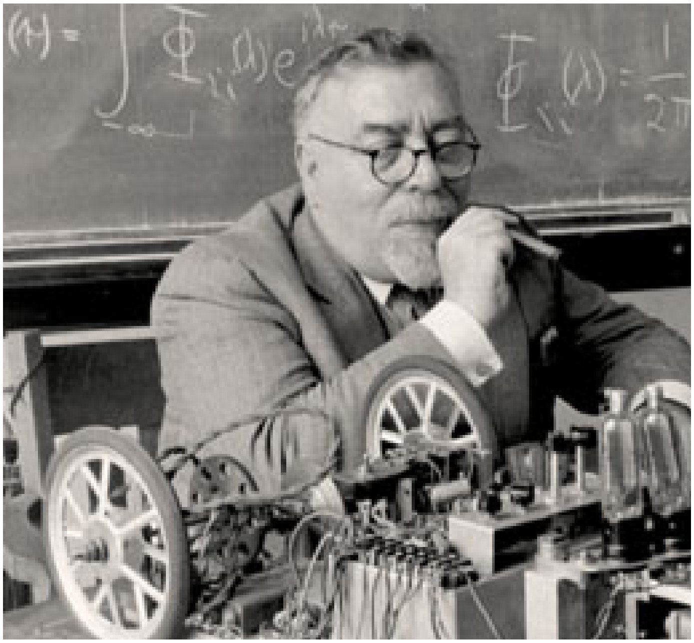
The seventh volume of *The Collected Works* contains the equations of mathematical physics in the monograph *Elements of Mathematical Phenomenology* (later in the monograph *Phenomenological Reflections*). The results of modelling explained by Petrović in philosophical sense are a highly important scientific contribution in applied mathematics. Moreover, it was in philosophy and other areas of humanities that his phenomenology with metaphors and allegories obtained a number of followers.

As an exceptionally educated mathematician, Mihailo Petrović dealt with mathematical research which can be divided, perhaps too freely, into *l'art pour l'art* research and motivated, i.e. applied research.

In a rather free interpretation, *l'art pour l'art* research concerns his doctoral dissertation and differential equations which he theoretically examines from the viewpoint of the theory of analytic functions, power series and appropriate algebraic problems, which he links to the complex structure of analytic functions.

Applied research is related to modelling, i.e. formulation of models, various physical or chemical phenomena. Moreover, in phenomenology to which he devoted a great number of papers and which he experienced as the most important part of his research, Mihailo Petrović tried to determine, through analytic dynamics, "the activity of causes" by establishing appropriate systems of equations and thus describe phenomena independently of the nature of objects and phenomena being modelled. Phenomenology arises from his algorithmic approach to mathematical problems. A monograph on this topic was printed by the Academy in Serbian in 1911. Petrović published the same work in an abbreviated version in Paris, at his own cost. Petrović analysed general phenomena arising from the minimum number of basic facts, causes, established analogies and arrived at conclusions based on a narrow, base set of data. This is a generally accepted procedure in mathematical research.

Metaphors and Allegories (a posthumously published monograph) elaborates on the ideas of phenomenological research. As unlike phenomenology, metaphors and allegories are significantly void of mathematical formulae, they offer, in addition to philosophy, a significant basis for the structural research of phenomena in other social and humanistic disciplines as well. For instance, in the phenomenology of Mihailo Petrović, in his metaphors and allegories, we also



Norbert Wiener (1894–1964)

find the contemporary line of linguistic research. Almost a hundred years after the publication of Petrović's works, Noam Chomsky, a foreign member of our Academy and the greatest world linguist, wrote that analogies or the establishment of similarities represent the key issues of language functioning. Academician Jasmina Grković-Major pointed out this and referred me to an interesting article by Dr Ivana Bašić from 2012.

Our literature perhaps overemphasises the fact that Mihailo Petrović was one of the first creators of phenomenology. There were before him many other authors of scientific theories attempting to use mathematical clarity and an axiomatic approach to the examination of general laws. Many our mathematicians believed that Petrović's phenomenology is the precursor of cybernetics. Dragan Trifunović's dissertation "Study of Modelling in the Work of Mihailo Petrović" dealt with phenomenology, metaphors and allegories. Somewhat younger than Petrović, world-renowned mathematician Norbert Wiener is considered the father of cybernetics. It is a real pity that the works of Mihailo Petrović were insufficently known at the famous Massachusetts Institute of Technology, where an entire group of brilliant mathematicians worked, headed by Wiener.

Let us emphasise that Petrović's descriptions of phenomena through the systems of relevant equations today still constitute a modern approach to exploring phenomena in nature, particularly in the context of the so-called motivated or applied mathematics, as the most important determinant of contemporary mathematical studies.

OTHER MATHEMATICAL WORKS

As we have already emphasised, the most important works of Mihailo Petrović belong primarily to the fields of analysis, differential equations, complex and real analysis.

Within the theory of analytic functions, he explored functions whose Taylor series do not have zeros in an appropriate circle of convergence. Landau, Hardy, Fejér, Montel, Pólya studied his works in this field, while Jentzsch developed them in his doctoral dissertation. Related to these works are his papers in the field of algebraic equations. Particularly interesting is his work on the geometry of polynomial zeros, which was also studied by this group of renowned mathematicians. Petrović determined the ring in which an algebraic equation has at least one root without using the Rouché's theorem. His paper from 1899, published in *Compte Rendus*, was the first paper determining the number of zeros contained in a given circle.

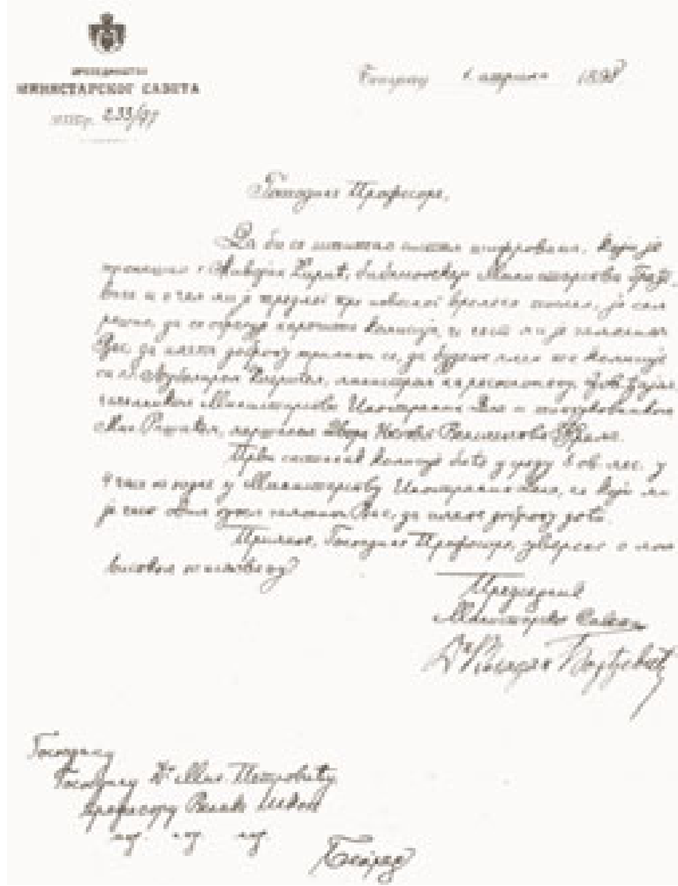
Let us also mention his contribution to numerical mathematics – for instance, the analysis of number differences and the instability of solutions, i.e. the interval analysis separately published in his textbook *Calculation with Number Differences*. Related to this research are his papers in which he dealt with the calculation of specific integrals through series, to which he devoted a particular textbook. In addition, his textbook *Elliptic Functions* is still popular today.

In an analogy with light spectres in physics, Petrović developed the theory of mathematical spectres which is not related at all with today's spectral theory of operators. Quite the contrary, it can be classified into the theory of numbers and cryptology – algorithmics, rather than analysis. During one term at Sorbonne, he held lectures on spectres, and published two monographs in this field in Paris. The main idea behind this theory is that infinite series of data are coded with infinite decimal numbers (with zeros and ones), translating mathematical operations with data into certain numerical or combinatory procedures. Relying on his exceptional knowledge of the theory of function series, he assigns to the appropriate analytic function the analytic expression – a number which with its decimals determines this function, i.e. its Taylor coefficients. His student, professor Konstantin Orlov focused later on this field, in his dissertation. Orlov was one of the most renowned followers of professor Petrović in the field of differential equations and numerical analysis. Unfortunately, the spectral theory did not find a significant place in mathematical literature. It seems to have appeared too early.

Petrović's inequality

$$\sum_{k=1}^{n} f(x_k) \leq f\left(\sum_{k=1}^{n} x_k\right)(n-1)f(0)$$

is the precursor of Jensen's inequality. It is related to the study of convex functions and was an inspiration to professor Mitrinović and his students in the examination of analytic inequalities and writing of a number of exceptionally well-received and cited books on functional equations and inequalities.



МИНИСТАРСКОГ САВЕТА

An invitation to Mihailo Petrović for participating in the work of the Encryption Commission, 1 April 1898

While serving in the army during the Balkan Wars and the Great War, Petrović dealt with cryptography and gained the rank of a reserve engineering lieutenant colonel. From the Balkan Wars until the start of World War II, he was a cryptographer at the Serbian army. Petrović took a lot of pride in his military rank.

As already said, Petrović completed in Paris the studies of physics and chemistry as well. He liked chemistry a lot. He probably attended lectures of Sima Lozanić, a professor at the Belgrade Great School and later an academician. He expressed particular creativity in his works in which he formulated models for various phenomena in mechanics and chemistry, Based on simple models, he constructed devices that produced solutions to equations through mechanical devices or chemical reactions. Analogous computers, and present-day cutting-edge quantum computers, were created based on the principles of electrical and physical-chemical reactions, which is the area of the state-of-the-art theory in computer science today. A possible link, in a rather free interpretation, with the ideas of modern quantum computers can be found in Petrović's results.

At the start of his career, along with mathematical modelling, Mihailo Petrović dealt with the creation of patents, range markers and depth gauges. He constructed the gear and eternal calendar. In 1900, he received the third award at the World Exhibition in Paris for the hydro integrator described in the *Journal of the American Mathematical Society* in 1898.

THE ECHO OF PETROVIĆ'S SCIENTIFIC RESULTS

There are few persons today who are well familiar with Petrović's doctoral dissertation and works. Objectively, mathematical results last as long as they are topical in contemporary research. After some time, only a small number of them become classical results and enter textbooks where authors are no longer mentioned, unless they carry the authors' names, as is the case, for instance, with the Pythagorean theorem, Newton-Leibniz formula or Petrović's inequality.

The works of Mihailo Petrović were cited in the time approximate to that when he wrote them, which is why these citations are not contained in statistics, which are modern, but quite often badly interpreted. Let us mention that the manuscripts published in *Acta Mathematica, Mathematische Annalen* and a number of other journals, which explore problems from his doctoral dissertation, and most other papers published in earlier specified journals in France, were correctly cited in the late 19th and early 20th century. The results of Petrović's first work in *Compte Rendus* were stated in their entirety in the famous, at the time the most valued Picard's monograph *Traité d'analyse*, while the results of his doctoral dissertation were cited in the *Encyclopedia of Mathematics*.

In the vortex of events during the Balkan Wars and World War I, Mihailo Petrović was increasingly less present in France, and thus in world circles within which new science was being developed.

Whether he wanted it or not, Mihailo Petrović was a rather lonely intellectual. He often did not develop his ideas in full, which is why others, inspired by his results and ideas, wrote much more in-depth and more widely cited works. Regardless of Petrović's brilliance and fondness of friendships, he did not have followers to highlight his scientific grandeur. The reason may be the fact that everyone – and here we refer primarily to mathematicians in France, aspired to their personal prestige. Given that there were very few mathematicians in Serbia at the time and that Petrović was our first mathematician who tackled these problems, he could not have an important mathematical support in Serbia, particularly in the period between the two wars. In addition, Mihailo Petrović was not much interested in being cited nor was he interested in having someone to follow him in mathematics, particularly after World War I. On the other hand, mathematicians from the late 19th and early 20th century did not have many options for citation and did not have the habit of such mutual communication in science. In Mihailo Petrović's thesis and works, the citing of results of other mathematicians did not take the form which is acceptable today. The citations in his dissertation are given through the presentation of results ascribed to Picard, Fucs and most certainly to Painlevé. He has in total eight references in his entire doctoral dissertation.

The certificate of membership in the Czechoslovakian Mathematical Society (SASA Archive, 14188/31)

Mihailo Petrović wrote all of his papers alone, apart from one paper which he wrote together with Karamata. This joint work is interesting because the authors corrected Poincaré's error. The reason for this "mathematical loneliness" may be his exceptional individuality, and perhaps, mathematical culture and mathematical knowledge that he possessed, which is why he did not need any associates.

Mihailo Petrović presented his works as a guest lecturer at a number of important international congresses of mathematics in Rome (1908), Cambridge (1912), Toronto (1924), Bologna (1928) and Zurich (1932), and many conferences of scientific federations of France (around ten times), Romania, Italy, Slavic and Balkan countries. He was widely esteemed. He was a member of the Yugoslav Academy of Sciences and Arts, Czech Royal Academy, Polish Academy of Sciences in Krakow, Academy of Sciences in Warsaw, Romanian Academy of Sciences, and many mathematical societies in Paris, Palermo, Bucharest, Leipzig, Prague, Lviv. He was also a member of several other scientific societies in Paris.

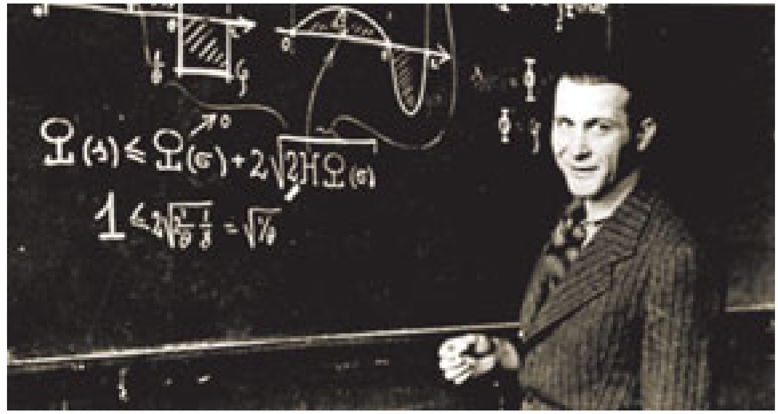
STUDENTS OF MIHAILO PETROVIĆ

Doctoral students of professor Petrović: Tadija Pejović, Konstantin Orlov, Dragoljub Marković (Faculty of Mathematics, University of Belgrade)

Mihailo Petrović came to Serbia with enormous knowledge that he gained in Paris. Learning from the best, he raised the educational system of mathematics to the highest possible level. He understood exceptionally well the classical mathematical fields of analysis and the theory of equations. His lectures are in fact the greatest contribution to the development of mathematics in our country. Petrović passed on to all of his students his excellent knowledge of classical mathematics, primarily of analysis. They continued to develop themselves further in the fields that best fit their preferences and talents, contributing to Serbian and world mathematics. His students, academician Jovan Karamata, professor Dragoslav Mitrinović and professor Tadija Pejović taught the greatest number of doctors of mathematics.

At the time when academician Petrović lived and worked, the most spectacular achievements were made in mathematics. Petrović's contemporaries were Lebesgue, Poincaré, Cartan, Hilbert, Dirac, Smirnov, Fichtenholz and many others. The functional analysis based on Lebesgue's integrals, Hilbert and Banach spaces, including the development of topological and algebraic methods of Poincaré, Hausdorff, Borel, Kolmogorov and many other greats of the time, largely enriched the areas of differential equations.

The majority of Mihailo Petrović's students continued his work in the area of equations, as already explained, and its quality is mainly on a par with Petrović. In addition, some of his followers in the field of analysis studied classical theorems on the mathematical analysis of real and complex functions – particularly the assertions of Abel and Tauber type, with many ingenious mathematical witty remarks, with exceptionally talented Jovan Karamata having a lead role. His proof of Tauber's theorem for the Laplac transform of the non-negative measure is still today largely important in mathematical literature, e.g. microlocal analysis. Karamata himself could not envisage that his theorem would be used nor was he familiar with all the areas in which the theorem is relevant. Of course, some of these areas were studied in his time as well. The works on the theory of analytic functions through professor Tadija Pejović, one of 1300 corporals, and his students, resulted in the creation of the outstanding *Belgrade School of Real and Complex Analysis*, which is still active today. Furthermore, professor Pejović taught distinguished professor Slaviša Prešić who, owing to his great erudition, initiated new mathematical research in mathematical logics and algebra. It is worth noting that Prešić created the so-called *Belgrade School of Logics*, the most famous in post-war Yugoslavia.



Doctoral student of professor Petrović:
Jovan Karamata
(Faculty of Mathematics,
University of Belgrade)

In his article about Mihailo Petrović, academician Tomić wrote that it seemed Petrović had hurried in his scientific work. As if he wanted to compensate for the enormous differences in the scientific level of the country that he came from and world science. It is possible that he therefore did not have time, and perhaps patience either, to fully develop his ideas, which was later used by mathematicians in world centres, who became famous with theories that had no mention of Petrović's ideas. We are referring here to works on differential equations, theory of analytic functions, the already mentioned phenomenology, and works on spectral theory which contain the roots of interval analysis. It seems that the reason for his perhaps rapid neglect of important results that he obtained was the fact that Petrović was lonely, without appropriate conversation partners or associates in the country who could with their questions or in discussions motivate him to go deeper into the analysis of his results. On the other hand, it was Petrović's great creativity that prompted him to quickly overlook some of his ideas. He was simply brimming with ideas.

Academician Petrović did not write papers with his students, but, being mathematically highly educated, he offered to everyone complete classical knowledge which they further used. The Belgrade School of Mathematics is fundamently connected to Petrović and Karamata. Historians of mathematics owe us the analysis of work of the *Belgrade School* before World War II and immediately after the war, in the context of world mathematics, and not in terms of turbulent events in Serbia only. It seems that there is no analysis of the impact of the German occupation. Some events are unreasonably mystified and those less favourable in the historical context of mathematics of the time are disregarded.

What is considered the greatest quality of academician Petrović is his closeness to ordinary people, sympathy with their suffering, as seen in numerous examples from his life. He participated as a Serbian soldier in the Balkan Wars and the Great War. Aged 73, he voluntarily joined the army in World War II. He was captivated and interned in a camp. Upon his return, he passed away in silence and sailed into our history as one of our greatest scientists.

Mihailo Petrović Alas in his late years
(SASA Archive, 14188/21)

PETROVIĆ'S CONTRIBUTION TO HIGHER EDUCATION

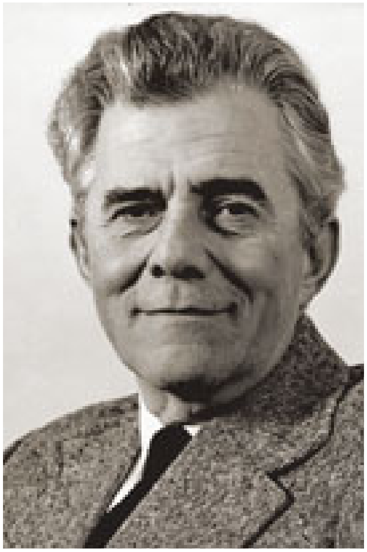
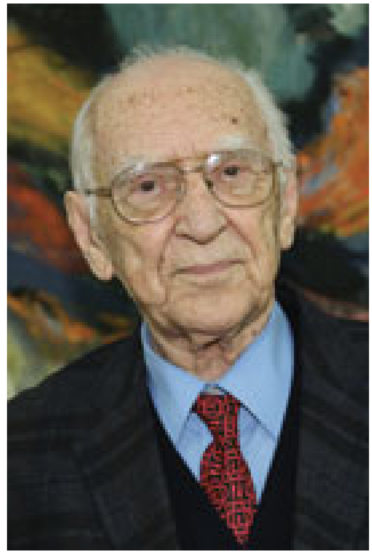
In the role assigned to him by a set of circumstances, academician Petrović was exceptionally important for the development of the university teaching of mathematics in Serbia. What we particularly emphasise and what is particularly important to all of us concerns Professor Petrović's mentorship in Serbia. Until World War I and between the two world wars, Petrović was developing the mathematical system to be applied at the universities of Serbia, almost on his own. As of 1894, he was full professor of mathematics at the Great School, and as of 1905 at the University of Belgrade. He was the only professor who in the 1912–1941 period mentored doctoral students in mathematics at the University of Belgrade. He did not publish many textbooks – only three, but his hand-written manuscripts are of exquisite quality and one still derives a lot of satisfaction from them. He held 16 different courses – ten courses on analysis and differential equations, two courses in algebra, three on numerical mathematics, and a separate course on mathematical phenomenology. He wrote eight teaching manuscripts.

He was Member of the Professorial Examination Board, Ministry Supervisor at maturity exams and President of the Main Educational Council of Serbia. He was also an advisor for the preparation of high-school textbooks and wrote several works on methodology.

Mihailo Petrović was a strict and principled professor. Graduate students of mathematics understood this very quickly, paying much more attention to the preparation of exams in mathematical fields. For instance, during one year, Mihailo Petrović held all mathematical subjects at the University of Belgrade. His students and Milutin Milanković later held courses and helped him in teaching. It has been noted that he held not a single public speech, which is not unusual among mathematicians. However, he left behind many interviews and several newspaper articles, describing mathematical life in Serbia.

(SASA Archive, 14197/II-22-1)

Together with Milanković, he launched the journal *Mathematical Publications of Belgrade University*, which later grew into today's *Publications of the Mathematical Institute*. The journal was established in 1932 by academician Anton Bilimović, one of our most famous mechanical engineers, who came from Russia after the October Revolution, and was a close associate of Mihailo Petrović who proposed him for an academician. In addition to the Mathematics Department of the Faculty of Philosophy led by Petrović, the Society of Mathematicians was established by professor Tadija Pejović. All this was very important for the development of mathematics in Serbia. Mihailo Petrović was an unavoidable or, one may say, one of the most important participants in all these activities.


Mathematical offspring of professor Petrović: academician Slobodan Aljančić (Archive of MISASA), academician Bogoljub Stanković (author: Dragan Aćimović, 2016) and professor Slaviša Prešić (author: Dragi Radojević, 2006)

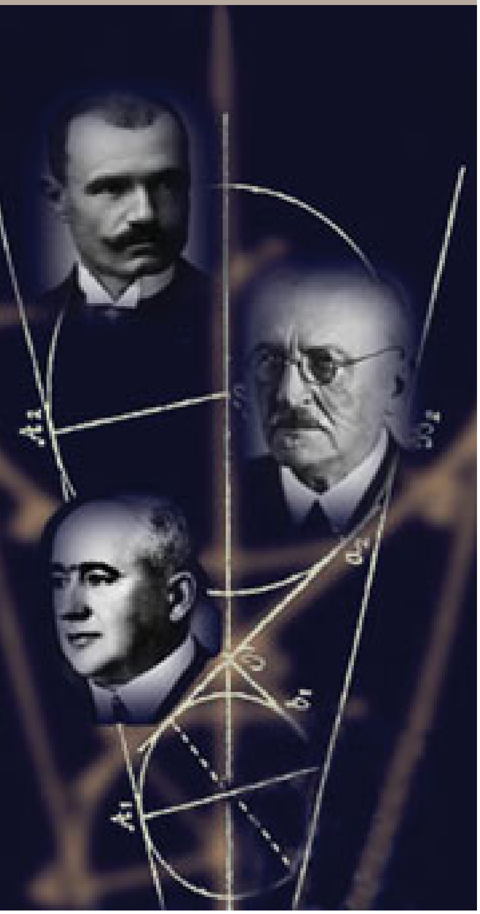
SCIENTIFIC SUCCESSORS OF MIHAILO PETROVIĆ

Academician Mihailo Petrović was to a great extent celebrated by his students, primarily professor Tadija Pejović, academicians Jovan Karamata and Vojislav Avakumović and professor Dragoslav Mitrinović, with a large number of students who, after them, further developed the enormous mathematical genealogical branch of academician Petrović.

The following result worth of great engagement in teaching is unknown for many our mathematicians, mathematical successors of professor Petrović – he is a mathematical predecessor for around 70% of doctors of mathematics in Serbia. The names of academicians, with each following being the mentor of the previous one, appear in the mathematical history of the author of this article: Bogoljub Stanković, Vojislav Avakumović, Jovan Karamata, Mihailo Petrović. In addition to students of academician Đuro Kurepa and mathematicians in Serbia who completed their doctoral studies abroad with different mentors, as well as geometricians whose mathematical roots go to professor Danilo Blanuša, there are many of those whose close mathematical predecessor is Mihailo Petrović.

Let us also mention the names of his students of the first generation: professor Mladen Berić, Sima Marković, Dragoslav Mitrinović, Konstantin Orlov, Tadija Pejović, Dragoljub Marković, Danilo Mihnjević, Petar Muzen, corresponding member of the Serbian Royal Academy Miloš Radojčić, and academicians Radivoj Kašanin and Jovan Karamata; second generation:

academicians Miodrag Tomić, Slobodan Aljančić, Vojislav Marić, Milosav Marjanović, Dragoš Cvetković, Gradimir Milovanović: third generation: Ivan Gutman, Olga Hadžić, Miodrag Mateljević, and the author of this article. I feel a great obligation to mention our exceptionally esteemed mathematicians, successors of academician Petrović, who passed away – professors Manojlo Marović, Ernest Stipanić, Tatomir Anđelić, Milorad Bertolini, Milica and Vojin Dajović, Petar Vasić, Slaviša Prešić, Zagorka Šnajder, Svetozar Milić, Zoran Ivković, Janez Ušan, Dušan Adamović, Dragoljub Aranđelović, Vladeta Vučković, Bogdan Bajšanski, Ranko Bojanić, Tatjana Ostrogorski, Zoran Popstojanović, Ljubomir Protić, Rade Dacić... All of them are related through academician Mihailo Petrović. Many names have not been mentioned. In the genealogy of academician Petrović we can find information about them and about those who are still active and whose names are therefore not mentioned.



Academicians Mihailo Petrović, Bogdan Gavrilović and Milutin Milanković (Faculty of Mathematics, University of Belgrade)

WORK AT THE ACADEMY

Mihailo Petrović Alas became a corresponding member of the Serbian Royal Academy in 1897 and its full member in 1899. The official admission took place in 1900, the same year when Jovan Cvijić became its full member. Petrović was highly active in the Academy's work. He was Secretary of the Department of Natural Sciences and member of the Executive Board of the Academy. He wrote overviews of the works of mathematicians who published in the Academy's *Glas* and also presented new candidates for membership. He published around 60 papers in *Glas*, thus significantly contributing to the Academy's reputation. According to the tradition practiced still today, which is also the tradition of the French Academy, the works of authors outside the Academy were reviewed by members of the Academy. Mihailo Petrović was devoted to these activities particularly because his students, doctoral students and later doctors of mathematics published a great number of their most important works in *Glas* of the Serbian Royal Academy. He promoted a number of academicians – Bogdan Gavrilović, Milutin Milanković, Anton Bilimović and Jovan Karamata.

In 1909, Jovan Cvijić and Mihailo Petrović invited Milutin Milanković to move from Vienna and take a teaching post at the University of Belgrade. In 1920, they proposed him for corresponding member and in 1925 he became a full member of the Academy. When Jovan Cvijić died in 1927, Mihailo Petrović was the most serious candidate for the Academy's president. However, due to the will of the authorities at the time and his friendship with Prince Đorđe Karađorđević, he was not elected. Slobodan Jovanović was elected instead.

Jovan Karamata became a member of the Serbian Royal Academy in 1939. He helped a lot his teacher Mihailo Petrović in preparing scientific publications. Owing to this and the great wish of Mihailo Petrović to bring his papers and legacy in order, we have today a plenty of information about his life and work.

NOVI SAD SCHOOL OF MATHEMATICAL ANALYSIS

Within the Programme Board for Marking the Jubilee, we agreed to prepare texts that connect academician Mihailo Petrović with today's situation of mathematics in Serbia and the areas that we belong to. The text that follows elaborates on the development of mathematical analysis in Novi Sad, functional analysis concerning partial differential equations and harmony analysis through the theory of generalised functions. The genealogical tree, through academicians Karamata and Avakumović, reaches academician Bogoljub Stanković, going to the contemporary research and the scope of mathematical analysis in Novi Sad.

As already stated, academician Jovan Karamata is our most famous and recognised scientist in the world of mathematics among students of academician Petrović. Introducing into mathematics the class of slowly varying functions, he earned a place in the history of world mathematics. This class of functions naturally lies between the class of constants and the class of polynomial functions with equations. It enabled him to achieve a number of brilliant results in the field of function asymptotic in the theorems of Abel and Tauber type for various integral transformations. The leading scientists in classical mathematical analysis in the first quarter of the 20th century, Hardy and Littlewood, were impressed with the elegance of his proof of the Tauber theorem, named after him the *Karamata Tauberian theorem*. Academician Vojislav Avakumović fit into the field developed by academician Karamata. In the already significantly developed theory of regularly varying functions, he defined new classes with appropriate Tauberian results, particularly in the estimate of the number of own values of elliptic operators. Their work was continued by brilliant mathematicians and pedagogues, students of Jovan Karamata – academicians Miodrag Tomić, Slobodan Aljančić and their students at the Faculty of Science and Mathematics. Academician Vojislav Marić used the class of slowly varying functions in the study of solutions of the Thomas-Fermi equation. Also, this class of functions was used by our mathematicians in America, Karamata's students Bojanić, Bajšanski and their successors.

The Novi Sad School of Mathematical Analysis was founded and led by recently deceased academician Bogoljub Stanković. He introduced modern mathematical analysis in Serbia through contemporary areas of functional analysis. Let us also mention that academician

Slobodan Aljančić wrote a high quality textbook on functional analysis at the Faculty of Science and Mathematics in Belgrade.

In the field of mathematical analysis, the 20th century was the period of functional analysis based on algebraic-analytic methods. In its integral part, it contains the theory of generalised functions. The text that follows contains names of important mathematicians, as well as years, in order to better determine the time of their scientific achievements and the development of scientific areas that we elaborate on.

The analysis of functional spaces began in the late 19th century with the works of Ascoli (1843–1896), Volterra (1860–1940), Arzelà (1847–1912), while the Lebesgue integral (in the dissertation from 1902) enabled a qualitatively new approach in all areas of analysis. The functional analysis in the works of Hilbert, Fredholm and Banach, was established in this context, through the synthesis of various areas of geometry, algebra and analysis. Worth mentioning are also Haar (1885–1933), Kolmogorov, Wiener (1894–1946), von Neumann (1903–1957). Particularly important for the development of functional analysis is the development of the modern topology of Borel, Fréchet (1878–1973), Hausdorff (1868–1942) and others.

Within functional analysis, the theory of generalised functions was developed, based on the duality theory. The first results of formal calculus with generalised functions in the solution of differential equations were found with Heaviside (1850–1925). Nobel prize winner Dirac (1902–1984) introduced (~1925) into mathematical physics the calculus with brackets, and Sobolev (1908–1986) introduced (~1930) the concept of the weak derivative in the examination of weak solutions of hyperbolic systems.

L. Schwartz (1920–2003) created (~1950) the theory of distributions, contributing to the development of functional analysis. He also published a monograph which is still today used at post-graduate studies in the theory of linear partial differential equations. Their theories gave an impulse to the theory of pseudo-differential and Fourier integral operators developed by Calderon, Sigmund, and particularly Hörmander, Gelfand, Stein, Bony, and a number of other, mainly European mathematicians. Another approach to the theory of generalised functions, based on the theory of complex functions of several variables and the cohomology theory, was introduced by Sato and his students Kawai and Kashiwara. In this context, we should mention Komatsu, who formulated the theory of ultradistributions (~1970) and Colombo who introduced (~1980) the nonlinear theory of generalised functions, with the aim of studying nonlinear problems.

After his stay in Paris, where he taught as a *maître de conférences* and attended lectures of great mathematicians, Bogoljub Stanković became aware of the importance of functional analysis. This was a period of somewhat older mathematicians than him – Dieudonné, Schwartz and later their students (J. L.) Lions and Grothendieck. Together with Mikusiński, based on algebraic methods in the theory of equations, they developed the theory of *Mikusiński's operator*. An exceptionally important aspect of the activity of academician Stanković was his work with young people, whom he familiarised with modern trends of analysis, with application in solving partial differential equations. The theories of distributions, ultradistributions and hyperfunctions developed in the second half of the 20th century, to which he gave a personal contribution, became a scientific language of an entire group of mathematicians making up the Novi Sad School of Mathematical Analysis, developed through the seminar held almost continuously over the past 60 years on Mondays, at noon at the Novi Sad Faculty of Natural Sciences. Distinguished world mathematicians were guests at the seminar. Students of academician Stanković presented their first scientific results there. In addition, as one of the first associates of the Mathematical Institute, after academician Aljančić, he led the Mathematics Department of the Mathematical Institute. The Novi Sad seminar is thematically primarily devoted to functional analysis, while the seminar of the Mathematical Institute was of a general nature, where our most important mathematicians taught, as well as world-renowned foreign mathematicians, such as Laurent Schwartz.

One of the first students of academician Stanković was academician Olga Hadžić. Her work on the fixed point theory and contribution to leading the journal *NSJOM* and international verification of scientific results of younger associates was exceptionally important.

Academician Stanković's cooperation with eminent world mathematicians Mikusiński, Vladimirov and others resulted in conferences under the general name of *Generalised Functions*. They served as an important impetus first to the structural analysis of various spaces of generalised functions and general integral transforms, and as of recently to application in the theory of partial differential equations and the microlocal analysis with the theory of pseudo-differential and Fourier integral operators. The International Association for Generalised Functions was set up, seated in Vienna, with mathematicians from Novi Sad playing a highly active role in its work. The majority of conferences on generalised functions were held in Novi Sad.

The Novi Sad group applied the strong theorems of functional analysis, particularly the theory of measures and various spaces of generalised functions, to solving equations with singularities to which classical mathematics does not have answers. Particularly important are contacts with scientific groups at universities in Vienna and Torino. Owing to cooperation of the author of this article and one of his first students Marko Nedeljkov (not to mention others) with H. Komatsu, F. Colombo, L. Rodin and M. Oberguggenberger and a number of brilliant mathematicians, new areas became the framework of the scientific work of the Novi Sad group.

Mihailo Petrović and Jovan Karamata

AREAS OF SCIENTIFIC RESEARCH OF THE NOVI SAD SCHOOL OF MATHEMATICS

In addition to the structural results of Avakumović, Marić, Tomić, Aljančić and their Belgrade followers, Karamata's theory of regularly varying functions gained in the works of the Novi Sad School its full meaning through the development of generalised asymptotics and Tauber-type theorems in the spaces of distributions, ultradistributions and hyperfunctions. The microlocal analysis of pseudo-differential and Fourier integral operators, including applications in the analysis of the spread of singularities through the wavefront, hypoellipticity and time-frequency analysis, are the dominant research topics. The framework and wavelet theory gives a clear perspective of applications in the analysis of signals. Research areas are maintenance laws and singular solutions of the so-called gradient catastrophe, with solutions containing distributions, as well as the fluid dynamics. Furthermore, the equations of evolution with the asymptotic of hypercyclical and chaotic orbits of semigroups, *H*-measures or microlocal defect measures and their natural generalisations of *H*-distributions and ultradistributions describe, for instance, the homogenisation of particular structures. The stochastic analysis and stochastic equations with uncontrolled noise (the so-called white noise), in initial conditions, i.e. stochastic perturbations of the ideal situation in a particular environment, are studied through stochastic differential equations within the Malliavin calculus and chaos expansions. Fractional differential equations with applications in the examination of models with viscoelastic materials, with the use of the distribution theory method, are also an important research area within the *Novi Sad School of Analysis.*

THE SERBIAN SCHOOL OF MATHEMATICS – FROM MIHAILO PETROVIĆ TO THE SHANGHAI LIST

Gradimir V. MILOVANOVIĆ[1], Miodrag MATELJEVIĆ[2], Miloljub ALBIJANIĆ[3]*
[1]*Serbian Academy of Sciences and Arts*
[2]*University of Belgrade, Faculty of Mathematics*
[3]*Metropolitan University, Faculty FEFA*

In 19th-century Serbia, there were six doctors of mathematical sciences. One of them was Mihailo Petrović[34], the founder of Belgrade School of Mathematics. Petrović and his successors–disciples contributed to the development of mathematical education in Belgrade, in towns and cities of the former Yugoslavia: Skopje, Sarajevo, Banja Luka, Zagreb, Podgorica, and particularly in university centres in Serbia: Novi Sad, Niš and Kragujevac. They contributed to the development of mathematics, each in his own way: through scientific work, education of young generations, excellent teaching, writing of textbooks, etc. In this text, we will try to give an overview of their respective contributions to this development, in different fields of work. When it comes to scientific publications, we have relied on the sources at *Zentralblatt MATH* (zbMATH)[35], *ZBL* in further text, for researchers belonging to the earlier period, as it also includes the period before World War II, and *Mathematical Reviews*

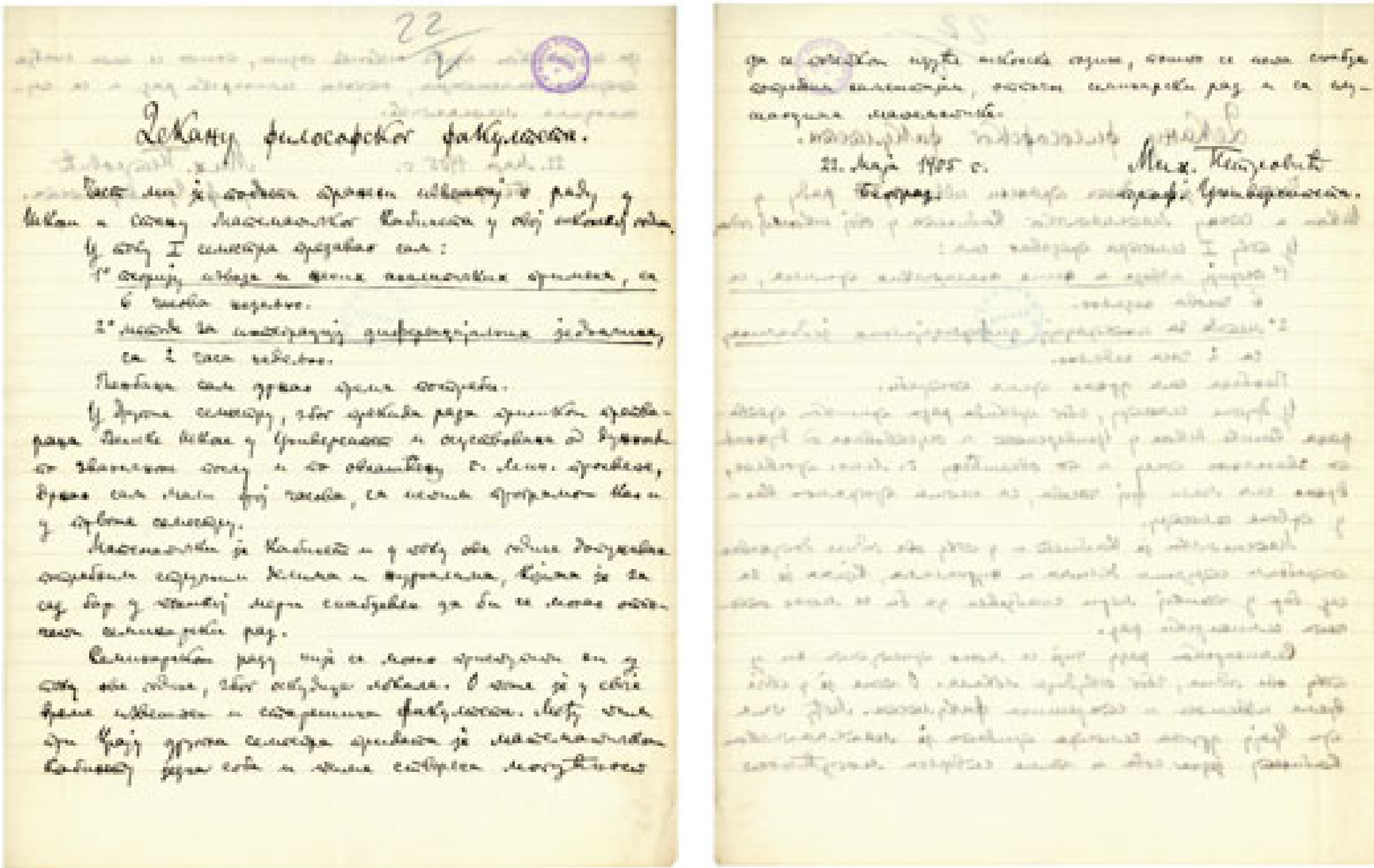
* The authors are part of the vertical line of the Belgrade School of Mathematics. Gradimir Milovanović obtained his PhD under the supervision of Dragoslav Mitrinović, and Mitrinović – under the supervision of Mihailo Petrović; Miodrag Mateljević obtained his PhD under the supervision of Vojin Dajović, Dajović – under the supervision of Tadija Pejović, and Pejović – under the supervision of Mihailo Petrović. Miloljub Albijanić obtained his PhD under the supervision of Miodrag Mateljević, Gradimir Milovanović and Miloš Arsenović.
Mathematicians' pictures were drawn by artist Bajo Luković.

(MathSciNet)[36], *MR* in further text, the database of the American Mathematical Society for more recent researchers.

There are 228 Petrović's publications, including 12 books (search under "ai:petrovitch. michel"), in the German referential mathematical journal *Zentralblatt MATH,* which is now edited by the *European Mathematical Society* and the *Heidelberg Academy of Sciences,* and the database of which incorporates, as of 2003, the entries of an earlier similar publication *Jahrbuch über die Fortschritte der Mathematik* (JFM) which existed from 1868 to 1942. His papers referred primarily to ordinary differential equations, complex variable functions, as well as sequences, series and summability. His associates continued to work in these fields of study, but they were evidently encouraged to work on the development of other areas of mathematics as well. This progress spread through the entire territory of former Yugoslavia and outside its borders on the one side, while on the other side it expanded through the introduction of new disciplines emerging in the world. A reverse process was also present, as mathematicians from the other places came to Belgrade, enriching our school of mathematics and adding to the existing knowledge. This refers primarily to the arrival of Anton Bilimović and Nikola Saltikov from Russia and later of Đuro Kurepa from Zagreb. Many of their successors are also to be given credit for the development of the Serbian school of mathematics, but here we will focus only on those who are genealogically connected with Mihailo Petrović.

In addition to choosing their area of work, associates were also given the opportunity to travel abroad in order to expand their knowledge and their views. These visits helped spreading mathematical ideas in the other countries as well. Mihailo Petrović was versatile with regard to his scientific and pedagogical work, establishing of journals, activities in the Serbian Royal Academy, etc. He also enjoyed some other activities such as music, fishing, travelling, writing travelogues and novels, etc.

As we have already said, the Serbian school of mathematics made an important impact in former Yugoslavia and the rest of the world. Since mathematicians gather around these ideas, the expansion of this school has continued to the present day through participation in the work of seminars, conferences and mathematical journals, such as today's *FILOMAT, AADM* and *MATCH Communications in Mathematical and in Computer Chemistry* (*SCI* list journals), as well as other *ESCI* list journals, etc. In recent years, our state universities have improved their ranking in the area of mathematics and they are now among the top 500 in the Shanghai list for mathematics, which is clear evidence of the progress of mathematical sciences in Serbia.



Декану Философског факултета.

22. Маја 1905 г.

Мих. Петровић

Mihailo Petrović's report on the work of Cabinet of Mathematics in school year of 1904/1905, submitted to Dean of Faculty of Philosophy, 22 May 1905

ROAD TO THE GRAND SCHOOL

Mihailo Petrović was born on 24 April 1868 in the vicinity of Princess Ljubica's Residence and the Cathedral Church, in the Kosančićev Venac neighbourhood on the Sava slope descending to the confluence of the two rivers. When he started elementary school, his father Nikodim died. His mother Milica and her father, Novica Lazarević,[37] took over care of his upbringing and education. Petrović respected his mother's father and preserved a fond memory of him. He often retold his grandfather's sayings and quips, and it is from him that he inherited his love for the ordinary people of the Savamala and Dorćol neighbourhood [Trifunović, 1969, 26].

After finishing elementary school, Mihailo Petrović enrolled in the First Belgrade Gymnasium, then located in the courtyard building of Captain Miša's Edifice. On the occasion of celebrating the 100-year anniversary of the Gymnasium, Petrović said: "If one of the alumni of the First Belgrade Gymnasium told you that he had spent 55 years in Captain Miša's Edifice, without changing during all that time either his vocation or his profession, you would look at that person with incredulity, wondering if that was even possible. However, not only is it possible, but it was actually the case for the writer of these lines himself, who entered this building at the beginning of school year 1878/79 as first class student of the Gymnasium and left it in school year 1937/38 as retired full-time professor of the University of Belgrade, with only one intermission during his education abroad, after finishing the Grand School which was located in this building, too [Trifunović, 1969, 32]."

In his book, *Chronicles of Life and Work of Mihailo Petrović,* Dragan Trifunović vividly conveys the impressions about education in the Gymnasium of the time. With his friend Pavle Popović, Petrović often commented on the literary abilities of the literature teacher, Andra Nikolić, presented to students by the then Minister of Education, Stojan Novaković. Interestingly, at a later time, Andra Nikolić and Mihailo Petrović were elected the members of the Serbian Royal Academy on the same day, and in 1905, when Petrović was appointed full professor at the University, Andra Nikolić was the Minister of Education. Petrović also had a particular liking for his chemistry teacher, Marko Leko. He studied this subject from a textbook written by Sima Lozanić for Grand School students. His teacher of mathematics was Sreta Stojković, who, as students used to say, was *mathematician by profession, poet by soul.* Mihailo Petrović's friends from the Gymnasium days included Paja Marinković, Jovan Cvijić, Jaša Prodanović, Vladislav Ribnikar, Ljuba Jovanović, and the others. They grew to become a generation that spearheaded the progress of science in Serbia in the decades to come.

Mihailo Petrović enrolled in the Grand School in 1885. A group of subjects was taught by professor Dimitrije Nešić (1836–1904), who heralded the dawn of Serbian mathematics. He was a teaching assistant at the Prince's Lyceum, long-time professor of the Grand School, the first member of the Serbian Learned Society. In 1887, he was elected among the first 16 full members of the Serbian Royal Academy. At that time, Josif Pančić, Dimitrije Nešić, Ljubomir Klerić and Jovan Žujović were in the Academy of Natural Sciences.

Professor Nešić's personality was such that he was able to convey to his students love for the subject; he was known for clarity of exposition; he directed his students' attention and taught them to be able to tell the important from the unimportant; he identified himself with the science he was teaching [Trifunović, 1996, 19]. Nešić began his studies at the Lyceum in Belgrade, then continued his education at the Vienna Institute of Technology and finally at the Polytechnic Institute in Karlsruhe. He was a true devotee of the temple of education, a humane, noble man, a man of *angelic soul.* He was considered to be an ideal man.

In 1873, the Faculty of Philosophy was divided into two departments: Department of Philology and History and the Department of Sciences and Mathematics. As of 1880, under the Law of the Constitution of the Grand School, it was determined that studies at the Faculty of Philosophy should last for four years. New subjects were introduced and the curriculum was expanded. In 1887, Bogdan Gavrilović, a young doctor of mathematical sciences was appointed at the Grand School. From this appointment until the end of his life (1947) and, from 1894 onwards together with Mihailo Petrović, he will, in a quiet manner of a scientist and an outstanding organiser, build our higher education and produce several invaluable contributions to the mathematical science.

PARIS SCHOOL OF MATHEMATICS

At the Grand School, Petrović received a general education in natural sciences. He had no speciality, but he did demonstrate an affinity for mathematics. In addition to mathematics, he was also an exceptional student of chemistry, taught by professor Sima Lozanić, mechanics, taught by professor Ljubomir Klerić, and history, taught by professor Srećković. The young Petrović was therefore well prepared for Paris. Paris was, at the time, the centre of scientific Europe. It was the hub of scientific and technological innovation. The mathematical school was particularly strong. A mathematician of this school emanated his universality. Such were, for instance, Henri Poincaré, Picard, Painlevé, Hermite, Darboux and the others.[38]

Thanks to Novica Lazarević, Mihailo Petrović continued his studies in Paris. He saw his grandson off with the following words: "I will see with the government that you get a state scholarship, your job meanwhile is to study." After an application by the minister of education to the French minister of foreign affairs, the preparation and sitting of the demanding entrance examination, Petrović continued his studies at *L'Ecole Normale Supérieure*.

After Dimitrije Danić and Bogdan Gavrilović, in 1893 the scientific environment in Belgrade was enriched by a new doctor of mathematical sciences, Đorđe Petković, who obtained his PhD in Vienna. The Serbian Royal Academy of the time had three mathematicians in its ranks: Dimitrije Nešić and Ljubomir Klerić were full members, while Petar Živković, the headmaster of the Gymnasium, was a corresponding member. Mihailo Petrović defended his doctoral dissertation *On Zeroes and Infinities of the Integral of Algebraic Differential Equations*[39] in Paris in 1894, before a commission made up of renowned professors and leading mathematicians of the time, Charles Hermite, Émille Picard and Paul Painlevé. "This young

Advanced Pedagogical College,
(l'École normale supérieure), Paris

Henri Poincaré

ace from Savamala and Dorćol, a masterful mathematician at the Grand School, the holder of two Orders of St. Sava, the author of appreciated student discussions, lived up to everyone's expectations. In Paris, he was ranked the best together with three other students of the school. Novica's grandson, his success in Paris, his doctoral title and other degrees in mathematical sciences were the talk of the entire town of Belgrade [Trifunović, 1969, 126]."

Mihailo Petrović was one of the best doctoral students of his generation in Paris. Emile Picard included his result on particular uniform integrals of differential equations into his textbook on analysis.

Belgrade Grand School (University of Belgrade) in 1907 (Belgrade City Library, fund F-1–191, inventory number 1593)

MIHAILO PETROVIĆ'S BELGRADE SCHOOL OF MATHEMATICS

In 19th-century Serbia, there were six doctors of mathematical sciences. They were **Dimitrije Danić** (1862–1932), who obtained his PhD in 1885 in Jena, **Bogdan Gavrilović** (1864–1947), who obtained his PhD in 1887 in Budapest, **Vladimir Varićak** (1865–1942), who obtained his PhD in 1891 in Zagreb, **Đorđe Petković** (1862–1934), who obtained his PhD in 1893 in Vienna, **Mihailo Petrović** (1868–1943), who obtained his PhD in 1894 in Paris and **Petar Vukićević** (1862–1941), who obtained his PhD in 1894 in Berlin [Kečkić, 1985, 3–6].

After professor Dimitrije Nešić left to assume state administration duties and his retirement in 1894, a vacancy for a professor of mathematics was announced at the Grand School. Competition was very strong: Dr Mihailo Petrović got 11 votes of the Academic Council of the Grand School, Dr Petar Vukićević – 10 votes and Dr Đorđe Petković – one vote. Mihailo Petrović succeeded Dimitrije Nešić as a professor of mathematics. His lectures were easy to understand, as he tried to maintain a level that was accessible to the listeners. He encouraged independent work in those who wished to expand their knowledge. He was direct, modest and had a cheerful spirit. He managed to bring the harmony of his spiritual qualities into everyday life. He considered scientific work to be the foremost duty of a university professor, as without science there could be no success in teaching and no progress at all.

Mihailo Petrović

He was elected corresponding member of the Serbian Royal Academy in 1897, at the proposal of his professors, now colleagues at the Grand School, Dimitrije Nešić, Sima Lozanić, Jovan Žujović and Ljubomir Klerić. At the proposal of the same group of academicians, he became full member of the Serbian Royal Academy in 1899. In the same period, he was elected member of several foreign scientific societies, and had already published an imposing number of scientific papers.

An important date in the development of education in Serbia was 19 February 1905, when University Law was enacted. The efforts of Grand School professors had finally borne fruit. Serbia got a University. At the proposal of the education minister, by a decree of the King of Serbia, the first eight full professors of University were appointed on 27 February: Ljubomir Jovanović, Dragoljub Pavlović, Milić Radovanović, Jovan Žujović, Andra Stevanović, Jovan Cvijić, Mihailo Petrović and Sima Lozanić.[40]

In 1909, at the proposal of Mihailo Petrović and Jovan Cvijić, the University of Belgrade invited Milutin Milanković, the civil engineer from Vienna, to work as professor of applied mathematics at the Faculty of Philosophy. As a result, Petrović and Milutin Milanković shared not only their office, but also the universal world of mathematics. Petrović taught theoretical mathematics and Milanković – applied mathematics. In 1920, Milanković was elected corresponding, and in 1924, full member of the Serbian Royal Academy.

In 1912, Mladen Berić defended the first doctoral dissertation in mathematical sciences at the University of Belgrade, before a commission made up of Mihailo Petrović and Milutin Milanković. A year later, Sima M. Marković defended his doctoral dissertation before the same commission. The letter which Petrović thereafter sent to the Council of the Faculty of Philosophy, as justification for appointing another lecturer to teach theoretical mathematics, represents the cornerstone of future scientific work at the University and the start of creation of the mathematical school.

Parts of this letter outline actual intentions to develop mathematical sciences: “By working alone, the teacher could only teach the indispensable elements, without having the opportunity to move on and introduce to listeners some of the branches of Mathematics that are developing today, that containing topical problems, that are, therefore, the most fitted for independent work, and that cannot be embarked upon until elementary things are mastered. However, it is precisely in these areas of Mathematics that we now have no time to embark upon that the system of today's scientific work is reflected and it would be of utmost benefit to introduce listeners into independent work on this terrain precisely, which in a science like Mathematics, where each part is the foundation for the next, is impossible to realize with the programme of work which has so far been applied."

After World War I, there was growing demand for teaching staff as the number of young people attending schools and of university students increased. Petrović dedicated his attention to this issue as well, aware as he was that a teacher of mathematics could, through his approach, either help students acquire love for the subject or turn it into a genuine "scare". For this reason, he advocated in the Council that it would be very useful for teachers to be well grounded in mathematical didactics and the methodology of teaching mathematics. He sent letters and requests to the authorities asking for an increase in the number of University professors in order to enable the spreading of scientific knowledge but also the introduction of pedagogical sciences to students so as to improve the quality of teaching. The lectures would thereby become clearer, more simplified and delivered in a systematized manner. This contributed to a greater interest in mathematics.

The period from 1921 onwards was significant for the development of mathematics at the University of Belgrade as two distinguished names of the mathematical science arrived in Belgrade: Anton Bilimović and Nikola N. Saltikov. There was also the younger generation: Vjačeslav Žardecki, Tadija Pejović, who obtained his PhD in 1923, and Radivoje Kašanin, who obtained his PhD in 1924 (both before a commission comprising Petrović and Bilimović), Jovan Karamata, who obtained his PhD in 1926 (the commission comprised Petrović, Bilimović, Saltikov), Miloš Radojičić, who obtained his PhD in 1928 before a commission comprising Petrović and Saltikov.

The year 1931 marked the start of publication of the journal *Matematički list za srednju školu (Mathematical Journal for Secondary School Students)* in Belgrade, edited by professors Radivoje Kašanin, Vojislav V. Mišković and Jovan Karamata. In the same year, together with his colleagues at the Faculty of Philosophy, Mihailo Petrović set up the famous mathematical journal entitled *Publications Mathématiques de l'Université de Belgrade*. Dragoslav S. Mitrinović obtained his PhD in 1933, before a commission comprising Petrović, Saltikov and Pejović, while Danilo Mihnjević and Konstantin Orlov obtained their PhDs before the same commission a year later. Petar Muzen obtained his PhD before the same commission in 1937, and Dragoljub Marković – in 1938.

Radivoje Kašanin and Jovan Karamata, both doctoral students of Mihailo Petrović, later became full members of the SASA, while Miloš Radojčić was a corresponding member. From the table above, it is evident that Mihailo Petrović's disciples for the most part wrote their doctoral dissertations in the area of differential equations.[41] Through their teaching and scientific work at the University of Belgrade, as well as their activities at the Seminar set up by Petrović and the publication of scientific papers in Belgrade and mathematical journals worldwide, this group of mathematicians, Petrović's students, made up the core of the mathematical school called the Belgrade School of Mathematics.

	Name and surname	year	Name of doctoral dissertation	Commission
1.	Mladen Berić	1912	Figurative Polygons of First-order Differential Equations and their Relation with the Properties of Integrals	Mihailo Petrović Milutin Milanković
2.	Sima Marković	1913	General Riccati Equation of the First Order	Mihailo Petrović Milutin Milanković
3.	Tadija Pejović (17 doctoral students)	1923	New Cases of Integrability of an Important Differential Equation of the First Order	Mihailo Petrović Anton Bilimović
4.	Radivoje Kašanin (two doctoral students)	1924	On Analytic Forms of Multiform Functions	Mihailo Petrović Anton Bilimović
5.	Jovan Karamata (12 doctoral students)	1926	On Certain Limits Similar to Definite Integrals	Mihailo Petrović Anton Bilimović Nikola Saltikov
6.	Miloš Radojčić	1928	Analytic Functions Expressed in Terms of Convergent Series of Algebraic Functions	Mihailo Petrović Nikola Saltikov
7.	Dragoslav Mitrinović (33 doctoral students)	1933	Investigations of an Important Differential Equation of the First Order	Mihailo Petrović Nikola Saltikov Tadija Pejović
8.	Danilo Mihnjević	1934	Structure of Partial Equations with Given Characteristic Integrals	Mihailo Petrović Nikola Saltikov Tadija Pejović
9.	Konstantin Orlov (nine doctoral students)	1934	Arithmetic and Analytic Applications of Mathematical Spectrums	Mihailo Petrović Nikola Saltikov Tadija Pejović
10.	Petar Muzen	1934	On Bases of Continuous Functions	Mihailo Petrović Nikola Saltikov Tadija Pejović
11.	Dragoljub Marković (one doctoral student)	1938	Limits of Roots of Algebraic Equations	Mihailo Petrović Nikola Saltikov Tadija Pejović

Mathematicians who obtained their PhD under the supervision of Mihailo Petrović

When Mihailo Petrović was awarded the title of honorary doctor of sciences of the University of Belgrade in 1939, he was referred to as the "engineer of mathematical sciences in our country, while the degree he received specified that his major achievement was the *Belgrade School of Mathematics,* whereby he was duly credited for his outstanding scientific work in all areas of mathematical sciences and the creation of a school of mathematics at the University of Belgrade [Trifunović, 1969, 412]."

The focus of Mihailo Petrović's work were differential equations and the function theory. It is interesting to note that the bulk of Mihailo Petrović's papers were published in Paris, but also that his papers were presented at the Paris Academy by great mathematicians of the time. Such approach to Petrović mirrored the approach of these great men of science to work in general. It was in fact an ode to the act of creation. Or as Newton said: "Hard work and dedication to learning are the highest hopes of humanity". Petrović applied the same style. He published and presented the papers of his doctoral students at the Serbian Royal Academy. This approach encouraged his students to improve and create further. Scientific work in Belgrade was therefore based on grand ethical postulates.

Petrović's qualitative analysis of solutions of differential equations was interesting and inspiring, particularly for that time. In this qualitative analysis, he did not look for the solution to the differential equation, which at the times can or cannot be found, while the quest for it is very complicated, but he rather tried to provide as much information as possible about the nature of the solution based on the properties of the equation itself.

Mihailo Petrović investigated the Riccati differential equation $y' = y^2 + f(x)$ and its generalization $(y')^2 = y^2 + f(x)$. Inspired by the papers of Mihailo Petrović, at a later time Sima Marković (also in his doctoral dissertation), Tadija Pejović (in his doctoral dissertation and a number of papers), Dragoslav Mitrinović (in his doctoral dissertation and in around 25 papers) and Milorad Bertolino (in about ten papers) studied the above differential equations. In Petrović's papers, the qualitative analysis of solutions to differential equations also included the so-called qualitative first integral, the nature of the solution and the properties, the number of zeroes and their distance, etc. He also ventured into the theory of numbers, the theory of polynomials, complex analysis, applications in numerical mathematics or chemistry.

We will quote an exceptional description of mathematics and Mihailo Petrović's place in it from his *Collected Works,* published by the Institute for Textbook Publishing: "Mathematics is a strange world in our universe. It is not known who its creator is – God or man. Therefore, it is not known whether a mathematician discovers the entities and their mutual relations in it or whether he himself creates them. Either way, it is both wonderful and exciting to be in that world especially to those who know how to discover it and enjoy in it. Mihailo Petrović spent many beautiful hours of his rich and diverse life in the mathematical world, discovering or creating, and he left a part of himself in it. This world is unique. Its classification into individual areas, which often overlap, is more administrative in nature. Mathematician Petrović was versatile.

He trod his roads, in pursuit of his ideas and visions. His papers, like memories from these travels, cannot always be classified into a single area. What is more important are the ideas that he had and the way in which he realised them [Aranđelović, 1999, 281]."

Petrović had a developed geometrical view on mathematics, which is particularly conductive to intuition or a sense of result. And whereas intuition can mislead, the element of creation is important for a creator. For instance, Petrović proved the following inequality:

$$\left|\sum_{k=1}^{n} a_k\right| \geq \cos\frac{\lambda}{2}\sum_{k=1}^{n}|a_k|$$

on condition that complex numbers $a_1, a_2, \dots, a_n$ are located in an angle with vortex at the beginning and measure $\lambda < \pi$, which is symmetrical with respect to the real axis.

Inequalities are one of the fundamentals of mathematical analysis. Well known is Petrović's inequality for convex functions at [0, a), $(a > 0)$ [Publ. Math. Univ. Belgrade 1 (1932), 149–156]

$$f(x_1) + f(x_2) + \cdots + f(x_n) \leq f(x_1 + x_2 + \cdots + x_n) + (n-1)f(0),$$

for which he is most often cited in literature. Even before, he considered this inequality to represent only a narrower class of functions (functions that can be represented by a power series with positive coefficients). Petrović's inequality has been used and generalised in different directions by a large number of mathematicians in the world!

Petrović also studied generalisations of some of Stiltjes formulas, but also many other issues relating to mathematical analysis. In the afterword to book 3 of *Collected Works* of Mihailo Petrović, a fine conclusion is drawn: "New mathematical structures are born out of a wish to cover everything that has been discovered so far; Petrović thus trod his own path, in pursuit of his own visions. He opened many new problems. He solved them impatiently, often not seeing them through. He generously left this for others to do. He was in constant search of a higher connection of mathematics and life, or at least a part of it. Because, finally, mathematics is life, too". [Aranđelović, 1999, 296].

In addition to differential equations, Mihailo Petrović also wrote some papers on the analytic function theory and inequalities, but his writings also included phenomenology, travelogues, etc. Like his professor Henri Poincaré, he wished to express his universality and thus be elevated to higher levels of knowledge. This is also the avenue for learning mathematics. Petrović considered metaphors and allegories subjective, but still belonging to a lawful form of human knowledge, spirit and consciousness. They had a deep sense and a deep root in human consciousness and corresponded to an instinctive and irresistible need of the spirit and the consciousness. They even tied themselves to certain facts and represented a specific expression of existence of these particulars. The methodology of natural sciences included, among other things, anticipation, analogical mapping, modelling and represented the development of thought in science, art and life in general.

A MAN OF INTELLECTUAL VIRTUE

This is how Milutin Milanković described his colleague Mihailo Petrović: "Petrović spiralled upwards to reach the peak of the pyramid of exact sciences already as a young man, when he left for Paris, in a bid to quench his thirst, tapping into the most abundant spring of mathematical knowledge, and then, imbued by the spirit of this science, he reached those boundaries of science where its new unsearched areas began. Already at the very start, he went past these boundaries with the doctoral dissertation which he defended in 1894 at Paris University before a commission comprising three most celebrated mathematicians of the time, Hermite, Picard and Painlevé. That year Mihailo Petrović returned to Belgrade as an accomplished and renowned scientist, and, appointed full professor at the Grand School, he also came to occupy the position of the greatest mathematician. This is the position he kept until his death – for one half of a century [Trifunović, 1969, 428]." Milanković spoke of Petrović's personality, of a man who received five Orders of St. Sava. Of how Petrović brought and sowed the seed of the mathematical science in Serbia and created there a sprouting ground of mathematical knowledge. Petrović's work was not limited to the education of secondary school teachers of mathematics only, but he managed to create scientists out of talented students and to prepare them for independent work.

"In terms of scientific work, he took precedence over us all. Since 1894, when his first study was published in the reports of the French Academy of Sciences, he published two and a half hundred scientific papers, of which twelve represent self-standing scientific works... He diligently compiled this treasure trove for the sake of the science, not for himself, because he never even thought of deriving from it any personal benefit, fame or celebrity. It was one of the most beautiful features of his character and of his entire work... He was a man brimming over with feelings, he knew how to enjoy everything beautiful that life had given to him. He loved company and music, his favourite sport (fishing) and travels... A tranquil, quiet, humanly simple, superhumanly gifted, Petrović was one of the greatest sons of our country [Trifunović, 1969, 429]."

Jovan Karamata also wrote about Petrović: "During his many years of fruitful work, Petrović touched upon almost all areas of mathematics, and though they differ significantly among them, he was guided by the procedures and considered them primarily from a viewpoint of mathematical analysis. [Trifunović, 1969, 362]." These papers cover the areas of algebra, arithmetic, integral calculus, theory of functions, differential equations, mathematical physics, chemistry and general phenomenology. Milan Bogdanović said: "The opus of Mr Mihailo Petrović is to the fullest possible extent contemporary, both with regard to its content and its spiritual orientation that goes hand in hand with both time and the spirit of time."

Tadija Pejović

DEVELOPMENT OF THE SERBIAN SCHOOL OF MATHEMATICS

Almost all doctoral students of Mihailo Petrović studied differential and functional equations, with the exception of Jovan Karamata, Dragoslav Mitrinović and Konstantin Orlov. Karamata studied sequences, series and summability, Mitrinović examined real, complex and special functions, and in particular inequalities, while Orlov engaged in numerical analysis of differential equations. This meant a step forward and the expansion of the mathematical school to include not only other people, but also other areas.

Some of the doctoral students achieved particularly noted results. One of the selected criteria is the number of further doctoral students, where Tadija Pejović, Jovan Karamata, Dragoslav Mitrinović and Konstantin Orlov stand out. Tadija Pejović was also exceptional for his textbook writing, Jovan Karamata wrote a particularly large number of scientific papers, earning him world fame, while Dragoslav Mitrinović stood out for the number of papers and the number of published books and Konstantin Orlov for his pedagogical qualities. All four of them together, and their successors, made an indisputable impact on the development of the Serbian School of Mathematics.

If we look at further development of mathematics along different branches, we will see that a number of areas developed in Serbia. For instance, *differential equations* were the object of study of doctoral students of Tadija Pejović, Dragoslav Mitrinović and Konstantin Orlov; *mathematical analysis* – of doctoral students of Dragoslav Mitrinović, Jovan Karamata and Tadija Pejović; *algebra* – of doctoral students of Tadija Pejović and Dragoslav Mitrinović; *numerical analysis* – of doctoral students of Dragoslav Mitrinović and Konstantin Orlov, etc.

The start of scientific and pedagogical work of Tadija Pejović[42] (Drača, 1892 – Belgrade, 1982) is marked by his doctoral dissertation *New Cases of Integrability of an Important Differential Equation*, which was completed in 1922 and defended on 6 February 1923 before a commission comprising Mihailo Petrović, Milutin Milanković, Anton Bilimović and Vladimir Petković (dean of the Faculty of Philosophy). The subject of the dissertation was the generalised Riccati differential equation $(y')^2+y^2 = H(x)$ which had even before been the subject of consideration by Apelle, Elliot, Mihailo Petrović, Liouville, and which was later studied in Belgrade by a number of mathematicians (D. Mitrinović, M. Bertolino, Lj. Protić). In his first scientific paper, Tadija Pejović also dealt with the problem of invariants of this same differential equation, which he will continue to examine in several later papers. When on 1 March 1948 the Society of Mathematicians and Physicists of Serbia was founded, Tadija Pejović was its first president (1948–1952). He taught at the Faculty of Philosophy and later at the Faculty of Sciences and Mathematics at the University of Belgrade. He was also the dean of the Faculty of Sciences and Mathematics.

In his pedagogical work, Tadija Pejović was particularly known for the publication of textbooks. He published *Mathematical Analysis* in five books and *Differential Equations* in three books, with the third book dedicated to the existence of solutions. The number of pages in the above textbooks is over two thousand, which is unusual in mathematical publications. All of the above books were reprinted, some for more than ten times (for instance, *Analysis I* was printed thirteen times).[43] He had 17 doctoral students including Vojin Dajović (1956), Ernest Stipanić (1957), Milorad Bertolino (1961), Nedeljko Parezanović (1962), Slaviša Prešić (1963), Zoran Ivković (1963), Milosav Marjanović (1964), Zoran Pop-Stojanović (1964), Petar Todorović (1964), Rade Dacić (1965) and Časlav Đaja (1967).

Vojin Dajović defended his doctoral dissertation *On the Existence of Limit Values of Some Classes of Analytic Functions* in 1956 at the Faculty of Sciences and Mathematics of the University of Belgrade, supervised by Tadija Pejović. After receiving his PhD, he developed **Complex Analysis** and continued the tradition of work with young and gifted students. According to MR, Dajović had 16 papers in the area of complex functions. He participated in the reform of the teaching of mathematics and physics at all levels and advocated a general promotion of mathematics as a fundamental science and one of the most important subjects at all levels of education.

Vojin Dajović wished to see a development of the impact of mathematics and mathematicians on the improvement of the educational system and the social reality at large. In particular, his contribution is reflected in his ability to recognise and foster gifted young mathematicians; he was the initiator of the idea and the founder of the Mathematical Gymnasium in Belgrade. He participated in the establishing of the Society of Mathematicians and Physicists of Serbia and in the founding of the Association of Mathematicians and Physicists of Yugoslavia. He successfully organised congresses of mathematicians. In particular, he advocated the introduction of the subject Methodology of Mathematics. At the Belgrade Faculty of Mathematics, doctoral studies of Methodology of Teaching Mathematics were introduced. Continuing Dajović's vision, the Faculty of Mathematics has organised symposiums entitled *Mathematics and Its Applications* each year since 2008.

Vojin Dajović had nine doctoral students, including Miodrag Mateljević with 82 publications (MR), Miroljub Jevtić with 79 papers (MR), Miloje Rajović with 54 papers (MR), Dušan Georgijević with 25 papers (MR) and Mioljub Nikić with 18 papers (MR). The most successful is Miodrag Mateljević, whose field of work is the geometric theory of functions and harmonic analysis and who has eight doctoral students including Vladimir Marković (the member of the

British Royal Society) with 43 papers (MR) and David Kalaj with 83 papers (MR), who works at the University of Montenegro. Mateljević chairs the Seminar for Mathematical Analysis which seeks to promote the geometric theory of functions.

Slaviša Prešić[44] obtained his PhD in 1963, with a doctoral dissertation entitled *A Contribution to the Theory of Algebraic Structures.* He published around 50 publications (MR). He began his scientific career with Dragoslav Mitrinović, with whom he wrote several joint papers in the area of difference and functional equations, for which he is recognisable. Slaviša Prešić was a very fruitful mathematician and he spearheaded further development of Algebra and Mathematical Logic in Serbia, but he also wrote papers in the other areas such as numerical analysis, geometry of polynomials and theoretical programming. He had 14 doctoral students who continued to work in these areas: Janez Ušan with 105 papers (MR), Žarko Mijajlović and Gradimir Vojvodić with 50 papers each (MR), Dragić Banković with 48 papers (MR), Svetozar Milić with 29 papers (MR), etc. With regard to the number of doctoral students, the most successful were Žarko Mijajlović with 12 and Svetozar Milić with seven doctoral students; their doctoral students for the most part continued and built their university careers in Novi Sad: Zoran Stojaković with 60 papers (MR) in Algebra and Combinatorics, Stojan Bogdanović with 149 papers (MR) in the Theory of Semigroups and Automata Theory, Siniša Crvenković with 64 papers (MR), Branimir Šešelja with 119 papers (MR) and Andreja Tepavčević with 89 papers (MR), published mostly in the area of Algebra and Mathematical Logic. Ušan and Vojvodić built their careers at the University of Novi Sad, and Banković at the University of Kragujevac. In late 1980s, Bogdanović moved to Niš where he organised a school in these areas. Among six of his doctoral students, the most successful is Miroslav Ćirić, now a professor at the Faculty of Sciences and Mathematics in Niš who has so far had nine doctoral students already and has published 127 papers (MR) in the area of Theory of Semigroups, Automata Theory and Theoretical Computer Science. The doctoral students of Žarko Mijajlović continued their university careers elsewhere – Slobodan Vujošević at the University of Montenegro in Podgorica, Miodrag Rašković at the University of Kragujevac, Milan Grulović at the University of Novi Sad, etc.

Zoran Ivković had 51 publications (MR) on probability theory and stochastic processes. He had 12 doctoral students, including Jovan Mališić with five doctoral students and Svetlana Janković with 52 papers (MR) in the area of probability theory, stochastic processes and differential equations. Among Mališić's doctoral students are Pavle Mladenović with 28 papers (MR) and 10 doctoral students, Tibor Pogány with 160 papers (MR) in the area of probability theory, stochastic processes and special functions (employed at the Rijeka University) and Biljana Popović with 31 papers (MR) and two doctoral students. Svetlana Janković and Biljana Popović work at the Faculty of Sciences and Mathematics in Niš.

Milosav Marjanović has 42 papers (MR) in the area of general topology, functional analysis, convex and discrete geometry and real analysis. He is the founder of topology in Serbia. He has also dealt with issues relating to the teaching of mathematics. He had eight doctoral students, including Rade Živaljević with 60 papers (MR) and Siniša Vrećica with 29 papers (MR). Žarko Mijajlović is also specified as one of the supervisors of Rade Živaljević.

Ernest Stipanić published 29 papers (MR). He studied the history of mathematics, summability of series and mathematical logic. Milorad Bertolino published 36 papers (MR) in the area of ordinary differential equations and the history of mathematics. Nedeljko Parezanović was engaged in the introduction of informatics and computing and had six doctoral students. Zoran Pop-Stojanović was at the University of Florida, USA, since 1965. He published 39 papers (MR) in the area of Probability and Stochastic Processes. From his retirement until his death (2011) he visited Serbia and organised stochastic seminars. Rade Dacić published 32 papers (MR) in the area of Algebra and General Topology. Časlav Đaja published 21 papers (MR). His doctoral student Miloš Čanak published 58 papers (MR) in the area of Complex Functions and Differential Equations.

Jovan Karamata

Jovan Karamata and Dragoslav Mitrinović made a special contribution to the development of **mathematical analysis** in Serbia. In 1926, Jovan Karamata (Zagreb, 1903 – Geneva, 1967) defended his dissertation *On Certain Limits Similar to Definite Integrals* before a commission made up of Mihailo Petrović, Anton Bilimović and Nikola Saltikov. He published 160 scientific papers (according to ZBL) in the following areas: Sequences, Series and Summability, Theory of Numbers, Fourier Analysis and other areas of mathematical analysis. Most joint papers he published together with Miodrag Tomić (6) and Bogdan Bajšanski (4).

Karamata's approach to the introduction of new subjects and the manner of their exposition was quite radical, which sometimes even made students protest. New scientific concepts and new theories are a difficult thing, and already accustomed opinion resists new ideas. At that time, not much discussion took place with regard to whether the faculty was a school for educating teachers or primarily a scientific institution. Karamata thought that science was the only goal of the faculty, so he attuned his lectures, even introductory ones, to this principle. Hence there is no wonder that the majority of students did not understand him. In order to have more time for contemplating mathematics and writing papers, he had the custom to hold all his lectures in a single day. A number of completely different courses: Elementary Algebra, Higher Algebra, Introduction to Analysis, Theory of Sequences and Series, and Descriptive Geometry – all in quick succession one after another.

During break time, Jovan Karamata's office was packed with students who brought in mathematical problems or asked for advice regarding their seminar papers. During his lectures, he also presented students with problems. Some were so difficult that at first students did not even understand them. There were students who spent days trying to solve some of the problems. Some dedicated all their work to interpreting his lectures. Others, by contrast, stopped attending his classes altogether. As he alone taught a large number of courses in the first year of studies, the number of those who fled mathematics after attending a few of his classes was certainly not small. He prepared some of his lectures in detail and there were moments when students left his classes with radiant faces. The first step when drafting a seminar paper was to study foreign sources, without which it was impossible to even conceive a seminar paper. Students thus began to see that it was not talent but hard work that mattered most.

Jovan Karamata achieved global fame in 1930, when he found a short proof of the Hardy-Littlewood theorem published in his paper *Über die Hardy-Littlewoodsche Umkehrungen des Abelschen Stätigkeitssatzes*. The paper had only two pages, but it caused perturbation in mathematical circles and brought his author immediate world fame. It is very interesting to read Vojislav Marić's testimony of this: "When I was visiting St. Andrews University in Scotland, I was introduced to the renowned mathematician Copson (E. T. Copson) from whose book many from my generation studied the theory of functions of a complex variable. He said: "I have so far heard of only one Yugoslav mathematician, Jovan Karamata. When I was studying under Hardy in the 1930s, I found him one day pacing nervously up and down his office. Without greeting me, and visibly excited, he said: *I got a letter from a young man from Belgrade who claims to have proven the Hardy-Littlewood theorem on two pages only. That is simply impossible*." This Karamata's paper not only brought a new, short and extremely elegant proof of the famous theorem, but also a new method that enabled many future results and applications.[45] Jovan Karamata set the cornerstone of the theory of regularly varying functions and was the author of several important Tauberian theorems. It was soon evident that these functions can successfully be applied in many branches of mathematical analysis and in the probability theory, wherever not only the fact of convergence itself is needed, but also other additional information.[46]

He was elected corresponding member of the Yugoslav Academy of Sciences and Arts in Zagreb, in 1933. The candidate report on Karamata was submitted on 20 February of the same year by academician Vladimir Varićak. In addition to a concise biography and the list of 37 papers published that far, the report also stated: "Though still young, our candidate, Dr Karamata, is already a well-known and reputable person in the mathematical world." He became a corresponding member of the Serbian Royal Academy in 1939, and a full member of the Department of Sciences and Mathematics of the SASA in 1948. Karamata continued his university career in Switzerland.

Jovan Karamata had 12 doctoral students, of whom nine in Belgrade, including Vojislav Avakumović, Miodrag Tomić, Slobodan Aljančić, Ranko Bojanić, Vladeta Vučković, Bogoljub Stanković, Bogdan Bajšanski and others, and three in Geneva, the most famous being Ronald Coifman, who is now a professor at Yale University. He has 170 papers (MR) published in the area of Fourier analysis and in a number of other areas, as well as over 30 doctoral students.

According to *ZBL* data, Vojislav Avakumović had 44 papers in the area of differential equations, operational calculus, sequences, series and summability, Fourier analysis and other areas of analysis. He had eight doctoral students, of whom six in Germany, including in particular Jochen Brüning with 107 papers (MR) and five doctoral students, Helmut Neunzert with 32 papers (MR) and 40 doctoral students, and Manojlo Maravić with 30 papers (MR) in the area of Fourier analysis, Summability of Series and Differential Equations. Maravić worked at the University of Sarajevo and had four doctoral students.

Miodrag Tomić published 72 papers (MR) in the area of Fourier analysis, Ordinary Differential Equations, Approximations, Special Functions and other areas of analysis. His papers on the geometry of polynomials have been greatly appreciated and highly cited.

Slobodan Aljančić defended his doctoral dissertation *On Asymptotic Expansion of A-Summable Linear Functionals* in the Serbian Academy of Sciences on 10 January 1953 before a commission made up of Jovan Karamata, Milutin Milanković, Vojislav V. Mišković, Radivoje Kašanin and Miodrag Tomić. He was elected corresponding member of the SASA in 1961, and full member of the SASA in 1968. He published 50 papers (MR), the majority in the area of Fourier analysis, sequences, series and summability and approximation theory, measure and integration theory, and special functions. In the 1957–1966 period, he published several auxiliary textbooks in the complex function theory, real functions, introduction to functional analysis and measures and integrations. He then published his most famous and influential textbook *Introduction to Real and Functional Analysis* (Belgrade, 1968), which was renewed in the next three editions. Aljančić's book, which is an excellent textbook from the pedagogical point of view as well, made an impact on the education of our mathematicians and represents a qualitative leap relative to standard courses in analysis. Slobodan Aljančić had 13 doctoral students, among whom Milan Tasković and Dušan Adamović were the most successful. Tasković published 92, and Adamović 32 papers (MR) in the area of mathematical analysis.

Ranko Bojanić and Bogdan Bajšanski continued their careers in the USA at Ohio State University, and Vladeta Vučković at the University of Notre Dame in Indiana, and they were very successful. Ranko Bojanić had 67 publications (MR) in the area of Approximation Theory, Fourier Analysis, Theory of Numbers, Partial Differential Equations, etc. One of his most successful doctoral students, and there were nine of them, is Ronald DeVore, who today is member of the American National Academy of Sciences and has 165 papers (MR) and seven doctoral students. Vladeta Vučković published 33 papers (MR) in the area of Mathematical Logic and Summability of Series. Bogdan Bajšanski published 25 papers (MR) in the area of Approximation Theory, Fourier Analysis and Summability of Series. He was more committed to teaching and had 11 doctoral students. In addition to Karamata, one of Bajšanski's supervisors was also Nikola Saltikov (Nikola Saltikov who in turn was the disciple of Vladimir A. Steklov and Aleksandr M. Lyapunov) who had five doctoral students, including Časlav Stanojević who continued his career in America. Stanojević made an outstanding contribution to the development of Serbian and Yugoslav mathematics by organising during the 1980s the famous *Kupari Conferences*, to which he gathered the most renowned global mathematicians of the time in the area of analysis, both from the East (Sergey Mikhailovich Nikolsky (Сергей Михайлович Никольский), Oleg Vladimirovich Besov (Олег Владимирович Бесов), Sergey Borisovich Stechkin (Сергей Борисович Стечкин), Boris Sergeevich Kashin (Борис Сергеевич Кашин), Sergey Aleksandrovich Telyakovsky (Сергей Александрович Теляковский...)) and from the West (Walter Rudin, Ronald A. DeVore, Richard Askey...), as well as our mathematicians, especially of the younger generation [Milovanović, 2013, 33–40].

The founder of Mathematical Analysis and Functional Analysis at the University of Novi Sad was Bogoljub Stanković. In addition to Karamata, Avakumović was also one of his supervisors. Stanković had ten doctoral students, including Olga Hadžić, Danica Nikolić-Despotović, Endre Pap, Stevan Pilipović, Dragoslav Herceg, Arpad Takači, Đurđica Takači, and others. Stanković published 163 scientific papers (MR), mostly in the area of Functional Analysis, Integral Transformations, Operational Calculus, Ordinary, Partial and Integral Equations, Complex and Special Functions, etc. He published the largest number of papers with Teodor Atanacković – 23 and Stevan Pilipović – 19.

Olga Hadžić published 160 papers (MR) mostly in the area of Operator Theory, General Topology, Differential Equations and Theory of Probability and Stochastic Processes, Danica Nikolić-Despotović published 34 papers (MR) in the area of Operational Calculus and Functional Analysis, while Edre Pap published 202 papers (MR), mostly in the area of Measures and Integrations and Functional Analysis, and had seven doctoral students. Olga Hadžić had four doctoral students, engaging primarily in General Topology, Operator Theory and Probability Theory and Stochastic Processes. They include: Mila Stojaković with 47 papers (MR), Ljiljana Gajić with 57 papers (MR) and Zoran D. Mitrović with 35 papers (MR), who works at the University of Banja Luka.

Stanković's most successful doctoral student and successor is Stevan Pilipović, who has so far published 352 papers (MR) in the area of Functional Analysis, Partial Differential Equations, Operator Theory and other areas of analysis, as well as several books and monographs. The famous publisher *John Wiley & Sons*, published in 2014 two monographs by T. M. Atanacković, S. Pilipović, B. Stanković and D. Zorica under a general title *Fractional calculus with applications in mechanics* and sub-titles *Wave propagation, impact and variational principles* and *Vibrations and diffusion processes*. Pilipović continued the tradition of work with young associates and he supervised 30 doctors of sciences. Dragoslav Herceg published 114 papers (MR) in the area of Numerical Analysis and had 10 doctoral students. Arpad Takači published 69 papers (MR) in the area of Functional Analysis, Integral Transformations, Operational Calculus and Differential Equations. Đurđica Takači had five doctoral students and published 48 papers (MR) in the area of Integral Transformations, Operational Calculus, Differential Equations and Numerical Analysis. Stevan Pilipović's doctoral students include: Mirjana Stojanović with 83 papers (MR), who unfortunately passed away prematurely, Marko Nedeljkov with 43 papers (MR) in the area of Partial Differential Equations, Mirko Kostić with 90 papers (MR) in the Operator Theory and Differential and Integral Equations, Nenad Teofanov with 37 papers (MR) in the area of Functional Analysis, Fourier Analysis and Operator Theory, etc.

The doctoral students of Dragoslav Herceg engaged in numerical analysis, linear algebra and optimisation: Ljiljana Cvetković has so far had 87 papers (MR) and three doctoral students, Nataša Krejić – 51 papers (MR) and seven doctoral students, Zorana Lužanin – 18 papers (MR) and four doctoral students, etc.

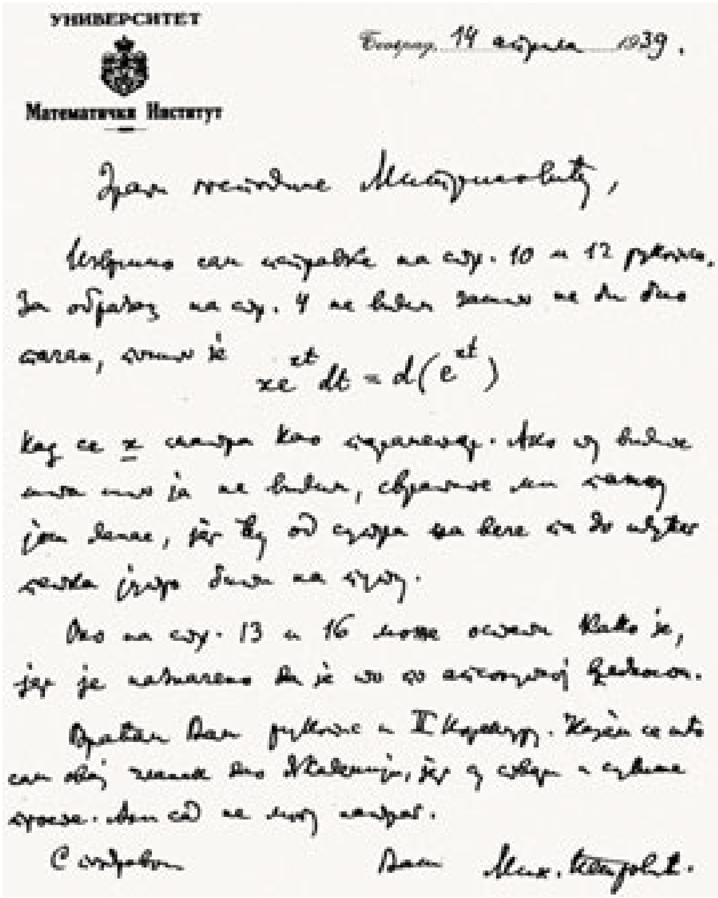
An exceptional student of Mihailo Petrović, Dragoslav Mitrinović (Smederevo, 1908 – Belgrade, 1995) obtained his PhD on 24 October 1933 in the area of differential equations, with a doctoral dissertation entitled *Investigations of an Important Differential Equation of the First Order,* before a commission made up of Mihailo Petrović, Nikola Saltikov and Tadija Pejović. From 1946 until his retirement in 1978, he worked as university professor in Skopje and Belgrade. He was the member of the Macedonian Academy of Sciences and Arts since 1991. The bibliography of Dragoslav Mitrinović has a total of 275 scientific papers and he has been cited 2310 times (MR). In addition, he published 30 other professional papers, 17 monographs, 35 textbooks and 12 other books. He published his papers in the journals at home and abroad, while his books and monographs were published by publishers in the country but also by the world famous publishers. He wrote mostly together with Pečarić, Vasić, Đoković and Kečkić. His main scientific areas included differential equations, functional equations, inequality theory, complex variable functions, special functions and a range of other areas of mathematical analysis.

Speaking of his prolific writing production, Radosav Đorđević accurately noted: "All this, without reprints, amounts to over 30,000 pages. The entire working life of Dragoslav Mitrinović lasted from 1931 until 1994, which is a total of 64 years, including 17 leap years. As this makes up close to 25,000 days, together with all religious and different state holidays, it is easy to conclude that Dragoslav Mitrinović, during 64 years, wrote on average more than one printed page per day, or even three if we take into account the reprinted editions. Or still more, if we exclude the five years of war [Milovanović, 2000a]."

He did not like to travel much, but he always returned from Paris satisfied because during his stay he had collected enough material for further research, for himself and for almost each one of his associates, in the libraries of the Institut Henri Poincaré and the École Normale Supérieure. By government order, after World War II, he was designated to set up the department of mathematics in Skopje. He was the supervisor of Blagoje Popov, the first doctor of sciences in Macedonia. He transferred there the school of differential equations which has been maintained to present day in Macedonia.

Upon his return to Belgrade, he became professor at the School of Electrical Engineering and head of the department of mathematics. He expanded the area of differential equations to include functional equations as well. Mitrinović set up three mathematical journals in Yugoslavia and initiated the publishing of several mathematical editions in the country and abroad. He established a school globally known as the Belgrade School of Functional Equations.

The most important Mitrinović's works were certainly on inequalities in the area of Mathematical Analysis. He considered many important classical inequalities including their generalisations. Especially, let us mention his work on the Steffensen inequality from 1969, as well as a joint paper with P.M. Vasić on an integral inequality ascribed to Wirtinger. In 1974, Mitrinović



УНИВЕРСИТЕТ

Математички Институт

Београд, 14 априла 1939.

Драги господине Митриновићу,

$$xe^{xt}\,dt = d(e^{xt})$$

С поздравом

Ваш Мих. Петровић.

A letter from Mihailo Petrović
to Dragoslav Mitrinović

and Vasić published one important paper about the history, variations and generalisations of the famous Chebyshev's inequality, and the question of some priorities relating to this important inequality. As far back as in 1965, Mitrinović published the book *Inequalities* on 240 pages; it was published by Naučna knjiga, Belgrade, as part of an edition entitled: "Mathematical Methods in Physics and Tehcnics." Five years later, in 1970, a grandiose work appeared – *Analytic Inequalities* published by *Springer Verlag* (Berlin – Heidelberg – New York).

Professor P. S. Bullen (University of British Columbia, Vancouver, Canada) wrote: "During his long and active life professor Mitrinovic not only did much original work in various fields, although mainly in inequalities. In addition he became famous for research into the obscure origins of many famous results. However his most abiding contribution are three. The famous book, done with the collaboration of professor Vasic, "Analytic Inequalities". It is, after the classic "Inequalities" by Hardy, Littlewood and P6lya, the most referred to book in the field of inequalities. He founded the journal *Publikacije Elektrotehnickog Fakulteta Univerziteta u*

Beogradu, serija Matematika i Fizika, an essential tool for working in the field of inequalities, and the almost complete run that I have is one of my most valuable possessions. Finally there are many students professor Mitrinovic brought along and who are now carrying on his work all over the world. It is no exaggeration to say that they are keeping him alive, and will continue to do so for many years to come."

The monograph *Analytic Inequalities* clearly had a very powerful impact both on the development of this area in our country and globally. It is, certainly, one of the most referred to mathematical books. Inequalities appear everywhere and have an important role in almost all areas of mathematics and other sciences. Dragoslav Mitrinović used to say that *equalities are rare and are almost always an exception, even in everyday life, whereas the inequalities are always met* [Milovanović, 2000b, p. 1–10]. The monograph *Topics in Polynomials: Extremal Problems, Inequalities, Zeros*, written by G.V. Milovanović, D. S. Mitrinović and Th. M. Rassias, and published by the famous publisher *World Scientific* (Singapore – New Jersey – London – Hong Kong), contains important results on the analysis of polynomials and their derivations.

Dragoslav Mitrinović organised a large-scale school of mathematics in Serbia and Macedonia. Under his supervision, 33 doctoral dissertations were written at the Universities in Skopje, Belgrade, Niš, Priština, Kragujevac and Sarajevo.

Blagoj Popov was his first doctoral student in 1952 in the area of Differential Equations and the first to defend a doctoral dissertation in Macedonia, not only in mathematics, but in any other area of science. Popov published 70 papers (MR) in the area of Special Functions, Differential Equations and Operational Calculus. Ivan Bandić was Mitrinović's second doctoral student (1958), also in the area of ordinary Differential Equations, while the third was Lazar Karadžić the same year. Bandić published 47 and Karadžić 30 papers (MR).

Mitrinović's first doctoral student in functional equations, Dragomir Đoković, was one of the most successful; he obtained his PhD in 1963, continued his career at the University of Waterloo and had seven doctoral students. He published, according to MR, 333 publications. His key areas were Combinatorics, Topological and Lie Groups and Algebra, Linear and Multilinear Algebra, Difference and Functional Equations, etc.

Petar M. Vasić obtained his PhD in the area of functional equations in 1963 and published 122 papers (MR) in the area of real functions, inequality theory, functional equations and special functions. Doctoral students in the same area included: Radosav Ž. Đorđević (1966) with 19 papers (MR), Radovan R. Janić (1968) with 53 papers and Ionel Stamate from Romania. Vasić's doctoral student Josip Pečarić, who later worked mostly with Mitrinović, left for Zagreb in late 1980s, where he founded the school of inequalities. He published a vast number of papers in the area of inequality (1193 papers according MR) and has so far had 18 doctoral students.

At the University of Skopje, under Mitrinović's supervision, Ilija Šapkarev obtained his PhD (1964) in the area of differential equations, Dragan Dimitrovski (1968) – in the area of Generalized Analytic Functions and Živko Madevski (1973) – in the area of Inequalities. Šapkarev published 45 (MR) and Dimitrovski 117 papers (MR) in the area of Differential Equations and Complex Variable Functions.

Jovan Kečkić obtained his PhD in 1970 and had 115 papers according to MR. He engaged in the area of ordinary and partial differential equations, difference and functional equations, complex variable functions, linear algebra, inequality theory, special functions, etc.

Dragoš Cvetković obtained his PhD in 1971 in the area of Graph Theory and is the founder of this area in our country. He has so far published 188 papers (MR) on Combinatorics, Graph Theory and Operational Research, including several books. He introduced the spectral graph theory and had three very successful doctoral students. Ivan Gutman works at the Faculty of Sciences and Mathematics in Kragujevac and this is his second doctorate, in addition to one in Chemistry. In the mathematical graph theory, Gutman published 536 papers (MR). Slobodan Simić has so far published 146 papers (MR), and Dragan Stevanović 112 papers (MR) and has already had six doctoral students. Three monographs by Cvetković, P. Rowlinson and Simić were published by the famous publishing house *Cambridge University Press*: *Eigenspaces of Graphs* (1997), *Spectral Generalizations of Line Graphs. On graphs with least eigenvalue-2* (2004) and *An Introduction to the Theory of Graph Spectra* (2010).

Mitrinović's doctoral students at the University of Niš include, among others, Ivan Lacković (1975) with 39 papers in the area of Real Analysis, Gradimir Milovanović (1976) with 292 papers (MR) in the area of Numerical Analysis, Approximation Theory, Special and Orthogonal Functions, and Real and Complex Analysis, Miomir Stanković (1979) with 57 papers (MR) in the area of Special Functions and Numerical Analysis, Miodrag Petković (1980) with 225 papers (MR) in the area of Numerical and Interval Analysis (iterative methods for zeroes of polynomials) and Igor Milovanović (1980) with 110 papers (MR) in the area of Real Functions, Inequality Theory, Graph Theory and Computer Sciences. Lacković's doctoral student at the University of Niš was Ljubiša Kocić (1980) with 78 papers (MR) in the area of Approximation Theory and Numerical Analysis.

Gradimir Milovanović has 13 doctoral students.[47] In addition to the above monograph *Topics in Polynomials: Extremal Problems, Inequalities, Zeros* (*World Scientific*, Singapore – New Jersey – London – Hong Kong, 1994), the famous *Springer* publishing house published the book *Basic Theory and Applications* he wrote with Giuseppe Mastroianni, while many generations of students throughout former Yugoslavia studied from his three-volume textbook *Numerical Analysis* (Naučna knjiga, Beograd, 1985). Milovanović's successful doctoral students include Predrag Stanimirović, with nine doctoral students and 166 papers (MR) in the area of linear algebra, optimization and computer sciences, Ljiljana Petković with 75 papers (MR), Miodrag Spalević with 59 papers (MR), Predrag Rajković with 53 papers (MR), Aleksandar Cvetković with 62 papers (MR) and Marija Stanić with 37 papers (MR) in the area of Numerical Analysis and Approximation Theory, Nenad Cakić with 34 papers (MR) and Gospava Đorđević with 28 papers (MR) in the area of Special Functions and the Theory of Numbers. Today, they are all professors at Universities in Niš, Belgrade and Kragujevac.

Miodrag Petković has so far had eight doctoral students, including Slobodan Tričković with 34 papers (MR) and Jovana Džunić with 25 papers (MR). Petković published a large number of books and monographs, including in particular: *Iterative methods for simultaneous*

Dragoslav S. Mitrinović

Konstantin Orlov

inclusion of polynomial zeros (Lect. Notes Math., Springer, 1989), *Complex interval arithmetic and its applications* (Wiley-VCH, 1998) (together with Lj. Petković), *Multipoint methods for solving nonlinear equations* (Elsevier, 2013) (together with B. Neta, Lj. Petković and J. Džunić).

Finally, the other three successful doctoral students of Mitrinović were: Dušan Slavić (1975) with 29 papers (MR) in Numerical Analysis, Petar Lazov (1977) with 49 papers (MR) in Ordinary Differential Equations and Vlajko Kocić (1981) with 63 papers (MR) in the area of Difference, Functional and Differential Equations, who now works in the USA.

Konstantin Orlov (Ufa, Russia, 1907 – Belgrade, 1985), obtained his PhD in the area of spectrum in 1934, before a commission comprising Mihailo Petrović, Nikola Saltikov and Tadija Pejović. He published, according to MR, 52 papers. His fields of work included Numerical Solution of Differential Equations, Ordinary and Partial Differential Equations, Spectrums and Numerical Analysis. He was also one of the first teachers of programming. At his lectures, he frequently expressed his view: "How much students learn is more important than how much they are taught." He published two monographs, *Finding a General Integral of Partial Equations of the Second Order, That are Not Monge-Amperes,* Serbian Academy of Sciences, Belgrade, 1948, and *Numerical Spectral Arithmetic and Algebra,* Society of Mathematicians of Serbia, 1973 [Zolić, 1998]. Konstantin Orlov had nine doctoral students. They included Mihail Arsenović, Petar Madić, Ljubomir Protić, and the most successful of them – Boško Jovanović with 12 doctoral students. He published 148 papers (MR) in the area of numerical analysis and partial differential equations. His most successful doctoral student is Endre Süli, professor at Oxford University. He published 160 papers (MR) and had 25 doctoral students. Boško Jovanović and Endre Süli published a noted monograph entitled *Analysis of finite difference schemes for linear partial differential equations with generalized solutions* (Springer, London, 2014).

As it is shown previously, Mihailo Petrović transmitted his knowledge to younger generations of mathematicians. He did not restrict their work only to those areas in which he himself worked, but enabled them to expand their scientific ideas further. Mihailo Petrović was the founder of the Serbian School of Mathematics and the teacher of an entire generation of our mathematicians. Though in late 19th century there were six doctors of sciences in Serbia, the development of our mathematical science began with the appointment of Mihailo Petrović at the Grand School. Scientific work was assigned special value, but also the assurance that it must be evaluated by European norms. At that time, Petrović brought experience and knowledge from Paris, the renowned centre of world mathematics where he received his PhD. The French school of mathematics was at the forefront of mathematical teaching of the time, not only because of its professors but also because of their disciples who spread the knowledge further on in the world. The transmission of this experience to Belgrade meant a lot for our country. This was happening at a time when the Grand School was being transformed into a University. The flourishing of new ideas in Belgrade led to a real progress of science.

It is clear from the above scientific and teaching activities that Petrović's school of mathematics expanded to include the entire Serbia, Macedonia, Zagreb, Sarajevo and some other centres. Many of his students left to work abroad – in America, Germany, Switzerland, etc. Hence this impact was allowed to spread beyond the borders of former Yugoslavia. All the centres set up fruitful, mutually beneficial cooperation with some developed centres abroad. Our mathematicians are often plenary speakers at important international conventions. Mathematicians from the other places came to Serbia, including Anton Bilimović and Nikola Saltikov from Russia and Đuro Kurepa from Zagreb. In Belgrade, Kurepa developed new areas of work. Our researchers, particularly young ones, travelled abroad for further education and acquired new knowledge. Some of them returned to the country and continued here their further scientific activities. A number of prolific mathematicians did not return, but they contributed to the expansion and development of mathematical ideas in global centres and have maintained ties and contacts with colleagues in Serbia.

Finally, it is important to highlight the role of the Mathematical Institute SASA which, throughout this period, and in the past three decades in particular, has, in its own specific way, taken care of the unique mathematical area of Serbia, bringing together mathematicians from all the centres to participate in joint projects grouped by scientific area. In addition to the traditional areas, new contemporary areas of work are introduced, with the involvement of primarily younger associates.

Finally, another novelty in the past years is the Doctoral School at the level of Serbia, endorsed by relevant state institutions from Novi Sad, Niš, Kragujevac, Belgrade and Novi Pazar, including the Mathematical Institute SASA.

The Shanghai list[48] is one of the criteria for assessing our mathematical school, which has evidently risen to a significant and respectable level globally. Four state universities in Serbia (in Belgrade, Novi Sad, Niš and Kragujevac) were included in the prestigious Shanghai list in the area of mathematics, published each year by the Jiao Tong University from Shanghai, and this is one of the most influential lists ranking the best universities in the world. The criteria taken into account include, among some other things, the number of winners of the Nobel Prize and of Fields medals among university alumni or staff, the number and quality of scientific papers. On the top of the list are Princeton, Paris, Stanford, Oxford, New York, MIT, Cambridge, etc. The above universities in Serbia are ranked among top 500 in the area of mathematics, and this success is based exclusively on the number and quality of scientific papers in referential international journals. This is the result of development of the Serbian School of Mathematics which, as has already been said in the Introduction, does not include only those who are genealogically connected with Mihailo Petrović, though their number is dominant.

In today's Serbia, there are mathematicians committed to teaching, to writing good mathematical textbooks, to producing prestigious scientific papers and publications. Their enthusiasm, their commitment and their work stand witness to future development of the Serbian School of Mathematics.

REFERENCES

[1] Miloljub Albijanić (2016), *Apstrakcija i primena matematičke analize*, Zavod za udžbenike, Beograd.

[2] Miloš Arsenović, Miloljub Albijanić, Miljan Knežević, Marek Svetlik (2014), *Miodrag Mateljević – vertikala Beogradske matematičke škole*, Simpozijum Matematika i primene, Matematički fakultet, Univerzitet u Beogradu, vol. V (1).

[3] Драгољуб Аранђеловић (1999), *Радови Михаила Петровића у анализи* у: Михаило Петровић, *Математичка анализа*, Сабрана дела, књ. 3, Завод за уџбенике и наставна средства, Београд.

[4] Неда Бокан и др. (ур.) (2012), *Српски математичари*, Зборник предавања одржаних на скупу "Српски математичари" у оквиру манифестације Мај месец математике.

[5] Раде Дацић, Миодраг Матељевић (2003), *Др Милош С. Радојчић (1903–1975)*, Живот и дело српских научника, књ. 9 (В. Ђорђевић, ур.), Српска академија наука и уметности, Београд.

[6] Jovan D. Kečkić (1985), *Serbian doctors of mathematics in the XIX century*, On the occasion of the 50th anniversary of Publications de l'Institut Mathematique, Nouvelle serie, t. 38 (52).

[7] Miodrag Mateljević (2014), *Fragmenti sećanja na kompleksnu analizu u Beogradu (1968–1980) i Vojina Dajovića – Izoperimetrijska nejednakost, Hardijevi prostori i Furijeovi redovi*, Zbornik radova 4. Simpozijuma "Matematika i primene", Univerzitet u Beogradu – Matematički fakultet.

[8] Градимир В. Миловановић (2000), *Драгослав С. Митриновић (1908–1995)*, Живот и дело српских научника, књ. 6 (М. Сарић, ур.), Српска академија наука и уметности, Београд.

[9] Gradimir V. Milovanović (2000), *Life and inequalities: D. S. Mitrinovic (1908–1995).* Recent Progress in Inequalities (G. V. Milovanović, ed.), Mathematics and Its Applications, Vol. 430, pp. 1–10, Kluwer, Dordrecht, 1998; MR1 609 996 (99k:01068a).

[10] Gradimir V. Milovanović (2013), *Memories on Časlav Stanojević and his scientific contribution*, In: Časlav Stanojević – Život i delo, Zbornik radova, Matematički fakultet, Beograd, 33–40 (Serbian).

[11] Драгослав С. Митриновић (1968), *Живот Михаила Петровића*, Мат. библиотека 38, 7–32.

[12] Драгослав С. Митриновић (1968), *О једној неједнакости*, Мат. библиотека 38, 93–96.

[13] Михаило Петровић (1999), *Сабрана дела*, Томови 1–15, Завод за уџбенике и наставна средства, Београд.

[14] Љубомир Протић (1999), *Петровићево директно проучавање решења диференцијалних једначина* у: Михаило Петровић, *Диференцијалне једначине*, други део, Сабрана дела, књ. 2, Завод за уџбенике и наставна средства, Београд.

[15] Драган Трифуновић (1969), *Летопис живота и рада Михаила Петровића,* Српска академија наука и уметности, Одељење природно-математичких наука, Београд.

[16] Драган Трифуновић (1996), *Димитрије Нешић – зора српске математике*, Архимедес, Београд.

[17] Ариф Золић, *Константин Павлович Орлов*, видети на сајту: http://www.matf.bg.ac.rs/files/KonstantinOrlov.pdf

PEDAGOGICAL WORK OF MIHAILO PETROVIĆ*

Vojislav ANDRIĆ
Society of Mathematicians of Serbia
Valjevo Gymnasium

> *"Serene, silent, modest, humanly simple, superhumanly endowed, Mihailo Petrović was one of the greatest sons of our nation."*
>
> Milutin Milanković

Various ranking lists with one hundred best, greatest, most important or most influential persons are often prepared across the world. Of course, such choices are invariably somewhat disputable and partially biased, depending on the authors making such lists. One thing, however, is certain – the list of the leading and most important Serbian scientists, greatest mathematicians, the most versatile and unusual people, and the list of one hundred most important historical figures among the Serbs of all time will certainly contain the name of Mihailo Petrović Alas – a man who significantly marked the time in which he lived and enriched the culture of our people with his prolific work in a large number of disciplines.

Even a fragmentary study of the work of Mihailo Petrović makes every researcher convinced that he was an outstanding person, teacher and scientist. Just as his entire life and work were marked by incredible and unusual versatility, his pedagogical work was also characterised by

* I dedicate this work to late professor dr Dragan Trifunović, who did the most to shed light on the interesting and dynamic life of Mihailo Petrović, his versatile personality and enormous contributions to the history and culture of our people.

House of Mihailo Petrović Alas
at 22, Kosančićev venac st.

many significant contributions to the future of our people and civilisation in general.

Of around twenty areas of mathematics and numerous disciplines outside of mathematics that Mihailo Petrović Alas successfully dealt with, mathematical science and teaching of mathematics are certainly among the most important, all the more so as the majority of scientists – teachers find it hard to say where teaching stops and science starts, because successful professional and teaching activity implies primarily a high degree of scientific expertise and a solid mathematical apparatus, including didactical skills of the lecturer.

The aim of this article is to elucidate the pedagogical work of Mihailo Petrović Alas and his indirect and direct contribution to the development of the teaching of mathematics in Serbia and the world, in his time and later, when the implications of his work became fully manifest in Serbia and Yugoslavia.

Mihailo Petrović's portrait, a drawing from
"Mihailo Petrović Alas" Primary School

TEACHING

Mihailo Petrović's pedagogical work deserves to be elucidated from all aspects, most notably his teaching efforts, the courses he taught, his direct influence on the creation of the Belgrade Mathematical School, contributions to the progress of several generations of his students and doctors of science in the fields of teaching and science, and the promotion of mathematics in Serbia and the region.

Very young, aged only 26, Mihailo Petrović became a full professor at the Faculty of Philosophy of the Belgrade Great School.[49] In the academic 1894/95 year, he taught mathematical analysis at the Natural-Sciences Department of the Faculty of Philosophy. The documents available to us show that professors at higher educational institutions were at the time obliged to take an oath. Newly elected professors would also be received in audience with the ruler.[49]

In the June term of 1895, Mihailo Petrović had an exam in mathematical analysis and in the academic 1895/96 year, he taught mathematics II and III at the Faculty of Philosophy and Technical Sciences of the Great School. In this period, Mihailo Petrović often presided at exams of his other colleagues, particularly Bogdan Gavrilović, and at exams in physics, including even the French language. From 1905 to 1909 (when Milutin Milanković was engaged to teach at the Applied Mathematics Department)[49], Mihailo Petrović was the only professor of mathematics at the Faculty of Philosophy and, together with Bogdan Gavrilović, the only mathematician at the Great School. At the time, following the model of French university studies of mathematics, Petrović began to teach his specialised mathematical courses.

It is worth noting that the rule on the limited number of teachers[49] was in force at the Great School at the time. It was only in 1912 that professor Mladen Berić was received as the first associate to Mihailo Petrović. Later, after World War I, the Mathematics Department was systematically reinforced. Teaching professors were Anton Bilimović, Nikola Saltikov, Radivoj Kašanin, Jovan Karamata and others.

Mihailo Petrović loved his teaching profession and dedicated to it a lot of time during his professional career (formally 44 years: from 1894 to 1938). His lectures were simple and appealing to students. The following quotation speaks volumes about him as a lecturer and pedagogue, who always held a piece of chalk: "His school produced forty classes of mathematicians, all of whom cherished and still cherish undivided respect and esteem towards their professor, who instilled them with knowledge at his versatile courses in mathematical analysis, which were presented with ease and reflected his deep thought and virtuosity of teaching. He gladly received, invited and taught everyone who showed interest in mathematics. He wanted to unselfishly devote himself to teaching and science in our country."[50]

At the start of his professorship, Mihailo Petrović was considered someone with a very strict grading criterion. The highest mark (five) was very rare, with a half of students often referred to the following teaching terms. Such criterion considerably differed from the hitherto practice and significantly encouraged students of mathematics and technical sciences to prepare themselves more systematically for the exams in mathematics and mathematical sciences.[51] Mihailo Petrović remained strict primarily towards himself and then towards others until the end of his life.

Petrović was a sincere and eager devotee to science in general, particularly mathematical and natural sciences. His biographers mention his great interest in all areas of mathematics and some restraint in terms of the contents of geometries and probability theories. In the course of his 44-year long career of a full professor at the Great School and University of Belgrade (from 1894 to 1938) and an honorary professor (from his retirement until April 1941), Mihailo Petrović taught a number of regular and specialised courses: analytic geometry in plane and space, higher algebra, differential and integral calculus, geometrical application of the theory of differential equations, calculation with numerical intervals, infinite order theory, elliptic functions, partial differential equations in mathematics and physics, second-order linear differential equation and its application, qualitative integration of differential equations, integration of differential equations by use of series, analytical problems for interpretation, theory of errors, theory of analytical functions, elements of mathematical phenomenology[52].

The range of areas and disciplines taught by Mihailo Petrović best testifies to his mathematical knowledge, and creative and teaching originality as many of the above courses were not classical, but were based on Petrović's long-lasting teaching and research work and innovations


Jovan Karamata (1902–1967)


Tadija Pejović (1892–1982)


Dragoslav Mitrinović (1908–1995)

that he continuously introduced in his lectures, based on his papers and papers published in foreign journals. Petrović's courses were simple, short, concise and methodologically well-designed, without any exaggeration in content.[53] Lecture notes followed each of the above courses, to assist students in learning.[54]

Mihailo Petrović carefully followed, detected and supported his best students. After completing the studies of mathematics at the Belgrade Faculty of Philosophy, his students would most often be referred to Belgrade gymnasiums as teachers of mathematics, while the best ones were engaged as assistants at the University. In the 1912–1938 period, eleven mathematicians defended their doctoral theses before Mihailo Petrović. In the decades before and after World War II, these mathematicians, together with their students and associates, played a great role in the development of mathematical science and the teaching of mathematics in Serbia and Yugoslavia.

Mihailo Petrović's doctoral students were the following (by order of gaining the doctoral title): Mladen Berić (1912), Sima Marković (1913), Tadija Pejović (1923), Radivoj Kašanin (1924), Jovan Karamata (1926), Miloš Radojičić (1928), Dragoslav Mitrinović (1933), Konstantin Orlov (1934), Danilo Mihnjević (1934), Petar Muzen (1937), Dragoljub Marković (1938).

The analysis of the available databases shows that as much as it is important to observe the doctoral students of Mihailo Petrović, so much it is important to analyse their students who continued their professors' curricular, extra-curricular and scientific activities. Mihailo Petrović currently has close to nine hundred mathematical "successors" given that his doctoral students, following in the footsteps of their professor, aimed to leave behind themselves numerous successors of their work and ideas. The most prolific in "producing" the scientific offspring were Jovan Karamata (12 doctoral students and 480 successors), Tadija Pejović (17 doctoral students and 193 successors) and Dragoslav Mitrinović (33 doctoral students and 120 successors).[55]

Although some connoisseurs of Mihailo Petrović's life and work are sceptical in terms of the scientific formation and continuous instruction of younger colleagues, there is no disputing that Mihailo Petrović, his closest associates Bogdan Gavrilović and Milutin Milanković, and his doctoral students and closest associates after the Great War, contributed the most to the creation and development of the institution designated in the history of our science as the Belgrade Mathematical School.

It is considered that the formal beginnings of the Belgrade Mathematical School date to the time when the first doctoral student of Mihailo Petrović – Mladen Berić began to work at the University of Belgrade. Berić defended his doctoral dissertation in May 1912 before a committee consisting of Mihailo Petrović and Milutin Milanković.[56]

Bogdan Gavrilović (1864–1947)

However, equally important for the functioning and activity of the Belgrade Mathematical School was the arrival of professor Milutin Milanković to Belgrade University (the Applied Mathematics Department – 1909), Anton Bilimović (1920) and Nikolaj Saltikov (1921). In the third decade of last century, a team of highly capable mathematicians and applied mathematicians gathered around Mihailo Petrović. They initiated many useful mathematical activities such as: replenishing the university mathematical library, set up by Mihailo Petrović, writing quality secondary-school textbooks, launching the Mathematical Journal etc., which will be elaborated in more detail hereinafter.[56]

Furthermore, the Belgrade Mathematical School was the precursor of the membership of our mathematicians in international mathematical associations. In late 1921, upon the order of the Serbian Royal Academy, Mihailo Petrović and Bogdan Gavrilović wrote a report, proposing and recommending that the National Section of the International Mathematical Union (IMU) be established within the Professorial Society.[57]

One of the most important contributions of the Belgrade Mathematical School was certainly the initiation of the mathematical journal in foreign languages – *Publication mathematiques de l'Université de Belgrade.* Upon the initiative of Mihailo Petrović and Milutin Milanković, the journal was established in 1932 to help interested mathematicians actively involved in research in the fields of mathematics and applied mathematics to publish their results. Until 1941, seven issues were published. The journal was, in fact, the gazette of the Belgrade Mathematical School. Owing to it, our mathematicians could present themselves, individually and collectively, to the international mathematical public[58].

The journal continued to be published in 1947 under the title: *Publications de l'Institut Mathématique (nouvelle serie)*. It is issued twice a year by the Mathematical Institute of the Serbian Academy of Sciences and Arts in Belgrade[59]. The new journal series has been published since 1961. The journal accepts papers of Serbian and foreign mathematicians. The main language is English. Papers in French, Russian and German are also accepted. The journal is one of the leading Serbian scientific journals in the field of mathematics and enjoys considerable reputation in the country and the world. It has a wide range of associates and a respectable editorial board.

"Mihailo Petrović rejoiced in his students' scientific success and did not impose upon them fields of research."[60] He was very circumspect and modest, never aiming to ascribe to himself the success and results of his associates, while at the same time not restraining himself from putting everyone in their place. Quite illustrative is his answer to journalist Krsto Cicvarić, published in the *Politika* daily on 24 June 1921 – Alas wrote that Cicvarić was not competent enough to assess (or even praise) him and his colleagues, some of whom did a lot for science and whom he (Alas) particularly esteemed and loved.[61]

Milutin Milanković (1879–1958)

Cover pages of other publications of Mihailo Petrović's textbooks

TEXTBOOKS AND LECTURE NOTES OF MIHAILO PETROVIĆ

It is known, and has already been highlighted in this article, that materials supporting the mathematical courses taught by Mihailo Petrović were mainly sheets (lecture notes). Unlike his colleague Bogdan Gavrilović, he long resisted the writing of a textbook.

In the period from 1909 until the end of Petrović's professorial career, his courses were supported by lithographic sheets (lecture notes). The notes were designed in the following way: students would take detailed notes at Petrović's courses and submit them to Petrović for analysis. He systematically inspected the notes and approved them for lithography (copying). The year 1924 was important as the notes for six Petrović's courses were lithographed. From 1925 to 1930, the notes for all 15 mathematical courses taught by Mihailo Petrović were issued. Unlike the previous ones, these notes were authorised and, in technical terms, the format of hand-written sheets was abandoned, with the text now taking the shape of typewritten books.[62]

The notes were divided into chapters. Within each chapter, Mihailo Petrović presented theory for each thematic unit (definitions, theorems with evidence), which was followed by solved examples and a significant number of tasks methodologically ranked from simpler to more difficult. In regard to theory, Petrović referred students in his notes to literature with detailed evidence, and in regard to tasks he referred them to more in-depth collections of tasks.[61]

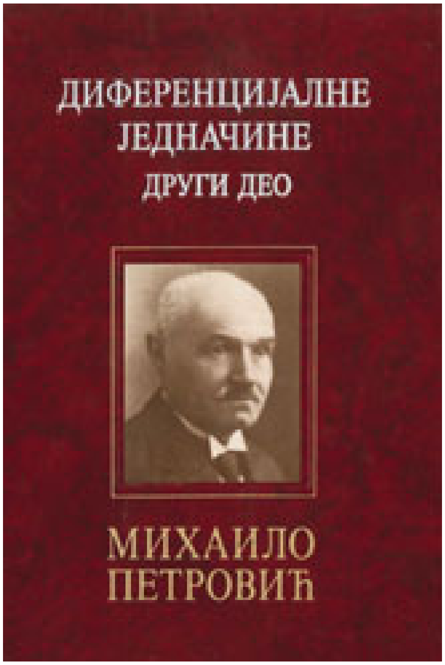
By the end of his career and life, Mihailo Petrović "yielded" and wrote several textbooks: *Calculation with Numerical Intervals* (1932), *Elliptic Functions* (1937) and *Integration of Differential Equations by Use of Series* (1938). Just like some of connoisseurs of his works, he considered his seminal work *Elements of Mathematical Phenomenology* from 1911 to be a textbook.[61]

In April 1932, Mihailo Petrović's textbook *Calculation with Numerical Intervals* was published, containing 193 + II pages, in 15.9 × 23.7 format.[61] The textbook was published by the Endowment of Luka Ćelović Trebinjac within the series "Lectures at Belgrade University", most probably in 500 copies.[61] Some mathematicians believe that this textbook has all features of a monograph, though this cannot be reliably claimed given the absence of the bibliography of works.

In 1937, Petrović's textbook *Elliptic Functions* was also published within the series "Lectures at Belgrade University", by the Endowment of Luka Ćelović Trebinjac, in 128 + III pages, 15.9 × 23.7 format.[63] It is interesting that students, not knowing that professor Petrović was working on the textbook, prepared and published the new issue of lecture notes on the same topic. It is noteworthy that a significant role in preparing the textbook was played by Petrović's doctoral students and associates Dragoslav S. Mitrinović, who carried out detailed proofreading, and Miloš Radojčić, who made the final review, together with his professor.

Already the following, 1938 year, the textbook *Integration of Differential Equations by Use of Series* was also printed, again by the Endowment of Luka Ćelović Trebinjac, within the same series[64]. It had 219 pages, 15.2 × 23.3 format.

These three textbooks of Mihailo Petrović were published three times ("Naučna knjiga") in 1969, as part of the programme to mark the 100th anniversary of the birth of this great mathematician. Dragoslav S. Mitrinović initiated the publication of the second issue of these textbooks. A significant contribution was also provided by dr Milorad Bertolino and dr Petar Vasić, who did the editing. In a succinct foreword, the editors gave the most necessary information about the interventions that they carefully carried out.[65]



Cover page of *Differential equations*, *pt. II*, *The Collected Works*, Book 2 (Digital legacy of Mihailo Petrović)

The third issue of Mihailo Petrović's textbooks was published within the seminal project *The Collected Works of Mihailo Petrović*: book 8 – *Interval Mathematics – Differential Algorithm* and book 9 – *Elliptic Functions – Integration by Use of Series*, prepared by dr Dragan Trifunović and dr Zoran Kadelburg. In addition to the prepared texts of Mihailo Petrović's textbooks, both books contain detailed forewords which systematically and highly illustratively elucidate the most important details concerning the publication of the three textbooks.[66]

The 15 books of *The Collected Works of Mihailo Petrović* were published owing to the Institute for Textbook Publishing and Teaching Aids in Belgrade and its then director dr Dobrosav Bjeletić, as well as thanks to dr Dragan Trifunović who was a tireless guardian of the personality and work of Mihailo Petrović, and the Faculty of Mathematics in Belgrade, Society of Mathematicians of Serbia, and editors and members of the Editorial Board.[67]

Anton Bilimović (1879–1970)

Nikola Saltikov (1872–1961)

MODERNISATION OF CURRICULA

Mihailo Petrović actively worked on upgrading the mathematics curricula at all levels, as illustrated by the following:

At the Fourth International Congress of Mathematicians in Rome from 6 to 11 April 1908[68], the decision was made to reach – through the comparative analysis of mathematics curricula in participating countries – a single curriculum to be applied in countries wishing to introduce it.[69] A working group was set up, led by well-known professor of Göttingen University Felix Klein. The group was to establish the International Commission on Mathematical Instruction (ICMI). Mihailo Petrović informed in detail professors of mathematics in Serbia about the events that followed in his inspired text "The International Commission on Mathematical Instruction" published in *Prosvetni glasnik (Educational Gazette)* in 1913[69]. The text shows that the International Commission had 43 full members from 25 countries of Europe, America, Asia and Australia. Our delegate was Mihailo Petrović[69]. The members of the Commission were respectable world mathematicians. Mihailo Petrović informed his colleagues about the Commission's meetings in Brussels in August 1910, Milan in September 1911, and Cambridge in August 1912.

The International Commission on Mathematical Instruction focused on two issues: a) systematic presentation of mathematics and its disciplines in secondary schools and b) theoretical and practical courses in mathematics for students of physical and natural sciences. Based on voluminous material obtained, Mihailo Petrović forecast the direction of courses in mathematics in the period to come (after 1913). For the sake of comparison with the present-day situation in teaching of mathematics in the country and the world, Petrović's "indications" are worth quoting: "an impetus should be lent to intuition and experiment in lower and medium-level teaching, with the least amount of formalistic elements; a purely practical, utilitarian character will be given to mathematics as the main auxiliary element in expert teaching; while a purely logical direction will be pursued in higher-level teaching, where mathematics is to be taught for the sake of itself, regardless of its role in other areas of knowledge".[70]

Mihailo Petrović then gives a short overview of the most important achievements in the teaching of mathematics in the world. He mentions British mathematics laboratories and textbooks focusing on clarity and intuition, the interest of teachers across Europe in reforming the teaching of mathematics (he mentions Germany and Romania as examples), and expresses his personal enthusiasm that all this took place spontaneously, without any official intervention[70]. He also mentions the partially designed *Bibliography of Mathematical Teaching*, which contained around 2000 titles, classified by content and type of school. Mihailo Petrović sincerely hoped that the reform movement of mathematical teaching would inevitably engulf Serbia and that the educational authorities in Serbia had to show interest in the project. He called upon the teachers of mathematics in Serbia to study the material of around 300 papers printed in 160 issues, and organise a national subcommittee of the ICMI.[70]

Of course, Mihailo Petrović's interest in this area was not limited to mere statements and observations. As was typical of him, he always actively participated in everything, writing papers in which he showed his interests and put forward interesting proposals for the introduction of elements of differential and integral calculus in mathematics curricula in second-level secondary schools.[71]

The International Commission on Mathematical Instruction was particularly active until World War I, but went into a decline between the two world wars, exerting no influence on the teaching of mathematics.[72] This, however, did not prevent Mihailo Petrović and his associates from advancing the teaching of mathematics in Serbia. In 1926, the Mathematical Club was created in Belgrade, led by Anton Bilimović. In 1937, the Yugoslav Mathematical Society was established (the precursor of today's Society of Mathematicians of Serbia), led by Tadija Pejović. The Society gathered around 100 eminent mathematicians and physicists from Belgrade, Zagreb and Ljubljana. Its work was interrupted by World War II.[73]

POPULARISATION OF MATHEMATICS

Real pedagogues in mathematics differ from those who are not so as they present mathematics by showing all its beauties and numerous applications, trying to popularise mathematics as much as possible, and give it an appropriate, significant place in the educational system. In addition to popularising mathematics among his students, post-graduate students, teachers and associates, Mihailo Petrović played an important role in many other activities aimed at familiarising the public with mathematics and pointing out the great importance of mathematics for civilisational development in general.

As he was an avid reader from his early childhood, Mihailo Petrović knew well what literature meant for the development of a science. As soon as he arrived in the Great School, he began to create a library.[85] Just before the foundation of the University of Belgrade, libraries were created at other departments as well. The first official data on the Mathematical Library of the Faculty of Philosophy of the Great School date to 1902. Judging by the then established inventory book of the Mathematical Cabinet, the library had over 300 titles. Until the inventory number 100, the library was led by Bogdan Gavrilović, with Mihailo Petrović taking the lead later on. Gavrilović and Petrović obtained mathematical literature through libraries in Belgrade, Paris and Vienna. The notes at the end of each inventory book suggest that they were carefully replenishing the library fund with books on elementary mathematics, applied mathematics, the history and philosophy of mathematics, analytical and higher geometry, higher algebra etc. Between the two wars, Mihailo Petrović and his associates carefully and continuously replenished the library, which was therefore very well equipped. In his lecture notes and textbooks, Mihailo Petrović recommended to his students to use the numerous books and collections from the library in order to deepen and broaden their knowledge. Unfortunately, the mathematical library and many other important documents concerning the work of this great scientist and his associates did not survive the war as they were burnt to ashes in the fire of 18 October 1944.[74]

Mihailo Petrović was also aware of the fact that “high” science had to be presented to pupils and students in a popular way, and that the University and the Academy of Sciences had to be constantly interested in mathematics curricula in primary and secondary schools. The mathematical literature between the two wars thus contains a number of articles that Mika Alas wrote for secondary school students, their

The book *Articles* which contains popular texts of Mihailo Petrović teachers and fans of mathematics in Serbia.[75] It is in this regard that we can observe three types of Petrović's contributions: articles – appendices to secondary-school mathematical textbooks, interesting popular texts in journals, and support to the publication of relevant mathematical literature.

Young mathematicians and teachers of mathematics certainly still find topical Petrović's articles: "Real and Illusory Geometric Impossibilities" (1937)[75,76], "Wrong Geometric Conclusions from a Carelessly Drawn Picture" (1938)[75,76], "Interesting Details in the Application of the Pythagoras' Rule" (1939)[75,76], "Indeterminate, Impossible and Incompletely Defined Planimetry Tasks" (1940)[75,76] and "Falseness of the Eye in Comparing Straight Lines and Surfaces" (1940)[75,76]. All these articles were published between 1937 and 1940 as annexes to textbooks on mathematics, i.e. geometry for the second, third, fourth, fifth and first year (in order of publication) of secondary school. The authors of these textbooks were Anton Bilimović and Tatomir Anđelić. It is worth noting that Petrović's article "Stereometric Inequalities" was written in 1941 for the geometry textbook for the fourth grade (the textbook was not printed due to the war) and published only in 1953.[75,76]

Mihailo Petrović wrote similar articles in the most widely read journals for teachers of his time: *Nastavnik (Teacher), Glasnik profesorskog društva (Gazette of Professorial Society), Srpski književni glasnik (Serbian Literary Gazette), Zbornik radova Akademije nauka (Collection of Works of the Academy of Sciences)* or other foreign journals. Particularly interesting are his articles "Absolute and Restrictive Mathematical Impossibilities" (1914)[75,76], "A Question from Logarithm Teaching" (1928) [75,76], "Squaring the Circle and Angle Trisection Before the Parisian Academy of Sciences" (1928)[75,76], "Mathematicians' Errors" (1933)[75,76], "Squaring the Circle" (1938)[75,76].

We wish to particularly highlight the article "About the Dependencies Among Magnitudes in Tasks", published by Petrović in 1932 in *Matematički list za srednju školu* (*Mathematical Paper for Secondary School)* (No 3–4, p. 37–44)[77]. The article is rather interesting – diminished caution in formulating tasks often leads to errors arising from the lack of knowledge about dependencies among magnitudes in tasks, so that the characteristics of mathematical objects are sought, which, with the defined characteristics, in fact do not exist.[77] It is also worth noting that the owner and editor of *Matematički list* was one of Petrović's students and closest associates – Jovan Karamata, and that the journal had a Yugoslav editorial board.[78]

All the stated, and other non-stated articles[79], were written in a simple, popular, comprehensible language and with an acceptable style. The themes of all enumerated articles were obviously very useful, very carefully chosen, highly interesting and topical, and largely interesting still today as part of the general mathematical culture of every young person. We shall give only one, but a very characteristic example. If some journalists had read Petrović's texts about the squaring of the circle or angle trisection, as his students did, it would have never crossed their minds to write in our daily with high circulation the truly "sensational" news: "A professor from Užice defeats Gauss and Descartes with his compasses and ruler".[80]

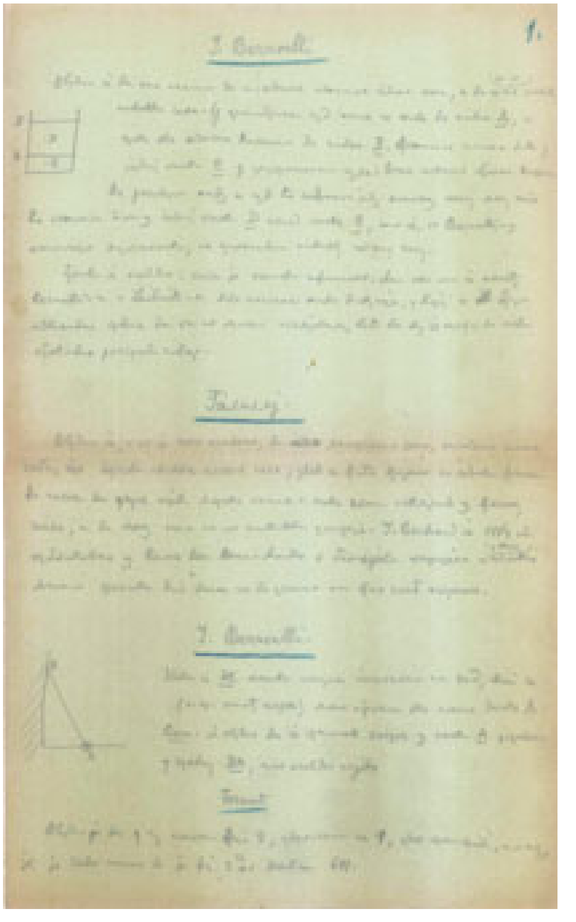
MIHAILO PETROVIĆ'S OTHER PEDAGOGICAL CONTRIBUTIONS

Mihailo Petrović was also active in other areas of pedagogical research. It seems he was a man on whom the then Ministry of Education and Church Affairs could always count, regardless of whether he was entrusted with tasks in Belgrade or the interior.

For a number of years, he was a member of the professorial exams committee. The first time, already in 1895, he and Bogdan Gavrilović had five candidates[81]. Somewhat later, in 1913, Mihailo Petrović was elected a member of the standing professorial exams committee with a four-year mandate. The importance attached to professorial exams at the time in Serbia is best seen in the composition of the committee, which also consisted of Pavle Popović, Jovan Skerlić, Aleksandar Belić, Jovan Cvijić, Bogdan Gavrilović, Sima Lozanić and other distinguished professors of the University[81].

Today, when many doubt the validity of the future national maturity exam, highly relevant is the fact that Mihailo Petrović was also a member of committees supervising final exams in Serbian gymnasiums such as the First Belgrade Gymnasium (1896, 1898)[81], Gymnasium in Kragujevac (1900, 1906)[81], Gymnasium in Niš (1902)[81], Gymnasium in Jagodina (1906)[81], Second Belgrade Gymnasium (1908)[81]. In terms of our present-day national maturity exam, it must be emphasised that back in 1902 Mihailo Petrović, owing to his experience in maturity exams in gymnasiums (which he praised), believed that only those students who passed the maturity exam could enrol in the University, as such exam was the best reflection of their actual knowledge[81].

Mihailo Petrović was a member of the Main Education Council of the Ministry of Education on several occasions (1896, 1898, 1900)[81], and served as its president in the 1909–1910 period[81]. As the Council's member and president, he could more decisively influence the development of the teaching of mathematics in secondary schools (curricula, textbook reviews etc.).



Handwritten notes and comments of Mihailo Petrović for one of his works (Society "Adligat")

Mihailo Petrović was occasionally appointed the supervisor in secondary schools (academic years of 1897/98, 1909/10 – the Third Belgrade Gymnasium)[81], which seems to have been a good practice. This method could be implemented today as well since the engagement of the most renowned university professors could significantly improve the cooperation between secondary schools and universities, and enable continuous monitoring, at least partial supervision of the work of schools, better understanding of their work, and greater possibility to upgrade all activities in schools.

It is worth noting that Mihailo Petrović also discharged several important functions at the University of Belgrade. He was the dean of the Faculty of Philosophy from 1908 to 1909[82], and its vice-dean three times – from 1909 to 1913[82]. He was proposed to be the rector of the University of Belgrade several times, but he refused these offers for the reasons of personal and political nature (although he never dealt with politics), which are not related to teaching or scientific factors[82].

The pedagogical contributions of Mihailo Petrović can be elaborated in even more detail since he exerted an undoubtable influence on the establishment of the Council of Yugoslav Students of Mathematics (1927)[82], formation of general staff officers in the area of code work (he wrote the textbook *Cryptography* in 1928)[82] and the establishment of the Department for Geometry (1930).[82]

At the end of this chapter, it should be noted that Petrović's pedagogical achievements were not related to Serbia only as his teaching influence went beyond the borders of our country. The most typical example is that from 1928, when Petrović was a guest lecturer during a summer term at Paris University, holding a one-term course on mathematical spectres.[83]

INFLUENCE OF MIHAILO PETROVIĆ'S PEDAGOGICAL WORK ON PRESENT-DAY TEACHING CIRCUMSTANCES IN SERBIA

The influence of Mihailo Petrović's scientific work on the present-day teaching practice in Serbia is doubtless significant. Petrović's greatest pedagogical achievement were his students. After World War II, his closest associates and followers did a lot for the development of mathematical science and teaching in Serbia and Yugoslavia. Several hundred mathematicians who are direct successors of Mihailo Petrović work today in Serbia, at universities, institutes and schools, carrying out the most important teaching and scientific duties in the Serbian Academy of Sciences and Arts, Mathematical Institute, Society of Mathematicians of Serbia, universities and faculties, Mathematical Gymnasium and other institutions. Also important for present-day mathematics in Serbia is the genealogical tree of dr Đuro Kurepa, which contains around 160 mathematical successors.

Worth mentioning is that all the above institutions and their present-day activity were designed by students and associates of Mihailo Petrović or their students (Jovan Karamata, Anton Bilimović, Tadija Pejović, Radivoj Kašanin, Miodrag Tomić, Dragoslav Mitrinović, Vojin Dajović, Milica Ilić Dajović, Slaviša Prešić etc.).

The analysis of Mihailo Petrović's teaching ideas[84] from his 44-year professorship at the University of Belgrade suggests that there is almost no segment where his exceptional work was not continued, as well as that, quite naturally, these ideas were further developed and improved in numerous fields. The table below contains one of possible comparisons only for Petrović's most important teaching activities:

Mathematics Department of Belgrade University	Faculty of Mathematics in Belgrade, mathematical departments and institutes at faculties of science and mathematics in other university centres
Mathematical Institute of Belgrade University	Mathematical Institute of the Serbian Academy of Sciences and Arts
Mathematical Library	Library of the Faculty of Mathematics and the Faculty of Science and Mathematics, University Library, National Library, Library of the Academy of Sciences and Arts, Library of the Society of Mathematicians of Serbia
Mathematical Club, later Yugoslav Mathematical Society	Society of Mathematicians of Serbia
Journal *Publication mathematiques de l'Université de Belgrade*	Journal *Publications de l'Institut Mathématique (nouvelle serie)*
Matematički list za učenike srednjih škola	*Matematički list za učenike srednjih škola* and *Tangenta* for secondary schools
Popularisation of mathematics	Society of Mathematicians of Serbia, Mathematical Society "Arhimedes", School for Lovers of Mathematics "Integral"

The present-day teaching of mathematics in Serbia contains areas not existing at the time of Mihailo Petrović, but it is worth noting that these ideas are mainly developed by his successors. These include: seminars for teachers and teaching conferences, mathematics teaching journals, publishing activity popularising mathematics, work with gifted students, mathematical competitions, summer and winter schools of young mathematicians etc.

*

The above sections of this paper contain a lot of information about the teaching work of Mihailo Petrović. Each footnote suggests the enormous effort that Mihailo Petrović invested in the education of young people and their familiarisation with science. At the same time, each of those footnotes contains a great story as it sheds light on a concrete event, endeavour or book, article, study, and life story that Mihailo Petrović left behind himself. Analysing the voluminous bibliography of his works[85], which contains around 400 bibliographical units (and numerous excerpts, overviews, quotations) and rich general literature[86] about Petrović, one can see that materials concerning the teaching of mathematics are so comprehensive that they can be used to write a number of papers, studies and even doctoral dissertations.

It is indisputable that each day of his professional career Mihailo Petrović worked on the mathematical enlightenment of the milieu that he lived in and that his teaching ideas and activities relating to the popularisation of mathematics have strong implications for the present-day teaching practice in the field of mathematics in Serbia.

Mihailo Petrović was a brilliant lecturer. He was correct towards his students. He respected their effort and supported their constructive proposals. Just as he was strict towards himself, he was strict at exams, but created an atmosphere of immediacy at his courses[87].

However, it would be unjust to observe Mihailo Petrović Alas only as a teacher and scientist as over a half a century he shaped the social life in Serbia of his time. Let us therefore at least briefly illuminate the social aspect of his personality and his unusual versatility. Petrović was at the same time both a sociable and withdrawn person. Although he was affable, he never imposed himself or showed off (he shunned great publicity). He was composed, somewhat shy[87], moderate (apart from work, travels, music and fishing) and modest[87]. He was obviously very pleasant, humorous, communicative, systematic[87] and disciplined. His general culture was at the highest level. The abundant preserved correspondence of Mihailo Petrović testifies to a highly accurate, well-organised and polite man, who did not leave his professorship work for a single day without informing beforehand his superiors and associates. He would leave no one's letter unanswered. Mika Alas was certainly a unique personality also judging by the fact that he socialised with people from the widest social strata[87] – fishermen on the Sava and Danube rivers, Roma musicians in Savamala, visitors and owners of numerous Belgrade taverns, his school friends and professors, students and associates, world-class scientists and members of the royal family. The extent to which his students, associates, acquaintances and friends liked him is best attested by numerous anecdotes from the life of old Belgrade, and even songs in his honour[88]. Mihailo Petrović was also a humane person. His biography illuminates many places and events showing that he often gave up on his pay, helped the poor and feeble, gave away the caught fish, or helped his friends to make ends meet.

Finally, let us also say that the life and work of Mihailo Petrović Alas, his patriotism and exceptionally rich work are an example of how to help one's nation and its future, as well as a very good chance for young people (those whom we educate and reveal to them the secrets of mathematical science) to be inspired by the work of Mihailo Petrović Alas, a wondrously original personality and one of the most important figures in the history of the Serbian people.

REFERENCES

[1] Драган Трифуновић, *Летопис живота и рада Михаила Петровића*, Српска академија наука, Београд, 1969, VIII + 361.

[2] Михаило Петровић, *Елиптичке функције* (друго издање), Научна књига, Београд, 1969, VIII + 113.

[3] Михаило Петровић, *Чланци* Друштво математичара и физичара Народне републике Србије, Научна књига, Београд 1949, V + 110.

[4] Сабрана дела Михаила Петровића – књига 9, *Елиптичке функције, Интеграција помоћу редова,* Завод за уџбенике Београд, 1997, 323.

[5] Сабрана дела Михаила Петровића – књига 8, *Интервална математика, Диференцијални алгоритам,* Завод за уџбенике, Београд, 454.

[6] Михаило Петровић, *Рачунање са бројним размацима* (друго издање), Издавачко предузеће „Научна књига", Београд, 1969, VIII + 169.

[7] Сабрана дела Михаила Петровића – књига 10, *Чланци, Студије,* Завод за уџбенике, Београд, 410.

[8] Владимир Мићић, Зоран Каделбург, Војислав Андрић и други, *Седамдесет година Друштва математичара Србије*, Друштво математичара Србије, Београд, 2018, 137.

[9] Група аутора, *Михаило Петровић Алас – родоначелник српске математичке школе (каталог уз изложбу)*, Српска академија наука, Београд, 2018, 159.

MIHAILO PETROVIĆ
IN PHILOSOPHY, LITERATURE AND PUBLIC LIFE

MATHEMATICAL PHENOMENOLOGY AND THE PHILOSOPHY OF MATHEMATICS*

Slobodan VUJOŠEVIĆ
Mathematical Institute of SASA

Twenty[89] papers discussing the representation of natural and social phenomena by mathematical means hold a noteworthy place in the corpus of work of Mihailo Petrović, founder of the mathematical school in Serbia. He published these papers throughout his career as professor, preparing the majority of them in both Serbian and French languages, since he, as a rule, wished to make them accessible to as wide an audience as possible. In them, Petrović laid the foundations of a general mathematical phenomenology, reminiscent of a philosophy of nature. The key philosophical assumption of this science is that each phenomenon can be mathematically represented, i.e. that it has its mathematics which can be determined. Its key goal was therefore to make a typology of analogous phenomena as a basis for a method for faithful representation of each individual phenomenon. And though Petrović was inspired by natural phenomena, what he had in mind was a very general science that would encompass all phenomena, natural and social, real and imaginary, including phenomena in literature and art. He believed such science was capable of becoming a "guiding principle in individual sciences,"[90] as well as shedding light on the "great problem of natural philosophy, the solution of which is the ideal asymptotic goal of all sciences, which consists of reducing to the least possible measure all that has to be assumed in order

* A revised and supplemented version of the paper initially published in the catalogue *Mihailo Petrović Alas: The Founding Father of the Serbian School of Mathematics* (SASA, 2018).

to understand natural phenomena as well as the number of propositions that encompass everything that occurs in nature"[91].

Petrović began forming his viewpoint on the role of mathematics in the phenomena of nature and the world during a period of a sudden blossoming of natural sciences and, at the same time, of an increasing presence of mathematics in them. Similarly to many philosophers and scientists of the time, he believed in the unlimited power of natural sciences, and thus of mathematics, as mathematics played such an important role in their success. In terms of his education, Petrović was not only a mathematician, but he also graduated in physics at Sorbonne in 1893 and engaged in the study of chemistry and other sciences as well; therefore, he had an opportunity to see directly that mathematics permeated and connected science and contributed to its unity. He perceived that one and the same analytical apparatus appeared in completely disparate areas of science, that "disparate phenomena" could be "analogous," that is, represented by the same mathematics. "One of the most important such analogies [...] exists among the phenomena of electric current, heat transfer and the flow of fluids. It is so complete that these three kinds of phenomena, with their multiple and diverse variations, represent from an analytical point of view one and the same problem, the solution of which only needs to be interpreted in three different ways." He believed that this connection was not random and tried to explain its regularities. Independently and without relying on any predecessors, he very patiently gathered an immense body of material and established a branch of science which he termed *mathematical phenomenology*.[92] He defined its conceptual apparatus, similar to that of mathematics,[93] in which it was possible to determine groups of analogous phenomena "from which the mechanism of this or that phenomenon could be extrapolated directly as the concepts which figure in them are assigned this or that concrete meaning. A set of such analogies, once there is a sufficient number of them, will make up a separate branch of natural philosophy, a kind of a general mechanics of causes which will, just like other mathematical disciplines and notwithstanding its vast general application, operate with a small number of general basic concepts". Hence the key problem of phenomenology is the "mathematical explication of particularities of disparate phenomena of all kinds and all concrete natures as necessary consequences of the similarities of (their) mechanisms."


СРПСКА КРАЉЕВСКА АКАДЕМИЈА

ЕЛЕМЕНТИ

МАТЕМАТИЧКЕ ФЕНОМЕНОЛОГИЈЕ

МИХАИЛА ПЕТРОВИЋА

У БЕОГРАДУ

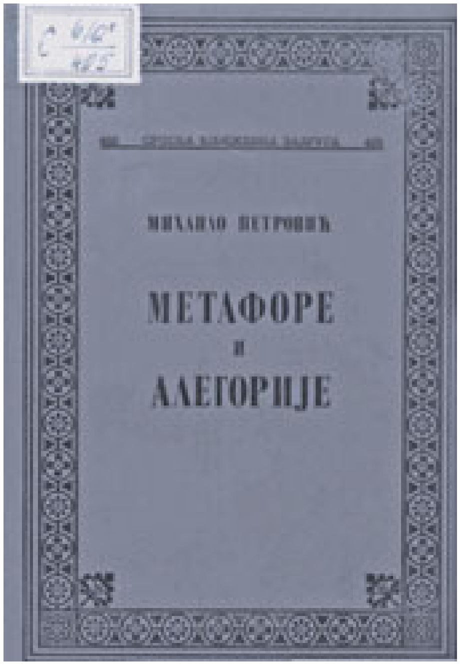
1911.

Cover page of the book *Elements of Mathematical Phenomenology*, published in 1911 (Digital legacy of Mihailo Petrović)

Petrović elaborated his science in a voluminous work of around eight hundred pages entitled *Elements of Mathematical Phenomenology*, published in 1911. He believed it could prove really fruitful in other disciplines as well, by helping them reconstruct valid mathematical models for natural and social phenomena. In order to transcribe a phenomenon into a mathematical form, he elaborated a method of phenomenological mapping: once its "mechanism" is determined, the phenomenon is represented as a "figurative point" in space and its mechanism is then mathematically described in the way mechanisms of phenomena are described in classical mechanics. He expounded this method in his monograph *Phenomenological Mapping* published in 1933. However, Petrović did not stop at phenomena in nature and some phenomena in society, but invested a great deal of effort to confirm the universality of his method in literature as well. The material he gathered for the accomplishment of this goal was published in 1967 in his book *Metaphors and Allegories.*

Phenomenon is undoubtedly the fundamental concept of mathematical phenomenology. And just as in mathematics, where its basic concept, the set, is not defined, in establishing his science Petrović did not define its key concept, the phenomenon, either. But whereas in mathematics the set has its clearly defined main interpretation, in a cumulative hierarchy, defined by a general inductive procedure, in mathematical phenomenology there is no detailed explanation of what is assumed under its fundamental concept. Phenomena are classified by analogy, but nothing is said of analogy as a relation between phenomena and its possible mathematical properties. The key properties of a set are its *extensionality* and *regularity,* i.e. sets are equal if they have the same elements and the relation *to be an element* has no cycles. By contrast to the set, the phenomenon is in all likelihood an *intensional* concept that is at the same time *irregular,* i.e. it allows for cycles, as there is no reason not to consider the manifestation of all phenomena a phenomenon. This actually means that there are phenomena in our world that cannot be fully represented mathematically, hence considerations of a quite generally understood concept of phenomenon are much closer to philosophy than to mathematics and positive science, to which Petrović generally had an affinity. During his schooling, the entire French educational system was based on a strictly rationalistic and entirely positivist spirit, to which each form of metaphysics was alien,

Cover page of the book
Phenomenological Mapping,
published in 1933
(Digital legacy of Mihailo Petrović)



The front cover page of the book *Metaphores and Allegories*, published in 1967 (Library of SASA, C 6/12;405)

Front cover page of the book *Mecanismes communs aux phenomenes disparates*, published in 1921 (Library of SASA, 687/12)

in science just as in philosophy. Aware that the key problem of his science, *the role of mathematics in our understanding of the phenomena of the world,* greatly surpasses the bounds of mathematics, Petrović who, in the spirit of his epoch, believed in the omnipotence of science, tried to shed light on this problem within the framework of a "general mechanics of causes which will, just like other mathematical disciplines and notwithstanding its vast general application, operate with a small number of general basic concepts." For some reason, he did not wish to see this as a purely philosophical problem, though that is what it essentially is.

Namely, already in German classical philosophy of the first half of the 19th century, in the period from Kant to Hegel, there were very detailed discussions about the phenomenon and its essence, and the circumstances in which we acquire knowledge of it, while in 20th-century philosophy this issue practically took centre stage in phenomenology as the prevailing philosophical orientation.[94] Thus, Kant made a distinction between noumenon and phenomenon. Noumenon is the essence of the phenomenon or the thing-in-itself, while phenomenon is that part of the noumenon that is accessible to consciousness. Science only deals with phenomena, i.e. with what is cognizable by the senses and fitted in apriori forms of consciousness, or more freely said, in mathematical and logical forms. Hence phenomenon is the collection of inteligible thruts about the noumenon, while the noumenon is unknowable and each rational attempt to determine its entity and the truth about it leads to a contradiction; science should, therefore, leave noumena to religious experience and theology. It is quite possible that Petrović's view of the phenomenon had some common points with Kant's understanding that our knowledge of phenomena is based on sensory data fitted into apriori mathematical and logical forms. Still, judging by the significance he attached to mathematics when studying phenomena, it is more likely that he perceived mathematics as an element of the very essence of the phenomenon, which, according to Kant, is not accessible to the rational consciousness.

Kant represented a kind of a turning point in the history of philosophy. After him, philosophy for a great part made a turn towards science, i.e. towards a positivist understanding of the world, by trying to describe and anticipate phenomena in the most accurate and comprehensive way possible, without asking questions about their sense. There was also a different point of view which disputed the view on the unknowability of the essence of phenomena in the world and continued to look for it through metaphysical means. The key representatives of this other orientation were Hegel and the phenomenological school of philosophy. In his *Phenomenology of Spirit,* Hegel developed a philosophical method to enable the knowing of the absolute metaphysical entity of the phenomenon, while the school of phenomenology, based on the works of Edmund Husserl, appeared as a form of resistance to the increasingly prevailing influence of a positivist position in science and philosophy. Interestingly, Husserl was an affirmed mathematician, inspired by similar ideas as Petrović, but when he embarked upon phenomenology or the "seeing of the essences", he distanced himself radically from positivism. He believed that the very essence of the phenomenon could be seen through the interaction of the consciousness and the phenomenon. In seeing the essence, it is important to liberate oneself from everything that is irrelevant for a given phenomenon, but also from everything that is irrelevant for seeing the phenomenon in the consciousness itself, i.e. from predispositions and apriori assumptions about the phenomenon. In a sense, Petrović's conceptual apparatus was designed so as to enable a sort of a "phenomenological reduction", but with the goal to mathematically represent a given phenomenon. However, Petrović's teaching undoubtedly had no common points with this school of phenomenology. He could have had some knowledge of it because he was friends with philosophers, primarily Brana Petronijević, at a time when this school was very current. In order to distance himself from philosophical teachings that were not close to him, it is possible that over time he began to use other names for his science, such as general mechanics of causes and similar.

If by education Petrović was connected with the positivist school in philosophy and science, the key problem of mathematical phenomenology, because of the mathematics present within it, turned him somewhat unexpectedly back to metaphysics. Namely, in the foundations of mathematics in the 20th century, it was necessary to raise questions about the philosophical status of mathematics and its abstract ideas; hence, a new branch of philosophy emerged – the *philosophy of mathematics.* It was dominated, though not exclusively, by an entirely metaphysical view of the nature of mathematical objects, mathematical *platonism*, and the philosophical assumptions that Petrović explicitly highlighted reveal that his teaching was close to this view.

Portrait of Mihailo Petrović Alas (work of Uroš Predić, oil on canvas, Belgrade, 1943 – Library of SASA, photographer Vladimir Popović)

The two assumptions that can certainly be viewed as fundamental to his mathematical phenomenology are:

1. *Each phenomenon in nature can be mathematically represented.*
2. *There is a minimal set of assumptions from which is possible to derive all laws of nature.*

The first assumption posits that each phenomenon in nature has its mathematics, i.e. that it determines a set of mathematical statements that are true for the given phenomenon. It should not be understood as a position that each science of a phenomenon can be reduced to mathematics, but that in the science of that phenomenon there is invariably a purely mathematical part comprising mathematical truths and a part comprising truths of the science itself about that phenomenon. Mathematical truths are universal and indispensable, they apply in this, but also in all past and future worlds. This cannot be said for truths of individual sciences regarding natural phenomena, as they are contingent, they hold true in this, but not necessarily in any other world, they are not eternal and cannot be derived from purely mathematical assumptions. In all likelihood, Petrović's first assumption was originally his and had not been taken over from another author. The second assumption is purely logical,[95] and had been taken over from Mill as a "problem of natural philosophy, the solution of which is the ideal asymptotic goal of all sciences." Petrović claimed that "there are general statements, the least numerous ones, from which it is possible to derive all regularities existing in nature" and he believed that this "asymptotic goal of all sciences" was achievable in mathematical phenomenology.

We will demonstrate that, taken together, both of Petrović's assumptions in the philosophy of mathematics belong to mathematical platonism. Metaphysics and platonism are not customary in the philosophy of nature, and they are completely alien to the mechanicism that all Petrović's commenters find in Petrović's work. Possibly because when Petrović mathematically represents a phenomenon, he relies on classical mechanics, but he uses mechanics as a form of a language for mathematical representation of phenomena in nature, because he does not view mathematics itself and its language as the universal language of science.

To a different degree, natural sciences aspire to base and present their research in the framework of a mathematical model. The more elaborate and perfected it is, the more complete, reliable and ultimately closer to reality do we consider its results to be. This also holds true for a substantial number of social sciences, the contemporary development of which greatly relies on the presence of mathematics in their research. Mathematics is omnipresent in science and the greater its presence, the more perfect a branch of science is considered to be. Science believes that each phenomenon, everything that changes in the world, can be clad in an appropriate mathematical form. The usefulness of this can sometimes be nil, or it may even generate a completely distorted image about a phenomenon, but in the main, and in the case of natural phenomena in particular, the advantages brought by mathematics are immeasurable.

The mathematical representation of a phenomenon usually emerges quite independently of mathematics, in concrete individual endeavours by scientists to explain that phenomenon. With time, this representation is adjusted for results in mathematics: the already existing results and theories are used, or mathematics is given an impetus to arrive at new results on its own and develop new theories. Such relationship between mathematics and other sciences has existed since antiquity and is really quite natural. To understand this relationship, one should first answer a purely philosophical question:

Does each phenomenon conceal immanent mathematics in itself?

If the answer is yes, then it is a scientist's job to discover and determine the unique inner mathematics of the phenomenon he is studying. Also, in this way, mathematics is given the key place in science – it is the foundation and the binding tissue of all sciences and ensures their unity.

If the answer is no, the mathematics in which a given phenomenon is represented is created by the very science of it, relying on its degree of development. This does not at all mean that each phenomenon has its own unique mathematics, as there can be more of them or none at all. They are more or less useful tools for studying phenomena in nature and the world at large, and it is up to a specific science which of the tools it will use, including the possibility not to use mathematics at all in studying some phenomena. It is left to the intuition of the researcher whether to choose mathematics or not, and this choice can be motivated not only by scientific but also by very specific cultural, ideological or utilitarian reasons. In this case, mathematics does not have a special place among sciences, and is only significant to the extent to which it actually contributes to them.

Petrović's mathematical phenomenology is based on an affirmative answer to the above question, i.e. the position that each phenomenon can be mathematically represented. This certainly refers to phenomena in nature. Even more than that, Petrović believed that there are always the tools to enable this representation. He constructed these tools and considered them completely universal, applicable to all phenomena in nature. Petrović, therefore, assumed that each natural phenomenon has its mathematics, i.e. that each such phenomenon has a corresponding set of mathematical truths about it, which can at all times be determined. Although he himself did not argue in favour of the uniqueness of this set, it actually follows from his assumptions. Namely, if there were two such sets that contradict each other and are at the same time true for a given phenomenon, the science of nature would be contradictory. Hence, each phenomenon has its unique mathematics. If this holds true for individual phenomena in nature, as there is a "minimal set of statements from which it is possible to derive all regularities in nature," than the same thing holds true for nature taken as a whole, i.e. the entire world is built upon unique mathematics. This is an entirely metaphysical view and it is very close to Plato's. In the *Timaeus,* his Demiurge uses mathematical laws to assemble the world from Plato's bodies. In the beginning was mathematics, interwoven into the physical world which is entirely subject to its abstract and ideal laws; mathematics is its metaphysics.

It is possible that Petrović did not wish to go that far, but this is where his assumptions lead. On the other hand, as we have already mentioned, starting from the time of Descartes and under the impact of his rationalism supported by an overall development of natural sciences which brought fundamental change to the life of man, metaphysics and all forms of platonism were systematically removed from philosophy. Today, only their traces can be found in philosophy and they are not welcome at all, while in sciences they are deemed entirely unthinkable. Still, Petrović's leaning towards Platonism has its justification in a part of the philosophy of mathematics.

Namely, when the question about the presence and uniqueness of mathematics in natural phenomena is limited to phenomena in mathematics only, i.e. to purely abstract mathematical objects and statements, it then gets the following form:

Does mathematics express the actual relations of actually existing objects?

The affirmative answer to this question determines the most acceptable philosophy in mathematics, *mathematical platonism.* Mathematicians who never asked themselves this question are platonists, hence a vast majority. But this can certainly not be said for philosophers of mathematics, who by nature of their work certainly asked themselves this question. They, with only very few exceptions, give a uniform negative answer to it. This answer, however, does not constitute a uniform view in the philosophy of mathematics that would contradict platonism. The denial of each form of actual existence of some (or even all) mathematical objects as a rule restricts mathematics and calls for a special interpretation of the meaning of its regularities. Depending on the extent of this denial and for some other reasons, a whole range of views emerged in the contemporary philosophy of mathematics, which is possibly infinite: constructivism, formalism, intuitionism, logicism, conventionalism, structuralism... With the exception of logicism, which is quite close to platonism, the denial of platonism is the common point of all these philosophies of mathematics, but what they also have in common is that they contradict one another and that each of them individually is incomplete. And whereas platonism is a completely natural and effective viewpoint, all of its disputations have had the character of academic objections that only rarely shed light on anything, most often bear no fruit, and are irrelevant for mathematics itself. Because they are reduced to a denial of platonism alone, they have made the philosophy of mathematics in which they prevail to a large extent useless for mathematics, much like Aristotelian logic interpreted in the scholastic tradition. The position on the uniqueness of mathematics in each individual phenomenon makes Petrović quite close to mathematical platonism. True, it does not follow necessarily from mathematical platonism, but it is quite in agreement with it and strongly supports it. Hence, the Platonism of Petrović's assumptions or their support to this position are not disputable at all when it comes to mathematics, though they may seem that way in the modern-day philosophy of nature to which all form of metaphysics is alien. Mathematical platonism was advocated by many mathematicians and philosophers from Petrović's time, including Cantor, creator of the contemporary set theory, Frege, who laid the foundations of the syntax of contemporary mathematical language, and finally Gödel, the greatest logician in history if judging by his works. He himself stated that platonism enabled him to understand the relationship of syntax and semantics on which he based his results.

Though they have a philosophical justification, Petrović's assumptions definitely call for additional explanations in the light of achievements of contemporary logic, i.e. in the light of Gödel's incompleteness theorems. The problem is that Gödel's theorems show that mathematics cannot be based on a simple non-contradictory set of assumptions. For Petrović's assumptions, this means that two questions need to be answered: *which part of mathematics is necessary to describe a given natural phenomenon, and which part is necessary to describe all phenomena in nature?* If that part of mathematics contains the summing and multiplication properties and is closed for mathematical induction, Gödel's theorems make this Petrović's thought impossible to be realised. By contrast, although contemporary science uses very complex mathematics, primarily the tools of mathematical analysis, i.e. the tools of second-order arithmetic, this still does not mean that they are necessary in the description of phenomena in nature. It is quite possible that this part of mathematics of natural phenomena is much less complex or even very simple. In physics, for instance, opinions are greatly mixed with regard to this question. If it is indeed possible to unify the general relativity theory and the quantum field theory, as postulated in the so-called *theory of everything,* we would obtain a complete description of all physical phenomena. Hence Petrović's other assumption, when reduced to physics, would get a significant justification, but would still not be confirmed. Stephen Hawking, one of its key proponents, believed that Gödel's theorems quite seriously put into question the theory of everything[96]. A similar view is held by Roger Penrose[97] because he believes that there are phenomena in nature that are not computable. On the other hand, a group of authors advocate a completely contrasting view, a variant of Church's thesis according to which all physical processes are computable. This position is justified by the fact that the speed of exchange of information in nature is limited by the speed of light. The theory of everything in which the tools of its mathematics would be limited only to the decidable fragment of arithmetic would entirely confirm Petrović's second assumption. In addition to the above, there is also a completely realistic possibility that there is a theory of everything, as a complete theory of the world of physics, while at the same time its mathematics is incomplete, i.e. the postulates that are not provable in this mathematics do not relate to purely physical regularities at all.

Petrović's aspirations to set the foundations of a science that could tailor an appropriate mathematical framework for each phenomenon, and his belief that the entire world is structured according to unique mathematical laws the assumptions of which can be determined, have not yet found their confirmation in science. Contemporary logic has undoubtedly proved that each language, even the universal language of mathematics, has a limited power of expression. To the extent that today's science relies on mathematics, where such limitations have been proved, it is natural to face the limited power of science as well. There is no justification for its omnipotence. One could rather say that this power is not even sufficient for a complete description of some more complex natural phenomena. This is possible in some ideal cases only, but by no means can a complete description of the entirety of nature be provided.

Mathematical phenomenology should be seen as a separate large-scale project in the total opus of Mihailo Petrović. Philosophers and scientists often work on several independent projects, spending some time on one, then moving on to another, while often some of the projects remain unfinished. In parallel with working on mathematics and philosophy, throughout his life Leibnitz worked on four such projects simultaneously: logic, ideal language, encyclopaedia of knowledge and a general scientific method.[98] They all remained unfinished and, logic excluded, the other three great projects by Leibniz strikingly overlap with and permeate Petrović's mathematical phenomenology. The ideal language is conceived as a universal symbolical language for science, mathematics and metaphysics, which would be "a basis of the calculation or the algebra of thinking," the encyclopaedia of knowledge was his systematic collection intended to enable the realization of the project of the ideal language, while within the project of the general scientific method, Leibnitz tried to formulate a procedure for an accelerated expansion of knowledge. Mathematical phenomenology fits quite well in the general scientific method project and it is possible that Petrović was inspired by the idea of accelerated expansion of science and knowledge of phenomena. Unlike Leibnitz, who found the cornerstone of his grand project of commonality of

Gottfried Wilhelm Leibnitz (1646–1716), German philosopher, painted by Christoph Bernard Francke, before 1729 (Herzog Anton Ulrich-Museum, Braunschweig) science in logic, but not necessarily in the logic of his time, Petrović sought the foundation stone of his science in classical mechanics. To him logic seemed empty, and the relationship between the cause and the effect too abstract, so he tried to formulate the concept of a cause in "concrete sciences" in which it "always appears inseparable from its substratum and its material nature." He gave his concept of cause "a natural-science-like form" implying that the cause is "every phenomenon which aspires to change some state of affairs or to introduce perturbation in some other phenomenon." At the time when he was laying his groundwork for mathematical phenomenology, contemporary logic was in its early days, hence logic was reduced to Aristotle's teachings about the four types of categorical statements. He had every reason to turn his back on such logic and base his mathematical phenomenology "closer to nature." He constructed a not entirely articulate conceptual apparatus with "active causes" and "necessary effects" on which he built a kind of a general mechanics of phenomena modelled on classical mechanics. Quite possibly, Leibnitz's and Petrović's attempts to reinstate unity to science and knowledge did not yield the expected results for the same reason, because of the Aristotelian logic. Petrović did not want to rely on that logic due to its perceived shallowness, but with this he also rejected contemporary logic which was developing powerfully in parallel with his phenomenology and which could have been useful to him. Leibnitz formulated the first contemporary logical systems and was the first to identify the significance of language for logic and science. Therefore it can be said that he tried to realize his project with some of the tools of contemporary logic, whose forefather he had been, but his hands were tied by the scholastic legacy in logic from which he could not free himself. Therefore in both Petrović's and Leibnitz's case, the problem was that, at the time when they were beginning their project of description of all phenomena, mathematics did not have its own language. In search of this language, Petrović relied on the language of classical mechanics, the expressive power of which lagged far behind the power of the contemporary language of mathematics. Leibnitz did try to establish such a language, but he had problems with its logical basis.

When Petrović began his project of mathematical phenomenology, the process of division of the existing branches of science, and the emergence of new disciplines, was in full swing. He held that for the benefit of science this process should not go too far, and believed that this fragmentation of science could be counteracted, and its former unity restored. In his view, mathematics was to play a key role in that and to become the main counterweight to the process of auto-fragmentation of science and knowledge in general. He was partially right in the sense that the language of mathematics and its syntax, created in the early 20th century in contemporary logic[99], became the theoretical basis for the development of the syntax of programming languages that enabled the realization of digital computers.[100] With them, mathematics entered the world of science in great style and perhaps created conditions for a renewal of its unity, to which Petrović had aspired.

MATHEMATICAL PHENOMENOLOGY BETWEEN MYTH AND REALITY

Nikola PETROVIĆ MORENA
Morena inženjering, Niš

Mathematical phenomenology is a relatively well-known term in Serbia owing to the works of Mihailo Petrović Alas. Even though none of Petrović's students or the other Serbian scientists continued the work in that field (or perhaps for that very reason), it acquired a certain mystical aura. At times, one could read (or, even more likely, hear) an opinion that Petrović's phenomenology has yet to be properly interpreted. However, searching the Internet for the term "mathematical phenomenology" yields just a few results, apart from the translations of Serbian papers, and contemporary encyclopaedias, for the most part, do not contain an entry referring to that topic. Does mathematical phenomenology exist at all or is it a part of national mythology nurturing the stories about the unrecognized grandeur of our scientists? If it exists, what is mathematical phenomenology concerned with and how can it be distinguished from mathematical modelling which, at least at first glance, seems very similar to it? And finally, what is the unique contribution that Petrović made to that field? This investigation represents an attempt to give some of the possible answers to those questions.

Immanuel Kant (1724–1804)

PHENOMENOLOGY AS A PHILOSOPHICAL CONCEPT

The word "phenomenology" is derived from the Greek words *phainómenon* ("appearance") and *lógos* ("study, research"), so one of the meanings of this term is "the study of phenomena". If we translate *lógos* differently in this compound word,[101] we will get alternative meanings of phenomenology – "the appearance (revelation) of the first principle, the all-encompassing law or spirit".

In *The Critique of Pure Reason* (1781), Immanuel Kant pointed out the difference between the "phenomena", man's interpretation of an object or an event based on the information apprehended by the senses, by reasoning or through experience, and the "noumenon", an object or an event in itself, which is inaccessible to man. According to Kant, human reason is actively involved in acquiring knowledge about the world, trying to put the phenomena into "matrices" that are already present in his consciousness. The main matrices exist in man a priori, that is, prior to experience.[102] Such a priori matrices include space and time. Science deals with the world of phenomena, with the apprehensible, and theology with the unknowable, the noumenon.[103] Kant has proved that striving towards the metaphysical truths of the noumenal world through reason invariably ends up in contradiction.

Hegel accepts Kant's distinction between phenomenon and noumenon, but denies his claim that the essence of things is not apprehensible by man. In *The Phenomenology of Spirit* (1807) and other works, Hegel develops an idea of phenomenology as a philosophical method which starts with what can be apprehended by the consciousness – phenomena, and by deepening the knowledge about the phenomena, ends in reaching the absolute, metaphysical spirit – logos, which lies behind phenomena.

By the end of the nineteenth and the beginning of the twentieth century, positivism had become a dominant trend in philosophy. Positivism aims to deal only with the things that are "positive", verifiable, and it banishes metaphysics from philosophy.[104] Within the positivist approach, phenomenology has been equated with the scientific view according to which it is vital that the phenomena of the so-called empirical world be described and anticipated as accurately as possible, without ever questioning their purpose.

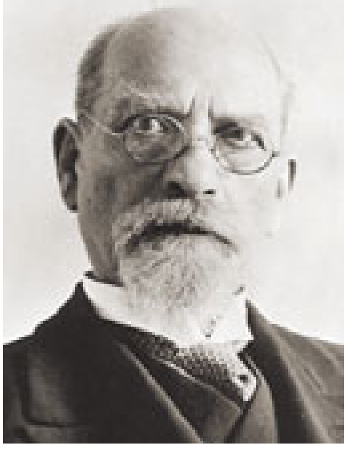
In the 1920's, Edmund Husserl, a German mathematician and philosopher of Jewish origin, laid down the foundations of phenomenology as an independent philosophical movement and made the term become more widely used. Husserl makes a departure from positivism, asserting that there exists a spiritual reality independent of the material world and that studying that reality is to be the basic aim of science, which has "strayed", especially in Europe, focusing only on the empirical and natural. The essence of things and phenomena (Husserl uses the term "essences") exist in our consciousness and we can grasp them by gradually discarding all that is variable in the phenomena. That process is called the *phenomenological reduction*. According to Husserl, phenomenology is a method of philosophical investigation which requires that the investigator eliminate all preconceptions and assumptions,[105] which enables him to look at things with an open mind and to understand their meaning through the interaction between his own consciousness and the observed object [Moustakas 1994].

Natural sciences are founded upon axiomatically accepted paradigms and hypotheses. An astronomer assumes the physics to be accurate and a physicist relies on the truthfulness of mathematics in the same way as a mathematician relies on logic. The advantage of a phenomenological method lies in the fact that the analysis begins with lived experience and does not require any a priori assumptions whose validity lies beyond the domain of a concrete investigation. Owing to that, a phenomenological investigation has got a fundamental character. One of the axiomatic assumptions of the natural sciences is the existence of reality that lies outside of man's consciousness and is independent of it. A phenomenologist rejects such an assumption. He does not deny the possibility that such a reality exists, he does not even doubt it, he simply refrains from passing judgement on that matter. A phenomenologist is trying to explain the world strictly by analyzing the experiences within his own consciousness through systematic reflection. Phenomenology is trying to build a framework for an objective, scientific studying of topics that are usually considered subjective, such as consciousness, reasoning, perception, or emotions, but using methods different from those applied in clinical psychology or neurology.

Unlike analytical philosophy, which is mostly concerned with analysis of utterances and sentences, phenomenology deals with experiences and their structure [Пивчевић 1997]. The structure of a linguistic utterance, as phenomenologists claim, cannot really be understood without analyzing the structure of experiences that lend meaning to those utterances.

The goal of phenomenology lies not in new empirical knowledge, but in understanding our fundamental relation to the world which

Georg Wilhelm Friedrich Hegel (1770–1831)



Edmund Husserl (1859–1938)

Martin Heidegger
(1889–1976)

Jean-Paul Sartre
(1905–1980)

Maurice Merleau-Ponty
(1908–1961)

comes before any empirical investigation [Zahavi 2008: 664–665]. It endeavours to describe rather than analyze, and that is the *fundamental phenomenological instruction* [Merleau-Ponty 1990]. Thus, in the Husserlian context, phenomenology acquired a markedly different meaning from the one used within positivism.

A similar view is held by a German psychotherapist Bert Hellinger, according to whom there are two ways towards an insight. One traverses the unknown, revealing to our reason the secrets of the world around us, step by step. That is the way of science. The other way requires of us to stop our trying to understand things and, in turn, let our attention become ever broader in scope, ever more expansive, until it is able to contain the whole instead of the parts. Such renunciation of analysis and giving priority to sensory experience represents the foundation of the phenomenological method.

The further development of phenomenology during the twentieth century has brought new, often critical interpretations of Husserl's views and new philosophical trends, including the existentialism. The most prominent philosophers of the twentieth century who based their doctrines on phenomenology were Martin Heidegger, Jean-Paul Sartre and Maurice Merleau-Ponty.

PHENOMENOLOGY AS A SCIENTIFIC METHOD

In science, phenomenology is a method which comes to the essence of a matter (noumenon) by using written accounts of lived experience (phenomena) as sources of knowledge [Conklin 2007: 275]. Phenomenological method is used in statistical sociological research, when the members of a target group in a research study are asked to respond to the questions presented in forms. Their responses are then coded in numerical values which are amenable to a mathematical treatment, mostly with a view to establishing an average value.

A scientific theory which mathematically expresses the findings of an investigated phenomenon, without considering the essence of the phenomenon (noumenon), which lies behind it, is called a phenomenological theory [Thewlis 1993: 248].

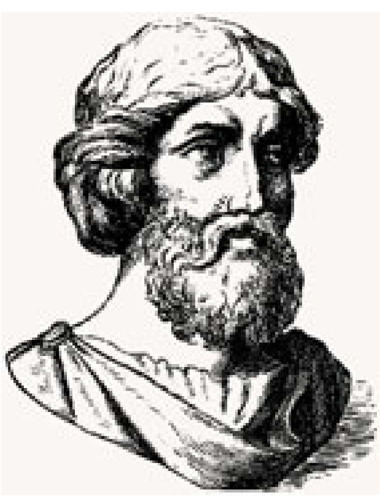
There are scientific subjects, such as astronomy, in which there is a very limited scope for conducting experiments. It is very hard to analyze the appearance and disappearance of a star in laboratory conditions [Божић 2005: 24]. In subjects of this kind, studying the phenomena is the basic scientific method.

PYTHAGORAS AS A PRECURSOR OF MATHEMATICAL PHENOMENOLOGY

If phenomenology is understood to be a process of reducing the phenomena to their "essences", and mathematical phenomenology is viewed as the kind of phenomenology that finds the said essences in numbers and their relations, then Pythagoras can be said to represent a precursor of mathematical phenomenology. He thought that the world had a mathematical character at its root. Studying music, he spotted the relation between the length of a string on a lyre, the most popular musical instrument in his time, and the frequency of the tone the string produces when it oscillates. Observing astronomical phenomena, the orbits of the planets, the length of days and nights, he found everything to be numerically related. That all things are number was the main Pythagorean dogma [Божић 2010: 53].

Moreover, Pythagoras and his disciples, the Pythagoreans,[106] had not only thought the world could be described in terms of numerical relations, but also that numbers as well as nature are governed by the same principle: both the infinite series of "natural numbers" and the whole universe represent the relation between the finite and the infinite – "peiron" and "apeiron". The Pythagoreans have taken over the terms peiron and apeiron from Anaximander, and they complemented his teachings with the thesis that the relation between apeiron and peiron is ruled by the principle of harmony. For instance, the infinite series of musical tones (apeiron) has to be limited in some way (peiron) in order to form a scale. However, we cannot choose any series of tones so as to produce a scale that is to be musically agreeable, harmonious. Only the tones whose frequencies are related by whole-number ratios sound harmonious! In the same manner, the universe and all living creatures in it are not created by a random combination of finite and infinite elements, but by elements combined in a harmonious way in numerical proportions, which all together constitute the cosmic order.

What the Pythagoreans meant by a number was what we now call positive rational numbers – natural numbers and their relations. When they had discovered that in nature there are values that cannot be represented by rational numbers, such as the diagonal of a square whose side is a natural number, their view of the world was shaken up to its foundations. That discovery was kept as a great secret of the inner circle of Pythagoras's followers. It is said that certain Philon divulged the secret, and was forced to commit suicide by jumping from a high cliff into the sea.



Pythagoras

In Pythagoras's reduction of everything to a number, we can sense the method that much later Husserl will come to call the phenomenological reduction. Pythagoras was not interested in the empirical (or "scientific", as we would now say) approach to phenomena, he was only trying to describe them using a mathematical model without entering into the analysis of the causes of a phenomenon. This is especially evident in Pythagoras's astronomical investigations. *The emancipation of mathematics from empiricism*" [Божић 2010: 126], which had been conducted by the Pythagoreans, enabled mathematics to start developing as an independent discipline.

THE PHENOMENOLOGY OF "MATHEMATICAL PHENOMENOLOGY"

Even though the ideas upon which mathematical phenomenology is based had appeared much earlier, the term itself started to be used in the late nineteenth century in parallel with the development of the positivist school of philosophy.

The claim that mathematics is the only solid epistemological paradigm and that every scientific finding has to be based on it (or even reducible to mathematics) was made among the first by a French philosopher René Descartes in his work *Discourse on the Method of Rightly Conducting One's Reason and of Seeking Truth in the Sciences* of 1637 [Божић 2010: 183]. A very influential work of Isaac Newton, *Mathematical Principles of Natural Philosophy*[107] of 1687, contributed to spreading the idea that all exact sciences have to be amenable to a mathematical formulation.

Among world-renowned scientists in the late nineteenth century, the term mathematical phenomenology was used by an Austrian physicist Ludwig Boltzmann, German physicist Gustav Kirchhoff, after whom the laws on the conservation of the quantity of charge in closed electrical circuits were named, as well as Heinrich Hertz, a German scientist after whom a unit of frequency was named [Hon and Rakover 2001: 9]. In the introduction to the book *The Principles of Mechanics,* which was posthumously published in 1894, Hertz wrote that physicists must focus on finding equations by which they can determine the development of phenomena in quantitative terms, without using any hypotheses, non-mathematical models or mechanical explanations. According to Hertz, Maxwell's theory is a classical example of such an approach. To the question – "What is Maxwell's theory?" – there is no shorter or more accurate answer than the following: "It is Maxwell's system of equations", wrote Hertz. Such an approach was very well-known to the positivists, who were on the rise at the time. Their ideal was not only philosophy freed from metaphysics, but also physics (and science in general) freed from "mythology", that is, from metaphysical systems that are trying to make sense of the world. The question "why" had been banished from science, and the pivotal place was taken over by "how much" and "in what manner". The majority of positivists later softened their original views, presenting them as a reaction to the historical moment in which philosophy had been dominated by German idealism.

René Descartes (1596–1650)

In the article on models for the tenth edition of *Encyclopaedia Britannica* of 1902, Ludwig Boltzmann explained that mathematical phenomenology represents a specific view on the nature of physical theories according to which the goal of a physical theory should primarily be the construction of mathematical formulae by which the observed phenomenon can be quantitatively described in a way that is closest to reality. Boltzmann named Kirchhoff and his school as typical representatives of mathematical phenomenology. "Mathematical phenomenology is a presentation of phenomena using mathematical analogies", wrote Boltzmann (1902).

Boltzmann also wrote about a radical variant of a mathematical phenomenology framework, according to which the equations describing a phenomenon are more important (or, at least, more purposeful) than an attempt to discover the cause of a phenomenon [Feuer 1989: 337]. According to that viewpoint, the hypotheses-paradigms based on which the equations are formed are not permanent and they change with the development of science, but empirically conducted and verified mathematical formulae describing physical phenomena remain valid even after a paradigm shift, except possibly in marginal scopes of measured values having lied outside of the domain of empiricism when the formulae were being created [Boltzmann 1901: 248–250]. Boltzmann has criticized the phenomenological approach by stating that it is impossible to understand nature by relying solely on the empirical. He has especially criticized mathematical phenomenology, claiming that no set of equations can ever give a complete description of a phenomenon.

Since the end of the first decade of the twentieth century, apart from the papers related to Mihailo Petrović Alas, very few references in literature related to the term "mathematical phenomenology" have been published. The most comprehensive world encyclopaedias (even those specializing in mathematics) do not contain entries regarding that field. According to data available on the Internet, the only higher education institution holding a course in mathematical phenomenology is "Waseda School of Science and Engineering" in Japan. However, it should be pointed out that even to this day there are many scientists, magazines and scientific conferences dealing with the relation between mathematics and phenomenology.

THE DIFFERENCE BETWEEN MATHEMATICAL PHENOMENOLOGY AND MATHEMATICAL MODELLING

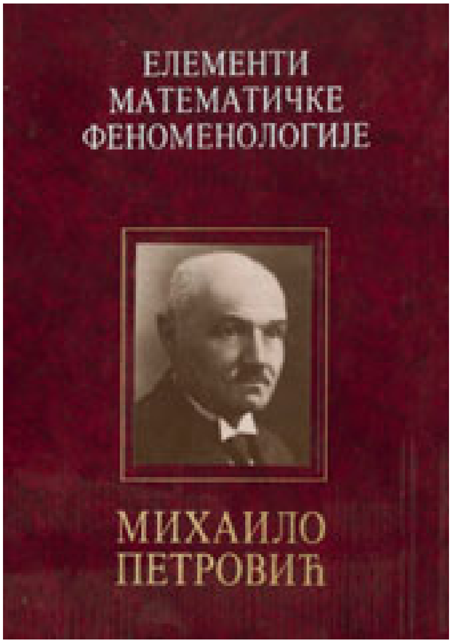
Unlike mathematical phenomenology, the term mathematical modelling is in widespread use. There are thousands of books and university courses dealing with mathematical modelling.

A mathematical model is a description of a system by using mathematical language and concepts. The process of creating a mathematical model is called mathematical modelling. The difference between this definition and Boltzmann's definition of mathematical phenomenology (the presentation of phenomena by using mathematical analogies) is a subtle one and lies at the level of linguistic and philosophical preferences.

It might be said that mathematical modelling is a practical skill within the domain of applied mathematics, whereas referring to mathematical phenomenology usually entails a philosophical viewpoint or at least a philosophical background. This distinction is not sharply defined, as Boltzmann himself [1901: 250] pointed out that mathematical phenomenology has got a primarily practical purpose. The author of this paper is of an opinion that between mathematical phenomenology taken in positivist context as defined by Boltzmann on the one hand, and mathematical modelling on the other, there is no essential difference. In Boltzmann's time, mathematicians were almost invariably philosophers, as well. However, the twentieth century saw many generations of mathematicians-craftspeople, and the term mathematical phenomenology was accordingly superseded by an intellectually less challenging term mathematical modelling.

Boltzmann's definition of mathematical phenomenology is not the only one. Some contemporary authors, such as professor Doorman from Utrecht University, think that *mathematical phenomenology refers to how mathematical ideas structure and organise phenomena* [Doorman 2005: 59]. Thus defined, mathematical phenomenology represents, in fact, the philosophy of mathematical modelling.

Ludwig Boltzmann (1844–1906)



The cover page of *Elements of Mathematical Phenomenology*, *Collected Works*, Book 7 (Digital Legacy of Mihailo Petrović)

MATHEMATICAL PHENOMENOLOGY OF MIHAILO PETROVIĆ ALAS

Mihailo Petrović Alas (1868–1943) is a renowned Serbian mathematician, physicist, travel writer, violinist and a fisherman. As a government scholarship holder, he obtained his doctorate in mathematics and physics in Paris. He spent his whole scientific career working at the University of Belgrade. The greatest impact on the community of experts was made by his papers in the field of differential equations. He is one of the first Serbian scientists whose works have been cited in Europe. His interests were very versatile and reinforced by his encyclopaedic knowledge [Трифуновић 1998: 366]. He travelled as a member of scientific expeditions to the North and South Pole. His virtuosity on the violin was recorded on one of the earliest audio files of that instrument produced by Radio Belgrade [Трифуновић 1991: 15]. It could be said that he was a polymath, homo universalis[108], one of the rare Renaissance people in modern Serbia (Божић 2005). Mihailo Petrović Alas had friends among people from diverse social backgrounds and in Belgrade downtown community he has been remembered as "our Mika".

Out of the fifteen volumes of his collected works published by "Zavod za udžbenike i nastavna sredstva", two volumes comprising about 1000 pages in total are devoted to mathematical phenomenology. They include twenty published Petrović's works (books, papers, speeches...) on that topic and list more than one hundred references in which other authors commented on Petrović's phenomenology. The editor entitled those volumes *Mathematical Phenomenology* and *The Elements of Mathematical Phenomenology*.

At the heart of Petrović's interest in those works lies the concept of phenomenological mapping. He realized that the phenomena from different areas of human experience (Petrović's term: "disparate phenomena") can be reduced to, that is, mapped onto the same abstract essence (Petrović's term: "a phenomenological type of facts") [Петровић 1998a: 13]. For instance, the phenomena of the height of tsunami waves being reduced as their distance from the place of origin increases, the waning of military power of a conqueror in the face of vast expanses, a reduction in the intensity of light in proportion to its distance from the light source, all of which represent disparate phenomena that belong to a common phenomenological type – weakening in proportion to expanding.

The turning of the tide or of the day into night have the same phenomenological type as some phenomena disparate in relation to those, like the menstrual cycle – periodical changes induced by a periodical cause. There is an obvious analogy with Husserl's terminology: phenomenological mapping refers to phenomenological reduction, and phenomenological types refer to essences. The roles (Petrović's term: "phenomenological beings") contained in the phenomenological type of facts are independent of the concrete nature of the holder of the role. In our first example, tsunami, military charge and light wave have the role of an impulsive factor, whereas the ocean, the expanse of a state that is being conquered as well as the space through which the light wave is spreading have a territorial role.

What is the goal of phenomenological mapping according to Petrović? The goal is to step a little closer to the ideal, ultimate goal of "positive philosophy", the reduction of an infinitely colourful view of the world to the most simplified sketch that underlies it, but such that the original picture could be reproduced from it by adding specific, phenomenologically insignificant details that do not contradict the sketch [Петровић 1998a: 17]. The significance of phenomenological mapping lies in the possibility of predicting the details in the phenomena whose phenomenological type we have identified even if we do not understand their essence. Those details are not exclusively related to the number and for that reason Petrović proposed founding a new subject whose methods would include all the details that could be completely abstracted from concrete phenomena and studied in themselves, as it is done in mathematics with abstracting numbers [1998a: 18]. Petrović defined that subject as "a new branch of the philosophy of nature that would comprise general methods for predicting phenomena based on the nature of the roles of the factors that represent the cause of a phenomenon" [1998b: 14]. What is the name of that new branch of the philosophy of nature?

A more attentive reader would be perplexed, even upon browsing through the volume entitled *Mathematical Phenomenology* – none of the five Petrović's works published in it contain in their title, or even in the chapter headings, the term mathematical phenomenology. The editor of these collected works mentions this illogicality himself in the afterword and he justifies it by saying he did not know how to name the field with which Petrović had dealt with in these works and that he gave that title to the volume "in order to arrive at a natural and necessary unity" [Трифуновић 1998: 420] with the title of the following volume, in which the central place is held by Petrović's work *The Elements of Mathematical Phenomenology.*

And indeed, it had not been an easy task for the editor. Although Petrović has kept the said definition of the subject of his research for about forty years of working in that field, the terms he used for that field varied a lot. In his inaugural address on the occasion of being elected a full member of the Serbian Royal Academy in 1900, Petrović named it "a mathematical theory of actionality", only to change the title into "a mathematical theory of the activity of causes" for the print version of the very same speech [1998c: 222]. Five years later, in the debate *An Attempt at a General Mechanics of Cause,* the field became "the general mechanics of cause" [Трифуновић 1998: 274], and in 1911 in *The Elements of Mathematical Phenomenology,* Petrović named it "mathematical phenomenology". However, he soon relinquished even that appellation

Branislav Petronijević
(1875–1954)

and in the book *Analogies as a Basis of General Phenomenology,* of 1922, he uses the term "general phenomenology". In his philosophically most complete work, *Phenomenological Mapping,* of 1933, Petrović is very careful to avoid using the term "mathematical phenomenology" and in general naming the field of his research, except in one place where he called it "mathematics in an extended sense" [Петровић 1998a: 18].

In spite of the proverbial saying "you can call a pot a jug if you will, as long as you don't break it", it is very likely that Petrović's undecidedness about naming that field was one of the reasons for the fact that his work did not have a wider reception. One has to admit, it is very difficult to popularize a field which even its pioneer cannot name. It is evident that mathematical phenomenology was only one of the terms Petrović had used and later discarded. Moreover, it is clear that Boltzmann's encyclopaedic definition of mathematical phenomenology (a presentation of phenomena using mathematical methods) refers only to a subset of Petrović's mathematical phenomenology as a new branch of the philosophy of nature, which comprises general methods for predicting phenomena based on the nature of the roles of the factors that represent the cause of a phenomenon. It seems reasonable to assume that Petrović had realized that the term mathematical phenomenology was already burdened with the other, narrower meaning, so he decided not to use it in order to avoid causing confusion.

If we accept the "fundamental phenomenological instruction" and pay attention to Petrović's descriptions instead of the terms he used, it is clear that it is philosophy based on phenomenological concepts. Which area of philosophy is this?

In *Phenomenological Mapping* Petrović wrote that phenomenological types of facts and phenomenological beings cannot be discovered within any single scientific field because, irrespective of how broad a field is, it is always concerned with one concrete nature of phenomena.

In order to reach the essence, “in our mind we should erase the boundaries between certain fields and view the world directly, the world in which one and the same details thread through the infinite colourfulness of external phenomena, their outer guise” [Петровић 1998a: 12]. Only then is it possible that, by direct observation, scientific analysis or poetic intuition, we can manage to abstract a common core out of the myriad of disparate phenomena. Therefore, Petrović’s method requires a return to the original philosophy, the first philosophy, such as it had been before specific sciences were derived from it. In spite of the fact that Petrović frequently refers to a “positive philosophy”, his science is the science of being – metaphysics! That may explain why he failed to “circumscribe” the field of his research by some term, and put it within the framework of mechanics, classical mathematics or any other specific scientific field.

The Swiss psychiatrist C. G. Jung would call it an instance of synchronicity that almost at the same time when Petrović was writing *Phenomenological Mapping* in Belgrade, only several hundred kilometres to the north, in the Hungarian plains, Béla Hamvas began his essay *Poetica Metaphysica*[109] with the following words:

> “There were times and there are peoples whose religion, science, philosophy and poetry were all one, and are one even now... However, at times like these, reality has different areas, layers, planes, and they are segmented from one another... The one who crosses from one field into another is looked upon as though he has made a false step. The one who breaks barriers, bring areas together, joins the planes, is simply said to have become insane.”

Was it the very fear of the reaction of the public and the loss of reputation of a positivist mathematician that prevented Petrović (and consequently the editor of his collected works) from defining this area more clearly as deeply philosophical? Or perhaps he did it out of consideration for his colleague from the University, the philosopher Branislav Petronijević, who had conceived as his life’s work precisely what Petrović managed to do to a much greater extent – to put metaphysics on the firm ground of logic, notwithstanding the prevalent view holding that even Kant in his time had proved it to be impossible?[110]

The fact that Petrović has made a solid philosophical system is also demonstrated by his original ontology. In his phenomenological works, Petrović has introduced over one hundred terms that are either completely new or he has given them a quite different meaning [Трифуновић 1998: 416–420].

Petrović's universality is also reflected in the fact that he was one of the rare philosophers who were both metaphysicists and practitioners. The book *Phenomenological Mapping* offers on hundreds of pages not only detailed instructions for inductive abstraction of phenomenological types of facts from versatile phenomena, but also quicker, deductive methods for their identification, followed by the methods for predicting future phenomena solely based on thus abstracted types, as well as the methods for inverse phenomenological mapping, which enables an abstract phenomenological type of facts to be mapped onto an insufficiently known concrete area (of nature, psychology, economy, etc.) and predict phenomena within it. For those who deal with artificial intelligence, a matter of particular interest is Petrović's vision of a general phenomenology as a tool which, once it is sufficiently developed, "will have that power to think for us and yield results surpassing human reasoning" [1998a: 20]. The other Petrović's seminal work from this area, *The Elements of Mathematical Phenomenology*, focuses on those types of analogies among disparate phenomena that can be expressed by the existing, classical mathematical apparatus, particularly by differential equations. With regard to that matter, Petrović is completely within the area of his specialized expertise, and extensive knowledge of mathematics is required in order to fully understand Petrović's accomplishments in that area.

Many ideas of Petrović, (probably) independently of him, have been further developed in such diverse areas as cybernetics, psychology, economics or mythology. For instance, Petrović stated that concrete myths of various peoples are frequently very similar, because their essence is represented by the same phenomenological type of facts [1998a: 197–208]. Jung has arrived at the same conclusions, only using different terminology – Jung's archetypes of collective unconscious are completely compatible with Petrović's phenomenological type of facts.[111] Petrović has gone even further than Jung, seeing in paradigms of contemporary science "scientific mythology", which is only another expression of the same phenomenological type of facts that is to be found in classical mythology. What is the modern scientific entity of "force" which pulls, pushes, attracts, but a manifestation of the same phenomenological being that also represents the essence of mythological Eros, concluded Petrović [1998a: 199].

Transactional analysis of a Canadian psychiatrist Eric Berne (whose most famous book –*The Games People Play* has been translated into Serbian as *Koju igru igraš?*) shows that social interaction functions in terms of small variations of a limited set of scenarios and roles. The said concepts are completely analogous with Petrović's phenomenological roles and types.

In investigations of "small-world networks" it has been noticed that many networks, like social networks (e.g. Facebook), neural networks in the brain or the Internet, have an unusual feature in that an average distance between two randomly chosen knots (a person, neuron, computer), measured by the number of knots between them, is much shorter than it could be expected considering the size of the network and that it is proportional to the logarithm of the total number of knots in the network.[112] That is only one illustration of what Petrović used to call mathematical analogies in disparate facts [1998a: 71].

An advantage of Petrović's general phenomenology in relation to the above-mentioned examples lies in its universality – while Jung's and Berne's investigations refer to isolated areas of human experience, and small-world networks look at one type of analogies among disparate phenomena, Petrović's phenomenology includes all the areas as well as all the types of analogies.

The term "mathematical phenomenology", as well as the word phenomenology itself, is polysemous. In the most frequent, positivist context, mathematical phenomenology denotes a presentation of phenomena by using mathematical analogies, which is very close to mathematical modelling. In another context, it refers to investigating the ways in which mathematical ideas structure and categorize phenomena, in which case it is associated with the philosophy of mathematical modelling. Mihailo Petrović Alas founded a new branch of the philosophy of nature, which comprises general methods for predicting phenomena based on the nature of the roles of the factors representing the cause of a phenomenon. That field of his research was at one period named mathematical phenomenology, but later he relinquished that term with reason, because a presentation of phenomena by using mathematical analogies represents only one component of his philosophy. Petrović's original contribution to phenomenology lies in developing universal, practically applicable methods of phenomenological reduction and inverse phenomenological mapping, which could also readily be applied in contemporary artificial intelligence.

REFERENCES

Boltzmann, L. E. (1901). "The Recent Development of Method in Theoretical Physics". *The Monist*, Vol. 11 (No. 2).

Boltzmann, L. E. (1902). "Model". In: *Encyclopedia Britannica*. London. Retrieved from http://www.muellerscience.com/MODELL/Definitionen/Encyclopaedia_Britannica.htm

Божић, М. (2005). „Математичка феноменологија Михаила Петровића". У: *Легенде Београдског универзитета* (23–27). Београд: Универзитетска библиотека Светозар Марковић.

Божић, М. (2010). *Преглед историје и филозофије математике*. Београд: Завод за уџбенике.

Conklin, T. (2007). "Method or Madness: Phenomenology as Knowledge Creator". *Journal of Management Inquiry*, 16.

Doorman, L. M. (2005). *Modelling motion: from trace graphs to instantaneous change.* Utrecht: Dissertation Utrecht University. Retrieved from http://igitur-archive.library.uu.nl/dissertations/2005–0311-094207/full.pdf

Feuer, L. S. (1989). *Einstein and the Generations of Science*. New Jersey: Transaction Publishers.

Hon, G. and S. S. Rakover (eds.) (2001). *Explanation: Theoretical Approaches and Applications (Synthese Library)*. Berlin: Springer.

Hartimo, M. (2010). *Phenomenology and Mathematics*. Berlin: Springer.

Merleau-Ponty, M. (1990). *Fenomenologija percepcije*. Sarajevo: Veselin Masleša/Svjetlost.

Moustakas, C. (1994). *Phenomenological Research Methods*. New York: SAGE Publications.

Петровић, М. (1998a). *Феноменолошко пресликавање*. Сабрана дела, 6. том. Београд: Завод за уџбенике и наставна средства.

Петровић, М. (1998b). *Елементи математичке феноменолоīије*. Сабрана дела, 7. том. Београд: Завод за уџбенике и наставна средства.

Петровић, М. (1998c). *О математичкој теорији активности узрока*. Сабрана дела, 6. том Београд: Завод за уџбенике и наставна средства.

Pivčević, E. (1997). *Na tragu fenomenologije*. Zagreb: Globus.

Poper, K. (2002). *Pretpostavke i pobijanja*. Sremski Karlovci: Izdavačka knjižarnica Zorana Stojanovića.

Thewlis, J. (1973). *Concise Dictionary of Physics*. Oxford: Pergamon Press.

Трифуновић, Д. (1991). *Бард српске математике Михаило Петовић Алас: прилоī интелектуалној биоīрафији*. Београд: Завод за уџбенике и наставна средства.

Трифуновић, Д. (прир.) (1998). *Сабрана дела Михаила Петровића Аласа,* 6. том. Београд: Завод за уџбенике и наставна средства.

Zahavi, D. (2008). "Phenomenology". In: *Routledge Companion to Twentieth-Century Philosophy*. London: Routledge.

MIHAILO PETROVIĆ ALAS AND MODERN COGNITIVE SCIENCE*

Đorđe VIDANOVIĆ
University of Niš, Center for Cognitive Sciences

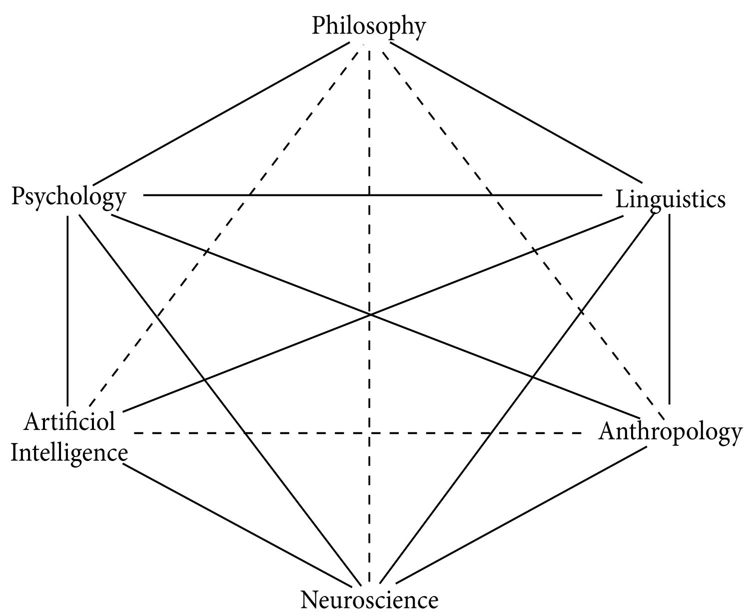
In this paper we plan to analyze and compare the work of our distinguished mathematician Mihailo Petrović Alas, from the early 20th century presented in his seminal book titled *Elements of Mathematical Phenomenology,* published in 1911, with a book authored by the American philosopher and cognitive semantician Leonard Talmy that came out in 2000. Our analysis will present a brief outline and the basic elements of the "expanded" mathematical theory of Mihailo Petrović Alas as well as the cognitive semantic approach to force dynamics by Leonard Talmy. In our paper we will try to keep track of significant similarities that connect the two authors and reflect on their deep and meaningful insight into the principles of human thinking in regard to Alas' and Talmy's interpretation of disparate phenomena, as well as the thought processes of metaphor, analogy and allegory, but most importantly, their like-minded interpretation of causality in an abstract multidimensional mental space, obviously representable by different vectors. Based on our findings, it can be concluded that Mihailo Petrović was a close forerunner of Talmy's conceptual understanding of important semantic phenomena.

This paper will focus on Mihailo Petrović Alas, a polymath with brilliant ideas and a synthetic mind, who worked on his doctoral

* This paper was written within the SASA project: "Comparison of stereotypical lexical-syntactic elements in the Serbian and the English language", No. 0-18-18.

dissertation more than one hundred years ago and defended it before a brilliant committee composed of the leading French mathematicians, which was a fact that helped him plunge deeply in the "philosophical" mathematical waters in France in the second half of the 19th century. At that time, as well as in the early 20th century, an important place in France belonged to a speculative interpretation of the Cartesian ideas dating back to Descartes, even though Leibniz's ideas were quite influential in France at that time. Namely, Leibniz originally composed his powerful essay on monadology, not longer than ninety pages of hand-written text, in the French language. Both Descartes and Leibniz were seeking comprehensive explanations of the Universe, and were continuing the scholastic search for a universal, general explanation of the World, the so-called *characteristica universalis,* which, Leibniz believed, was the monad, a constitutive unit of everything in this world and a basic ontological element of the Universe (Leibnitz, 1714/1978).

It is at this point that we can find a connection between Mihailo Petrović and modern cognitive researches and theories. It appears that any comprehensive attempt of explaining the world and human cognition about how the world is construed and how it functions is nowadays in the focus of the science of cognition, i.e. cognitive science, which is not a singular, separate whole, but rather a coherent, although loosely defined, scientific paradigm that is best described by way of a corresponding diagram (see below).



Connections among the Cognitive Sciences
Kn: Unbroken lines = strong interdisciplinary ties
Broken lines = weak interdisciplinary ties

The diagram depicts philosophy, which is a general, theoretical and abstract science, as a fundamental cognitive science, with epistemology, as its part, accounting for the ways in which we acquire knowledge about the world, whereas the neuroscience is at the bottom of the diagram as the true opposite of philosophy, looking into specific issues on how the human brain works and accesses facts about the outer world. The dash lines which connect philosophy with psychology and linguistics indicate that there are very strong interdisciplinary ties among them in terms of their interests, goals, and even their research methodology. Indeed, one can notice that philosophy, via epistemology, looks into the processes of making conclusions and learning facts, just like linguistics does (how people acquire language and how such an exosomatic mechanism can help them gain knowledge about the world easier), or psychology (how children develop and acquire different concepts, how one creates beliefs and attitudes towards society and oneself, and one's own subjectivity). Both psychology and linguistics are also closely related (connected with the dash line) to each other, and to neuroscience, as well. This is fully in line with what we already know: namely, there is an established science termed "psycholinguistics", but there are also sciences such as "neuropsychology" or "neurolinguistics", which try to find a connection between their deductive, mainly theoretical, postulates such as Chomsky's transformational-generative syntax or Piaget's theory of cognitive development on one side, and their cortical equivalents within the research of the central nervous system which is a part of the cerebral cortex (neocortex).

In addition, linguistics is directly related to anthropology, the science which studies customs and traditional cultures of certain communities and which, in various ways, often customary, contributes to linguistic expression which can be specific, depending on the environment in which the community lives and works. For instance, it is interesting to grasp, both for linguists and anthropologists alike, to what extent the Inuit language (the most frequent language among the Eskimos), influenced the perception of the world with its lexicon. Thus the Eskimos have a much richer description of winter and various types of snowfall than the rest of the world, and, consequently, different customs and oral and written narratives stemming from the mentioned circumstances (Li and Abarbanell, 2018).

Alan Turing (1912–1954),
British mathematician, philosopher

John Searle, contemporary
American philosopher

In a certain way psychology is in direct relation with the research in the area of artificial intelligence, as it uses computer-generated models for describing the functions of the human brain, which can serve as a guidepost for psychology to take the right direction when describing spiritual and emotional life, nevertheless this process is also reverse: algorithmic descriptions, especially neural networks, replicate the existing cognitive psychological hypotheses and test their descriptions. About twenty years ago, a special case shed light on the interaction between psychology and artificial intelligence, showcasing how several similar and mutually related disciplines looked into the semantic awareness of the subject who answered questions asked by the examiner who did not know whether the answer would come from an entity that possesses artificial intelligence or from a human (the Turing test for checking the knowledge of a natural language, Chinese in this specific case). During this thought experiment which philosopher John Searle devised to determine whether it is enough to answer the asked question meaningfully, or it is necessary to be aware of the process itself at the same time, and not to give answers without prior self-reflection and understanding of the nature of the offered answers, which eventually led to the rejection of the so-called functionalistic model of human cognitive abilities (Searle, 1980).

We believe that this important thought experiment is worth mentioning, given the fact that the discussion initiated by philosopher John Searle 38 years ago, by mentioning the so-called "Chinese room" mystery, is still ongoing. Nevertheless, the diagram which depicts multiple interdisciplinary connections that exist in the cognitive sciences, shows, at least partially, how various approaches are included in the common attempt to unravel the mystery of cognition, i.e. of the cognition process. In addition, the "Chinese room" example directs the attention of the motivated scientists towards the existing ties between philosophy and neuroscience, because it is possible to investigate to what extent the success of computer intelligence in the Turing test coincides with the existing human neurosynaptic connections, and which areas of the cortex can be included in the processing of syntactic and semantic properties (Bostrom 2014).

Leonard Talmy, contemporary American cognitive scientist and linguist

MIHAILO PETROVIĆ'S PHENOMENOLOGY OF MATHEMATICS WAS A PRECURSOR OF SOME CURRENT POSTULATES OF COGNITIVE SEMANTICS

From 1902 to 1906 Mihailo Petrović was investigating and writing about ideas which he would later transform into a voluminous book titled *Elements of Mathematical Phenomenology* (published in 1911 in the Serbian language, printed in the Cyrillic alphabet), (Petrović, 1911). This last fact should be specifically emphasized, because it had a logical outcome: unlike most of his works published during his stay in France, Mihailo Petrović published his most speculative, and, in a way, most ingenious and most original book in a little-known language and which came out printed in the Cyrillic script. For this reason, his prophetic work, of great importance and scope, was unavailable to a broader global scientific community, which was not an issue at that time, because Petrović was constantly in touch with a narrow circle of his French colleagues, with whom he discussed what he wrote in his book. This is the reason he felt that he had had a productive exchange of ideas on an international level, and, being a young scientist, he was pleased with this fact. He definitely could not suppose what would be happening much later, and this constitutes the focal point of our paper in regard to his scholarly labor.

It just so happened that Leonard Talmy, the distinguished American cognitive scientist, linguist and semanticist (but also an author who majored in mathematics), wrote a two-volume book, his *magnum opus*, titled *Toward a Cognitive Semantics, volume one: conceptual structure systems,* which was published 89 years after the Petrović's book *Elements of Mathematical Phenomenology*, which means in 2000 (Talmy, 2000). Over a thousand pages long Talmy's two-volume set offers detailed descriptions of mechanisms in language related to cognitive semantics, by using the fundamental concepts of space, force, time and the so called *embodiment*. To understand the importance and academic influence of Leonard Talmy's work, we should mention that on January 7, 2019 the mentioned book had 7,021 citations at the *Google Scholar* scientific internet forum, whereas Talmy's overall influence on the international scientific community is also huge, given the fact that the total number of citations from his works and books exceeded 35,000 at the previously mentioned web search engine. Definitely, these facts would not be so relevant if it was not for significant conceptual overlapping found between the ideas which Mihailo Petrović Alas elaborated on in his book from 1911, and the ideas presented by Leonard Talmy in his book printed 89 years later.

We will follow a chronological order and summarize what Mihailo Petrović Alas was saying in 1911, and try to show similarity, or, in some cases, to draw an analogy between the ideas of these two authors.

First, in his book *Elements of Mathematical Phenomenology*, Mihailo Petrović develops the so-called "extended mathematics" which is a logical consequence of his wish for mathematical explanations to leave their abstract, hermetic world and start opening up to other areas of life in reality. This was in line with the ancient search for *universal characteristic* of order in the Universe (let us remember the scholastic need to find *characteristica universalis*, as well as later Leibniz's monadological description of the construction and interpretation of the World). We should point out that it is a mistake to look for the connection between Plato's understanding of mathematics and the universal characteristics of medieval scholastic philosophy. Namely, Plato, instead of seeking the elements which make the total picture and construction of the World, sees arithmetic and numbers as permanently given entities which are unchangeable and totally inaccessible to people due to their divine characteristics. On the other hand, Mihailo Petrović Alas, like other mathematicians of that time in France, although under great influence of the French mathematical philosophical school, believes that there is no significant "separation" between the researcher (mathematician) and the object of research (any object, either in the domain of numbers and equations, or in social, political and psychological sphere of life), and claims that it is logically justified to include mathematics in the research of unknown objects, with the purpose of finding any common element whatsoever, seemingly different, disparate (as Mihailo Petrović used to describe it), which would

Jean-Baptiste le Rond d'Alembert
(1717–1783), French philosopher
and mathematician,
one of the encyclopaedysts

be comparable to the language of mathematics. Petrović does that, although the idea of separating the observer from the observed object is old, and dates back to the Presocratics, first from Protagoras, whom Plato quoted, stating the colloquial sentence that "man is the measure of all things" (Shoemaker 1988 for further clarification).

Mihailo Petrović Alas emphasized that in such situations it is useful to use a "mechanicistic" procedure, or, in a broader sense "Newtonian" mechanics, in order to eventually draw up a scheme for explaining the phenomena in literary language and its frequent use of stylistic devices such as allegories, metaphors or analogies, when there is a successful figurative mapping of disparate phenomena from one mental space into another.

Naturally, when we mention *allegories* and *metaphors* (it is well-known that in 1933 Alas wrote a book titled *Metaphors and Allegories,* which was published in 1967 by the Serbian Literary Cooperative), we note that both stylistic devices are based on the cognitive mechanism of mapping (translation) from one mental space into another. Much earlier, between 1902 and 1906, he developed a mathematical function in order to have an adequate tool for later analysis of figurative speech, which, actually, often does not have to be literary language only. He was convinced that human spirit requires recognition in order to fulfill this function and stimulate knowledge, once the activation of mapping happens. Various phenomena or observed disparities speak in favor of the fact that even seemingly different phenomena still can be sorted, classified and recognized.

It is useful to briefly mention another cognitive act: *the cognitive process of analogy*, which, one way or another, can be sufficient to raise cognitive awareness of an object which we notice with our senses or describe via a literary formulated report. It is an analogy, which, historically, in ancient Greece was considered a part of mathematical technique or skill based on proportion (hence the Latin translation of Greek word "analogy", *proportio, secundum, rationem,* and even *regula*). Although analogy was primarily a mathematical procedure of recognition and categorization, it was a useful research tool for various complex language forms at the time, especially in terms of closed morphological sets such as flective forms, where classical philologists dealt with simple proportions such as $A: B = A^1: X$ ($X = B^1$), (Mattiello, 2017).

Owing to such cognitive processes (metaphors, allegories and analogies), Mihailo Petrović managed to formulate and mathematically describe similarities and parallelisms among mechanics, electrics, electric motors, biological phenomena, human body, medicine, along with social and psychological conditions, drawing the adequate, primarily analogical, but also metaphorical and allegorical, comparisons of disparate phenomena. In order to get a deeper insight into Alas' work and concretize it, we will state several illustrative examples:

> "That was the key which opened the door to his conscience"; "He was rummaging through the ashes of his past"; "His fear burst like a soap bubble"; "Try to heighten your cultural level"; "In his heart a storm was raging"; "All his efforts were shipwrecked".

When we consider the quoted examples from the book *Metaphors and Allegories* (1933/1967) it is easy to notice some disparate phenomena, which belong to different physical domains, but also to something which, in cognitive science, is called different mental spaces (see author Gilles Fauconnier, mathematician who wrote the first and the most influential book on mental spaces, Fauconnier, 1985/1994). The fact that some mental spaces are available to human senses, and some are not, at the same time indicates that some phenomena cannot be noticed sensually, nevertheless deduction leads to the conclusion that they do exist, and, furthermore, that it is possible to talk about them. In previous examples, this is easily illustrated by a concept we passionately talk about, and even with understanding, nevertheless it is a concept categorized and constructed in one mental space, while its existence in another, non-perceptual space can only be discerned – we are, definitely, talking about the implicit mentioning of "time", when, in metaphorical sense, it is possible to talk about rummaging through ashes of the past, which is mentioned in one of the Alas' examples. Of course, many mathematicians are acquainted with the fact that in the 18th century a French philosopher, encyclopedist and mathematician, d'Alembert was probably one of the first scientists who advocated the introduction of the fourth dimension, dimension of time, which should be described with the language of space, nevertheless, even though Petrović's professor, Henry Poincare, showed interest in such an idea, he remained passive about it, so, unfortunately, it was realized rather late, in the early 20th century, when German physician Hermann Minkowski, in his short article consisting of only few pages, formalized the discussion on the invisible time which can be described by space (Minkowski 1915/1907). As for many everyday conversations in common, even colloquial language, this was trivial, because natural spoken languages have long been doing exactly this – talking about time within the frame of space. So, what we mentioned was actually happening, and this case reflects two mental worlds, two mental spaces in which elements of one can be replaced by the elements of the other. It is easy to talk, and this goes back to ancient Greek, about the "arrow of time" which once released, following its spatial trajectory, marks the flow of time. Natural language takes this for granted, and hence the paremiological narrative that time flies, runs, or, as it is said in Latin, "tempus fugit" or "tempus volat".

Hermann Minkowski (1864–1909), German mathematician and physicist

SIMILARITIES AND COMMON ELEMENTS OF ALAS' MATHEMATICAL PHENOMENOLOGY AND TALMY'S COGNITIVE SEMANTIC APPROACH TO FORCE DYNAMICS

In order to describe the work of Leonard Talmy as clearly and straightforwardly as possible, it would probably be best to start with an adequate example:

"The door is closed".

Such a sentence is fully neutral from Talmy's force dynamics point of view, because there is nothing in it which is opposite to something else. Simply, we are faced with a homeostatic description of inactivity and with position that can be described as the "status quo" within the frame of dynamic relation. There is neither a doer, nor an opponent. However, the following sentence is different:

"The door cannot be opened".

In this other case, there is an element of force that is present, because one element is opposed to something else, i.e. to another element. Thus, we can notice that there are **two** roles, one that is active, which in Talmy's cognitive semantic approach to force dynamics is called ***the agonist***, while the other one, which opposes the activity or the change of standstill, is known as ***the antagonist***. In this example, it becomes clearer what Talmy's cognitive semantic approach to force dynamics is about, so we can say that in Talmy's theory there are *two roles*, one which is active, known as *the agonist,* and another one, which opposes the beginning of action and change, known as *the antagonist.* Thus, in the previous sentence, we can mark the formal subject "the door" as the agonist, while the other, in this case unidentified, force is called the antagonist. If we talk about the action potential of a certain state, Talmy says, then we should start from the initial status in which the action potential is intrinsic characteristic of the agonist. On the other hand, the antagonist opposes action and has a tendency towards passive preservation of *the status quo*, and its intrinsic characteristic is inclination to inertia.

In order to initiate a short discussion on Talmy, we will start from the construction of the sentence similar to the one we used at the beginning of this chapter:

"The door is closed".

Such a sentence, from Talmy's cognitive semantic approach to force dynamics point of view, is neutral, because it does not have elements which contradict one another in cognitive-semantic sense. However, *the situation totally changes* if we slightly modify it and say:

"The door cannot be opened".

Here we are dealing with different sentence, because there are dynamic forces which are opposing one another. The essence of the cognitive semantic approach to force dynamics is that there are two roles in the sentence, which we are already familiar with, the roles of – *agonist* and *antagonist*. This means that in the previous sentence the door was the agonist, whereas anything that prevents the door from opening is the antagonist, force opposite to action and the change of dynamic status quo (that could be wind, draft, a person on the other side of the door who prevents the doors to open, etc.).

If we look into, for example, the second chapter of Mihailo Petrović's book *Elements of Mathematical Phenomenology,* and, especially, the descriptive table of contents on "the mechanisms of phenomena" which is shown below, it is clear that in the very descriptions of chapters and sections some entities are mentioned which, by their essence, and in terms of terminology used (agonist, antagonist, activity, obstacle to action, cause, etc.), precede the "new" terms and names that Talmy first mentioned in the 1990s, and which appeared in a more updated and refined form in his seminal two-volume book in 2000. We have an example in which we can easily identify overlapping ideas of causality, mechanical, "Newtonian" causation, and, strangely, the same or almost the same terminology that is used in the description written almost 90 years after Petrović's book *Elements of Mathematical Phenomenology* was published.

ДРУГА ГЛАВА.

МЕХАНИЗМИ ПОЈАВА.

In terms of its structure, Talmy's concepts of cognitive structuring and semantic conceptualization are very similar to Alas', especially his descriptions and explanations of cause and effect, so in this segment of the description of cognitive mechanisms, their works overlap in many aspects. Interestingly, Talmy's work was modified and complemented by probably today's most prominent active cognitive linguist Ray Jackendoff, first in his book from 1996 titled *The Architecture of the Language Faculty* (Jackendoff, 1996). Although at first sight there is some inconsistency, chronological of course, we should keep in mind that

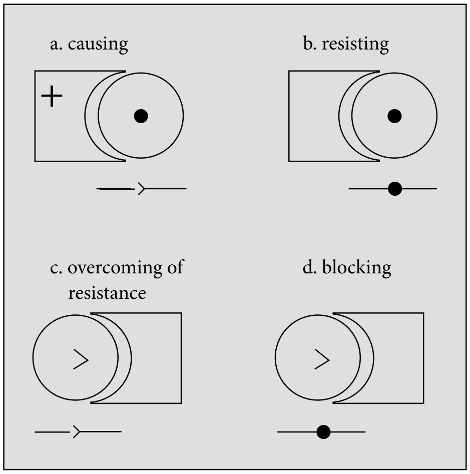
Talmy's work on the description of force dynamics is based on hierarchical, vertical linguistic and semantical analysis that starts from the interpretation of meaning and its foundation on the determination of concepts in terms of space, time and vector, so we can say that his earlier articles and monographs, written in the early 1990s, were in similar tone and had continuity that preceded his book from 2000. Therefore, it is no wonder that Jackendoff influenced later Talmy's work, regardless of the chronology of publication of the two major books by these authors.

We will try, again with appropriate examples, to better explain the mechanism of explanation of cognitive force dynamics in Leonard Talmy's work. Here are the examples which should above all be interpreted as agonist-antagonist and by using vectorization and unintentional appearance or use of force at the given moment. As for vectorization, it is nothing but a frequently used mathematical term convenient for the representation of some mechanic or automated terms, e.g. any more or less intelligent machine, similar to Alas' prototype of analogue intelligent machines. When describing the work of such mechanical or electronical devices by using natural languages such as Serbian or English, vectors are successful, and, thanks to their indexicality (displayability which is sometimes semi-iconic, which means almost picturesque, they are easily understandable in a large number of cases). It has to be perfectly clear that relative positions of vectors of the same terms in different languages (in our case Serbian and English) are conceptually transparent because they represent the original form of communication, which humankind used before the appearance of discursive language forms of communication (today's natural languages). **Simply put, language semantics is, in many ways, represented by vectors within different *geometric relations in multidimensional Euclidean spaces.***

We can remember that, while talking about the cognitive mechanism of analogy, we emphasized that it was originally defined mathematically, and that analogy was seen as a sort of proportion. Thus, we want to emphasize that the perception of an analogy is primarily a mathematical concept in which, during the interpretation, we look for similarities of disparate phenomena. Of course, analogy is just one of the elements that both Mihailo Petrović and Leonard Talmy share in their approaches to the interpretation of meaning. The most frequent and, perhaps, the most important common element is *metaphor*, which Petrović usually interprets as a phenomenon of recognizable disparity, and Talmy as a similarity in different mental spaces. Common for both authors is the issue of causation which is directly related to the role of force, coercion or obstacle and hurdle to the agonist's intentions or action. Several examples of Talmy's work will confirm this:

> "A gust of wind made the pages of my book turn"; "The appearance of the principal made the students calm down"; "When the dam burst, water gushed out from the artificial lake"; "The abating of the wind slowed down my sailboat".

Since the above examples are typical even of this huge corpus of sentences, although metaphorical ones, in Alas' book titled *Metaphors and Allegories*, we can talk about the common set of formal features attributed to Petrović's and Talmy's theories:



Regarding the mechanics of force dynamics the accounts of the two authors are not strictly physical, which can be noticed in Petrović's interpretation of metaphors and allegories, where he often used examples from literary or colloquial language. For instance, Petrović accepts that it is possible to "expand" mathematical explanation of causality by analyzing psychological states such as "convincing" or "coercion", although he is aware that, in its final philosophical frame, it will lead him to interactive Cartesian dualism. The same or similar comments can be applied to Leonard Talmy's work, as is evident in the previous diagram.

INSTEAD OF CONCLUSION

At the end of this condensed outline of Petrović's brilliant insight into a "hidden" connection between 'expanded mathematics' and modern cognitive science that uses multidimensional spaces in order to explain meaning in language, including modality, deontic logic, as well as causality, we shall look at one specific example which should elucidate and enhance this claim even further:

"Jovan was dragging his feet down the street, lacking the strength to walk".

In the previous sentence we are facing reflexive causality where **one and the same body (Jovan) is both the agonist and the antagonist at the same time**, implying that body and will are separate, which, as we stated, can be interpreted as an interactive type of duality that Mihailo Petrović quite possibly discussed with his senior colleagues Henri Poincare and Cartan during his academic stay in Paris.

Thus it seems plausible to contend that Talmy's cognitive-semantic theory, in essence very similar to Mihailo Petrović's ideas, can be accepted within modern interpretation as an overall explanation of causality, based on the functions of the neocortex constantly calculating the minimal muscle vectorizations and their probable outcomes, making other doable or undoable decisions that lead to action or inaction (activity or passivity).

REFERENCES

Bostrom, N. (2014). Superintelligence: Paths, Dangers, Strategies, Oxford University Press, 2014), *The New York Review of Books*, vol. LXI, no. 5, pp. 52–55.

Leibnitz, G.W. (1714/1978). *La Monadologie*. Paris: Librairie Delagrave, Paris, 1978, éditionannotée, et précédéed'une exposition du système de Leibnitz par ÉmileBoutroux.

Li, P., and Abarbanell, L. (2018).Competing perspectives on frames of reference in language and thought, *Cognition,* 170: 9–24.

Fauconnier, G. (1985/1994). *Mental Spaces*, 2nd edition. Cambridge University Press.

Jackendoff, R. (1996). The Architecture of the Language Faculty (Linguistic Inquiry Monographs). Cambridge, MA: The MIT Press.

Mattiello, E. (2017). *Analogy in word-formation: A study of English neologisms and occasionalisms* (Trends in Linguistics: Studies and Monographs 309). Berlin and Boston, MA: De Gruyter Mouton.

Minkowski, Hermann (1915) [1907]. „Das Relativitätsprinzip". *Annalen der Physik*. 352 (15): 927–938.

Петровић, М. (1933/1967). *Метафоре и алегорије*. Београд: Српска књижевна задруга.

Петровић, М. (1911). *Елементи математичке феноменологије*. Београд: Српска краљевска штампарија.

Searle, J. (1980). Minds, Brains and Programs. Behavioral and Brain Sciences. 3: 417–57.

Shoemaker, S. (1988). On What There Are. *Philosophical Topics*. 16(1): 201–223.

Talmy, L. (2000). *Towards a cognitive semantics. Volume I: Concept Structuring Systems.* Cambridge, MA: MIT Press.

Talmy, L. (2000).*Towards a cognitive semantics. Volume II: Typology and Process in Concept Structuring*. Cambridge, MA: MIT Press.

ON FISHING AND LITERARY WORKS OF MIHAILO PETROVIĆ ALAS*

Mihajlo PANTIĆ
Faculty of Philology of the University of Belgrade

There is but one Alas. Among all his interests and preoccupations, for which two lifetimes would hardly suffice, Mihailo Petrović Alas had given undeniable primacy to an age-old passion – fishing. It is this passion for fishing, for that matter, that earned him the nickname by which he has been permanently remembered in Serbian history, science and culture. (According to certain testimonies, the nickname, used ironically at first, was given to him by Milutin Milanković, another prominent Serbian scientist of that era, but later on that other name of Petrović's came to be used as a term of endearment which, according to the customs of Serbian cultural milieu, emphasizes general affection and popularity; it is enough to say – Alas! – and everybody knows the name and to whom it refers.)

The power of Petrović's pastime, which, over time, had turned into a specific worldview and had imperceptibly become his own brand of an idiosyncratic, identificatory, applied and embodied "philosophy of life", is substantiated not only by a large body of data and evidence, but also by some direct Alas's gestures (in a veritably eccentric "fishermen's style"). The son of Alas's friend Mladen St. Đuričić, a writer – Predrag

* A revised and supplemented version of the paper initially published in the catalogue *Mihailo Petrović Alas: The Founding Father of the Serbian School of Mathematics* (SASA, 2018)

Portrait of Mihailo Petrović from 1921 (*Collected Works*, Book 14) (Digital Legacy of Mihailo Petrović)

Đuričić, a publicist, who had remembered the renowned Serbian scientist since his childhood, wrote in the article "The Personality of Mihailo Petrović Alas in Memories and Anecdotes" that it was his father who drew his attention to the odd character of Alas, showing him "the study of Mihailo Petrović, in which he had only one diploma hung on the wall above the desk – it was a master fisherman's letter, written in a clumsy, coarse handwriting by the president of their committee, stating that on such and such a day Mihailo Petrović, professor, passed the master's exam for a river fisherman, the fishing craft, as hereby acknowledged and confirmed by the undersigned committee" (Ђуричић 2005: 70)[113]. Notwithstanding all the academic accolades, honorary doctorates and memberships in the most prestigious scientific associations, Alas saw himself, first and foremost, as a committed fisherman, turned towards the water and the brotherhood of men with a passion for fishing.

Apart from the voluminous and versatile interdisciplinary scientific work which, though systematized a number of times, requires permanent study and re-examination, which has been facilitated by the most comprehensive edition to date in the *Collected Works*,[114] throughout his life Mihailo Petrović Alas had been concerned with both the theoretical and practical aspects of ichthyology, the study of fish. Having had a lifelong, almost fated, fascination with the world of fish, he knew down to the last detail everything concerning the nature, tradition and techniques of river fishing in general, and on the Danube and Sava rivers in particular. As a committed member and practitioner of the river fishing community, he perseveringly, one could rightfully say – even obsessively! – investigated, described and interpreted the customs, language, tools, ordinary and extraordinary scenes from the lives of that numerous, seemingly marginal, but then, at the turn of the century, and even afterwards, before and after the Great War, very colourful and authentic social group.

At first and any other glance, one could say that Mihailo Petrović Alas was *a man of two callings,* a mathematician and ichthyiologist, and also a man of exquisite, very well nurtured and complex passions, a passion for travelling, scientific research, music, and yet, above all else – the passion for catching fish. "If I had not obtained that one additional vote at the competition for the position of a professor at the Belgrade Grand School, I would never have pursued a career in mathematics. I would have been living on the rivers across Serbia, not on a ship, but in a boat" – he wrote once (Мијајловић 2018).[115] When we talk about people with two callings, we usually, according to certain unwritten rule, look for the answer to the question as to which is the first, and which the second activity (vocation) of such a person, because two competencies, and in equal measure to boot, are rarely acknowledged to anyone, at any time or place. A man can be only one thing, not both, such is the ingrained, rigid stereotype. In the vein of that denigrating and yet active stereotype, Mihailo Petrović could be said, most probably with no great inventiveness, to have been the best mathematician among fishermen, and the best fisherman among mathematicians: competence always lies elsewhere, never there where it could undermine a hierarchy established by the power of customs, especially one that is artificial.

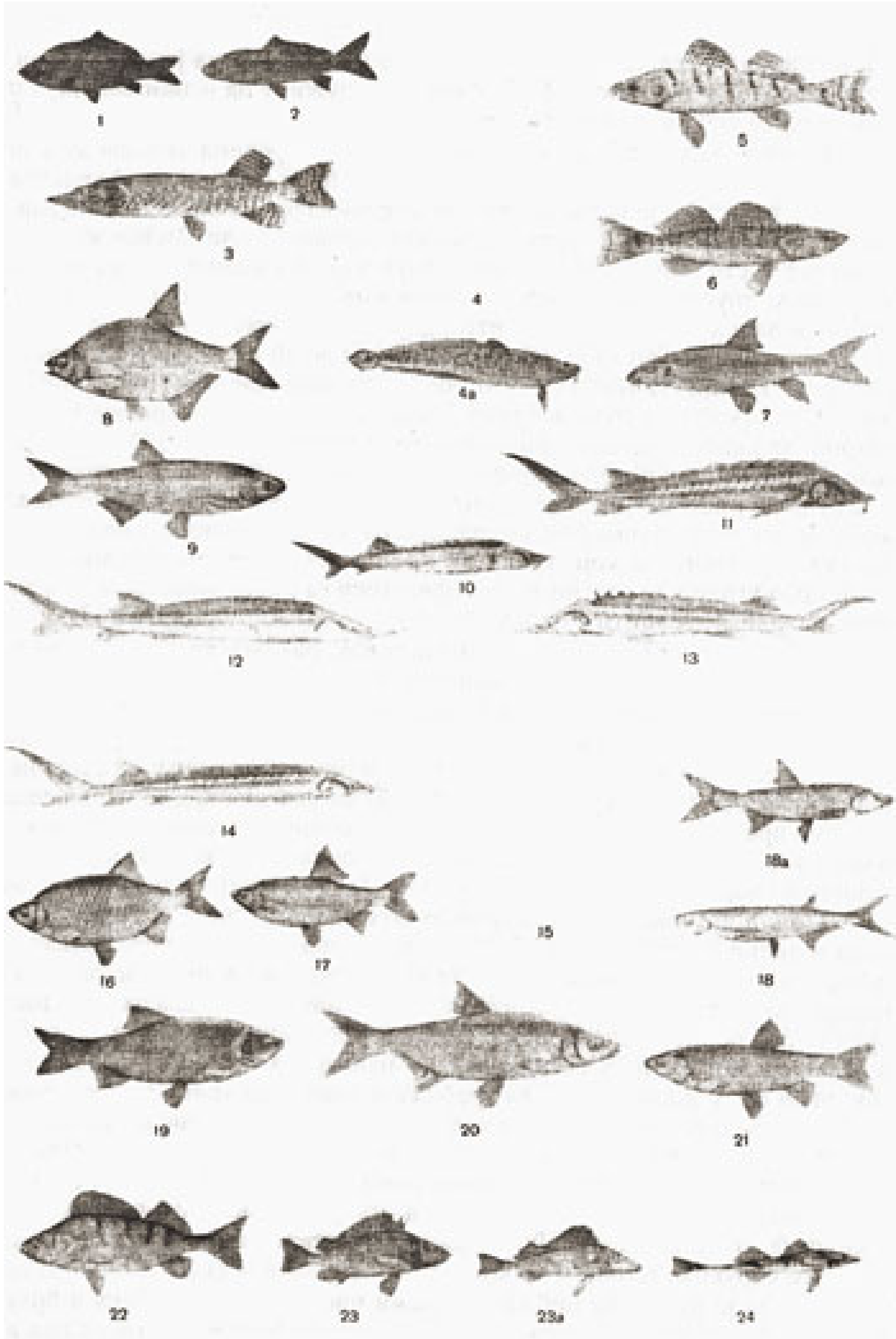
However, Mihailo Petrović Alas could not content himself with even those two callings. When we read his book, by which I mean primarily the travelogues and articles on fishing, we cannot fail to discover his third talent and skill – the one for writing. We come across a writer not in the common, fictional sense of that term. Alas is a first-class "applied" writer, he is able to flourish at what many writers of fiction labour under, even though that is their main calling – in a word, he knows how to narrate engagingly about lived, not imaginary, events, and to articulate linguo-stylistically an area of experience he has undergone, to give a document fullness it does not have outside of the framework of the story, even though an unskillfully prepared document is what it is, a dry archive material. Whatever he reports about or gives testimony of, Mihailo Petrović Alas does it in a straightforward manner, seductively drawing the reader in. He has the capacity for the magical weaving of a tale, which, it should be noted, has its roots in oral heritage and, as is easily associated, in the famous fishermen's tale-telling and oneupmanship (competing in telling tall tales, banter, derision, and getting one up on each other) in a tavern or outdoors, by the fire. In his writings Alas himself often mentions both of these situations (tavern and fireplace), in which story and story-telling are a mandatory side act, almost like a natural phenomenon. Viewed from a more expert, formal and stylistic aspect, one could say that Mihailo Petrović Alas is a representative of the Belgrade school of style, which was promoted exactly at the turn of the nineteenth into the twentieth century, and whose main feature is narrativization, structured and fluent presentation of even the most complex speculative content. Only bad writers and pretentious scientists, it was thought at the time, and it is not any different now, tend to mistify their competences (or, rather, pseudo-competences), whereas the representatives of Belgrade style, Mihailo Alas among others, present what they have to say in a simple, meaningful, visual and always engaging manner. The travelogues and fishing-related writings of Mihailo Petrović Alas belong to a corpus of truly representative texts of the said type, then, as now.

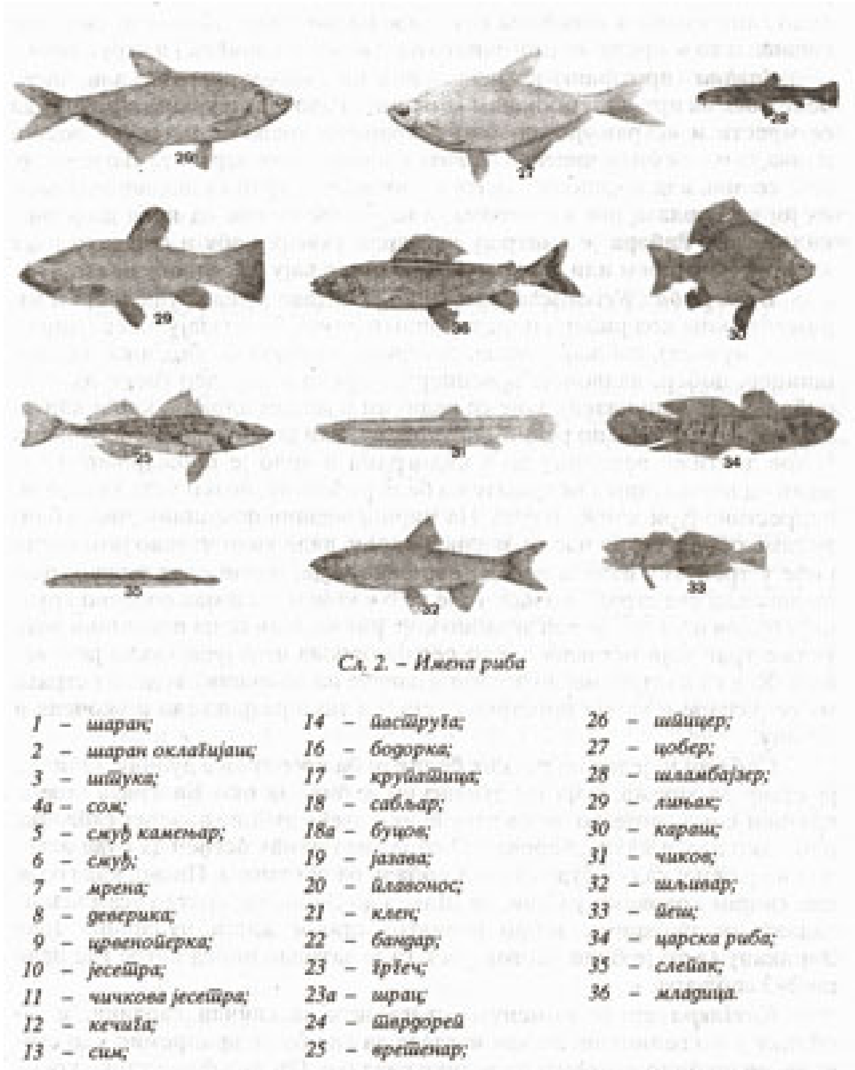
Belgrade Fishery Association confers a diploma to its founder Mihailo Petrović on 12 July 1942
(SASA Archive, 14188/33)

Of course, one should not exaggerate and accord Alas canonical status and relevance that he objectively does not have, but, equally, one should not lose sight of his literary work in our literary history, which had practically been the case before the publication of his *Collected Works*. The inner circle of readership, truth be told, were familiar with several Petrović's travel books that had been published at the time in the editions of „Srpska književna zadruga". However, few people knew of the texts scattered about the periodicals and in less accessible publications, which were collected in Volume XIV of the said publication, under the title *Fishing (Ribarstvo)*. (Alas being the only "fisherman" to get his own edition of *Collected Works*!) Special attention should here be given to the editorial work of Dragan Trifunović, because editing the book *Fishing,* composed of insufficiently known Petrović's papers, required applying a whole set of complex editorial procedures and following a specific methodological principle, which is primarily reflected in the fact that the individual texts, published for the first time in their integral version, are organized in a harmonious way, with an inner logic to them and a natural order, which lends the book a dimension that even Mihailo Petrović Alas himself did not have in mind. Some of those texts, though, were not hard to find and be made available to the public again, as they had originally been published in the regular editions of the Serbian Academy of Sciences and Arts (SASA), but a certain number of texts had firstly been printed in small papers and magazines or the ones that are now antiquarian. These diverse texts, which only have the same general topic, while everything else is different, were to be unified in a proper way intrinsic to the material, and this is exactly what the editorial team headed by Dragan Trifunović did. Petrović's book *Fishing*, thus, is not only a collection of texts entertained and organized around the theme of fishing, but something else besides, a novel, so to speak, an unintentional, spontaneously created “novel”, such as is usually written out of obsession, out of an all-encompassing passion, for passion, when it is true, stands in for the whole world. I do not think I am qualified enough, though I am not entirely unqualified, either (I share the same passion with Alas, and passion is what makes me qualified), to speak of the purely scientific, ichthyological aspect of Petrović’s *Fishing*. Let others, who are more professionally equipped, have their say about it. And perhaps for the very reason that *Fishing* does not appeal to me only and exclusively from its scientific aspect, which is not at all to be underestimated (on the contrary!), I see in that book a great story, a history, in fact, a special chronicle of these parts seen from an unexpected, unconventional, and yet ordinary angle, from the perspective of fishing, which treasures and reflects the glimpses of bygone times. (It seems to be similar in kind to the intentions of Philippe Ariès to write a history of the Middle Ages based on facts from the daily life of common people, and not on the so-called pivotal events or biographies of privileged individuals.) That conditional “novel” of Petrović has got an anthropological, ethnographic, geographical-descriptive, chronicling, and documentary dimension, which, taken collectively, in their comprehensiveness, represent the very quality that gravitates towards literature.

Nowadays, we also read *Fishing* as a testimony of a time gone by, as a story about people that once inhabited the banks of the Danube and Sava rivers, and, in harmony with nature, lived off plentiful fish stocks in those great expanses of water. The way in which they, metaphorically speaking, picked the fruit of those mythical rivers changed and improved over the centuries, preserving something from the olden days of one of the earliest crafts in the world, and building up around it a whole system of very convoluted customs, facts, beliefs and superstition. That established system is clearly and strongly related not only to the category and type of geographical region, but also to the character and customs of the people inhabiting it. Jovan Cvijić, another prominent Serbian scientist, wrote and spoke in his time of how the natural landscape shapes the character, mentality and the way of life of the domicile population. That thesis, precisely that scientific truth, is indirectly confirmed in the book *Fishing* by Mihailo Petrović Alas. If a “novel” represents, as it does, some sort of a non-systematized encyclopaedia, with deeper, poeticized intellectual and conceptual system, Alas’s *Fishing* is also a special, though non-alphabetized, encyclopaedia in which the interested reader can get very detailed information about numerous names, aspects, facts and subtleties not only concerning fishing, but also the daily life in the history of Belgrade of earlier and more recent times.

Fishing is an ancient practice, so old that it delves into the collective unconscious, with deep archetypal and symbolic meanings, and thus it is natural that this practice reflects universal, characterological, geopolitical, ethnographic and psychological features of the nation and individuals that are involved in it. In the book *Fishing* we can find, among other things, that, at least as far as fishing is concerned, the Turks have left us a legacy containing some positive features. In accordance with the power and might they had wielded, Alas instructs us, during the late Middle Ages, the licences and rights for fishing on the Sava and Danube were in their hands, but, after the Serbian uprising, they passed on all their knowledge, or rather, gave it away reluctantly to


1
2
5
3
6
4
8
4a
7
9
11
10
12
13
14
18a
16
17
15
18
19
20
21
22
23
23a
24



Types of fish at Belgrade hunting ground (*Collected Works*, Book 14) (Digital Legacy of Mihailo Petrović)

Serbian fishermen, who in the nineteenth century went on to fish in those still enormously rich waters of the two large bordering rivers, and fishing at the border is fraught with challenges, dangers, resourcefulness and adventures, which makes it all the more suitable for being turned into a good novel. (While we are on the subject of the richness of the fish fauna of the Danube and Sava, let me add that virtually none of the travel writers from the previous centuries who passed through Belgrade failed to mention its rivers replete with fish as well as the abundance,

Folding of a large fishing net (a dragnet, *alov*) into a boat (SASA Archive, 14197/II-6)

diversity and cheapness of its fish markets. Alas mentions those travel writers and passers-by extensively in his writings.) The techniques for catching fish and the equipment for fishing had been used according to the Turkish tradition up until the great regulation of the Danube, in the 1890's, when they were gradually changed into their present condition, having been adapted and innovated in accordance with the Central European style of fishing, and indeed since then, due to more limited options for natural propagation, as well as to the forthcoming industrialization, the fish stocks have become increasingly reduced...

In the book *Fishing* a reader can also find out the origin of the surname Balugdžić, Balugdžija and the nickname Balug, which in Turkish refer to – a fisherman. This should be pointed out because a special linguistic value of Mihailo Petrović's narrative style, who might as well have been nicknamed Balug, is represented by extremely interesting and extensive lists of fishermen's nicknames, to such an extent that we can infer that the onomastic treasure of the said kind speaks volumes about the character of that marginal, but certainly very authentic part of the Serbian society. During the eighteenth and nineteenth century, that demi-monde, free from social conventions, and rudimentary in the more positive sense of the word, populated the right, lower bank of the Danube in Dorćol, having their back turned towards the town, facing the river. In the twentieth century, Belgrade completely lost its touch with the water, and moved away from the banks of the Sava and Danube, cramming them full of warehouses, railway line, industrial facilities, dumps and slums. The connection had been lost between the old townspeople and the people from the river banks, who passed their whole lives on and by the water, from their birth to death. (That world is evoked by Ivo Andrić in the story entitled "A Rabbit" („Zeko").) And Alas describes how, at the time of favourable water coming in, the inhabitants of Belgrade had the opportunity to witness true spectacles of catching fish, as at the foot of the Kalemegdan fortress lay one of the richest fishing grounds in the whole flow of the Danube river. A fisheries association had been established and, in agreement with the state, it imposed strict regulations upon that branch of economy. Mihailo Petrović writes the history of fishing in a well thought-out and at the same time enthused manner, naming the types of fishing, the fish fauna and the most favourable fishing grounds on the Danube and Sava, but at a certain moment of reading his unusual, spontaneously created "novel", you come to realize that in the whole story the fish is not important in itself, but as a symbol around which the whole community of people has organized themselves, and which serves to confirm life in its fundamental aspect, in its deepest elementary form. Apart from that, it is common knowledge that one of the most prominent Christian symbols, the symbol of fish, is in essence pre-Christian, ancient, and that it is rooted in the deepest layers not only of collective memory, but of the individual, subconscious memories, as well – it is enough to mention Lepenski vir and their fish-like stone idols. It touches the deepest past of the humankind and has an unequivocal ontological significance. After all, a man in one of the phases of his uterine development represents a fish, as a phoetus has gill openings. Through that universal dimension, as suggested by the book *Fishing*, we indirectly discover the basic values of man's life in the community, on the water, in the quoted fisherman's *boat*, under the open sky, which is exactly the moment in which the Kantian categorical imperative begins to take shape within us.

Upon publishing an anthology of Serbian stories about fishing entitled *Reading Water (Čitanje vode)* (Пантић 1998)[116], which represented the result of twenty years' "targeted" investigations of Serbian prose, during which I had been collecting stories related to that topic, I remembered Mihailo Petrović Alas among others, and included one chapter of his unusual travelogue, imbued with cosmopolitanism

Mihailo Petrović with a caught catfish weighing 124 kg, on 5 December 1913 (SASA Archive, 14188/28)

– *With Deep-sea Fishermen… (Sa okeanskim ribarima...)*. In that extract Mihailo Alas describes the time he spent with cod fishermen during his stay aboard a fishing vessel, on Newfoundland's plate. The deep-sea fishermen used to spend six months aboard the vessel during one harvest in extremely severe weather conditions, compared to which modern "extreme sports" seem like child's play. Many of them were buried at sea, while in their homeland, on the European coast, cenotaphs were erected in their honour. And that narrative passage of Alas's is certainly one of the best in the thematic anthology *Reading Water*.

The theme of fishing was introduced in the Serbian literature in the late nineteenth and early twentieth century. (In the story „Švabica" Laza Lazarević writes about how to "gut a fish" in the Serbian household (Лазаревић 1986: 216).[117] The fish, particularly among the people from earlier times, had an almost sacral status, predominantly because of their way of life, because of a great number of fasting periods, as well as because of a great tradition of fishing and fishery among the Serbian people; one merely needs to mention the toponyms derived from that root word. Alas himself in his book *Fishing* reconstructs the said past with a whiff of nostalgia.) It is well known that Stevan Sremac intended to devote one of his stories to the description of Belgrade fishing community. The credits for the discovery of the aforementioned fact go to Mihailo Petrović, who wrote an article about it entitled "An Incomplete or Lost Story of Stevan Sremac" („Jedna nedovršena ili izgubljena pripovetka Stevana Sremca"), and published it in *Prilozi za književnost, jezik, istoriju i folklor,* of 1938. (The article was reprinted in the book *Metaphors and Allegories (Metafore i alegorije)*, Volume XIII of Petrović's *Collected Works*.) Stevan Sremac, a great writer of people's daily life, in accordance with his non-academic method of observing reality and frequenting utterly quaint places, also liked visiting taverns in which fishermen gathered, and there he used to diligently take notes about anything that might be of use to him in composing a story with such a theme, which had never been published, and it is uncertain whether he had written it at all. Giving primacy to verisimilitude and "objectivity" in his writing, Mihailo Petrović is getting closer to the realistic tradition, which he particularly appreciates, and he speaks about it directly in the aforementioned text. Thanks to Alas himself, this idea, somewhat indicative of Sremac's poetics, has remained remembered; it is, namely, in accordance with the writer's principle of the close observation of life and the subsequent authentication and transposition of thus acquired experience.

Following the period of realism, fishing becomes an increasingly frequent topic in Serbian prose. An interest for it is sparked by none other than Petrović's books. In spite of being a non-fictional writer, Alas gives the topic of fishing in the twentieth century a considerable impetus and a literary, symbolic value and weight. Moreover, fishing represents the topic that was readily and fairly often taken up predominantly by the writers who were fishermen themselves, who had that passion, and an increased interest in writing such stories is probably the result of a fast-paced urbanization of life, in which the natural environment and fishing occur as some sort of a relaxing, liberating counterpoint, as a verbal stress relief therapy. The writers who wrote about fishing are, among others, the following: Isidora Sekulić (a superb description of a fish market in Bergen in the book *Letters from Norway*), Miloš Crnjanski, Rastko Petrović, Miodrag

Borisavljević, Antonije Isaković, Stevan Raičković, Danilo Nikolić and others, ending with my generation.

In the world of fishing, in an environment which seems rather odd and eccentric compared to the daily life, just like Mika Alas now apears to us, story is an inescapable part of the whole fishing ritual. It is this, precisely this, and many other things, that Mihailo Petrović Alas speaks about in an inimitable way in the book *Fishing*, a string of narrative medallions, details described with inspiration, verbal records covered with patina, well found, well chosen, and still better composed anecdotes and documents.

Writings about fishing by Mihailo Petrović Alas can provisionally be divided into four groups. The first group is ichthyological in the strict sense of the term (the list and description of fish species). The second deals with fishing skills, ways of harvesting, the descriptions and locations of fishing spots, cataloguing equipment and baits, technology of fish preservation and treatment, and finally, guidelines for fish breeding. The third group refers to the history of fishing in Serbia, since the ancient times, across the Turkish era during which fishing was very well developed, up until Alas's time, including the history of the association and legislation of that branch of economy, while in the fourth he gives different kinds of literary accounts of his existentially privileged passion (travel pieces, travelogues, popular articles on the topic "Do fish sleep?" and the like, whimsical historical fragments, for instance: "An Extraordinary Fishing Adventure at the Court of Emperor Napoleon the Third").

Front page of *The Eel Novel*, published in 1940 (Library of SASA, C 7/12;11)

A special group, indirectly related to fishing, is represented by Petrović's works on hydrography and oceanography, and the one that stands out in every sense is the most voluminous and best known popular science writing – *Eel Novel (Roman jegulje)*. Defining the genre of that at first glance unusual title does not refer so much to the longest fictional form as it points to a broader meaning of the said literary concept. Under the label of a "novel", we read in that book of Petrović's about an exciting, real, but at the same time mysterious, duly stylized and formalized account, with a clearly defined theme, a specified time and space, as well as an immaculately formed role of the narrator.

The *Eel Novel* begins with an explanation of the reason why it has been written. Alas notes: "The eel has since time immemorial been considered as a living creature that nobody can make heads or tails of. The question as to how the eel is engendered has been a riddle piquing the curiosity and imagination of natural scientists and philosophers of all times. It also interested Aristotle, who, thinking about it, found it to

Three portraits of Mihailo Petrović as a fishery master on the Sava and the Danube rivers.
Petrović maintained an extensive correspondence with the world. He used his seal to secure a postal package or an envelope. His stamp depicts a "fish eating a fish" with MP initials, which certainly isn't a coincidence (the stamp was made in Paris out of silver and yew tree wood – Taxus baccata, in 1898). (*Collected Works*, Book 14) (Digital Legacy of Mihailo Petrović)

be an unsolvable conundrum, just as much as the question of how an eel ends its life. The mystery baffled the world to such an extent that an opinion has been formed, when it became evident that no one can say anything about it, that it is elusive, inaccessible to the human reason for eternity and that it is part of the mysteries of religion. Writing about the eel, Herodotus has said that it is a holy creature of whom only a deity can give account" (Петровић 1998: 177).[118]

In the Middle Ages, Petrović continues, eels were also considered to be the evidence of the divine. Such evidence, he says, "had to be convincing even for the most hardened non-believer". And every mystery, not only of the eel, as has long been known, has its poetic flavour. Else, there would be no mystery at all. Izaak Walton in *The Compleat Angler* (1653), undoubtedly the most world-famous book that deals with the subject of fishing in a literary fashion, in an attempt to shed light on the mystery of eel breeding says, among other things, that "they breed (as some worms do) of mud […] or out of the putrefaction of the earth […] like some species of bees and wasps"; that they are "bred out of a particular dew, falling in the months of May or June on the banks of some particular ponds or rivers […] and some of the ancients have called the eels that are thus bred the offspring of Jove", and he adds that "her brood come alive from her, being then little live eels, no bigger nor longer than a pin" (Walton/Cotton 1897: 185–188).[119]

Petrović's *Eel Novel* presents to a wide readership a retrospective of the proposed solutions to the aforementioned two-thousand-year-old mystery of the eel in easily understandable terms, starting with the discoveries of the naturalists Redi and Mondini in the seventeenth century, followed by Sirevi and Jacobi in the second half of the nineteenth century, as well as Grassi and Fesersen by the end of the same century, ending with the pre-eminent Johannes Schmidt, whose scientific research carried out in the first half of the twentieth century lasted for twenty-five years. After all the reconsiderations, challenges and enigmas, that research resulted in an in-depth scientific description of the breeding of that mysterious fish, in whose creation maybe there is no divine, but there is undeniably an inexhaustible source of supreme creative imagination of our holy Mother Nature. With a view to giving a detailed account and interpretation of the numerous questions encountered in ichthyology to do with unravelling the mystery about the eel, Mihailo Petrović Alas has himself undertaken trips to the eel spawning areas in the Atlantic Ocean. In his "novel" he included a series of outstanding reports, replete with extraordinary, sometimes intriguing, sometimes almost unbelievable details, as well as facts established by experience and then scientifically confirmed and explained, and last but not least, some striking impressions. It was such impressions that enabled Petrović's text to overcome the limitations of the documentary genre and make for a compelling read, with a worldview of an individual-empirical provenience transposed into writing with great inspiration and conveyed by the matching, recognizable, nicely modulated narrative voice of the author. This in turn begs the pivotal, previously partly suggested, question of whether Petrović's prose of the said type, at the secondary level at the least, could be read and considered within the context of literary art.

Indeed, it could. Even though he did not consider himself a writer, dismissing the thought that other people see him as such, and being a self-proclaimed rationalist – that is, a scientist who believes in facts and practical evidence – expressing scepticism regarding the human need for fantasizing, particularly towards flimsy "poetry" and the dissembling art of the "theatre", Mihailo Petrović Alas has nevertheless shaped a considerable part of his opus by literary methods and devices, by following an inherent rule that the portrayed world is conditioned by the one who writes the story. Not only in a great number of travelogues, which are an eminently literary genre, but also in texts of another, non-scientific character, Petrović proves himself to be a writer with an engaging style. That style is neither expressive, nor lyrical, after the fashion of the best writers of his time, but very suggestive; the subjects and themes thereby exposed instantly grip the readers' attention.

Many critics wrote about the literary qualities of Petrović's texts (Milan Bogdanović, Milivoje Pavlović, Dragan Trifunović, Slobodanka Peković, and others). They are united in the view that Petrović is a master of the functional, reporting narrative style, an engaging and beguiling narrator focused on presenting his key subject with utmost precision. He is completely in charge of the form and the narrative rhythm, he controls the relation and proportions holding between the main course of action and the digressions, strikes a balance between a document and his own reaction to it, all of which is suggestive of the writer's rational, scientific character, of his emotional restraint, and clear, positivist penchant for exact facts apprehended by common sense. (This could be brought into question up to a point: Mihailo Petrović Alas was, namely, an avid reader of adventure and science fiction novels, with a particular interest in the relation between science and poetry, which is the subject of one of his studies.) Alas's language is accurate, mildly archaic at times, and an added value is represented by his lexis, always pertinent to the subject of narration, specialized when it is required, and colloquial when describing life itself (for example: fishermen's nicknames, an index of tools, professional jargon, etc.). What testifies to this are his travelogues and texts related to fishing, as well as certain essays and articles, some of which represent full-fledged stories without an excess of poetic stylization ("A Great Muslim Pirate" („Jedan veliki muslimanski gusar"), "Memories from the Grammar School" („Gimnazijske uspomene"), "An Experience from the Djerdap Gorge" („Jedan đerdapski doživljaj"), "Mija Jagodinac, a Bandsman" („Muzikant Mija Jagodinac")).

Standing apart from, but indirectly related to the other scientific and existential interests of Mihailo Petrović, is his incomplete phenomenological study *Metaphors and Allegories,* published posthumously in 1967 by „Srpska književna zadruga", the publisher of many of Alas's books. Starting from the description of the properties of metaphor, that universal, generative literary trope, and allegory as a particular artistic mechanism of conveying meaning and crystallizing sense, Petrović deals exhaustively with the issue and types of analogies, that is, sets up a network of analogies, which represent the basis of the scientific and creative thinking alike, and rounds off his work by systematizing typical events and their roles, in an attempt to get as close as possible to an otherwise elusive principle of the phenomenological "absolute adequacy". As Dragan Trifunović, editor of the first edition of the book *Metaphors and Allegories*, finds, "Petrović is adept at grasping the sense of those figures of speech and develops an intriguing theory of affinity as yet another contribution to his 'extended mathematics'" (Петровић 1998: 233).[120]

Let us now go back to the beginning. Petrović's inimitable creative nature presents itself to us from a distant past, since which time the world has rapidly accelerated and transformed in a way that our ancestors could not even have dreamed of, as an epitome of a harmonious, optimal realization of the full potential of a creative individual, who was in all things second to none. In saying that, we take account of his sense and passion alike. Petrović's passion, fishing – and every passion truly beggars all description – belongs to the deepest, archetypal, collective instincts of humankind. By realizing the instinct for hunting, that passion returns to the general mysterious laws governing the existence of the whole living world and (re)turns to be its integral part. Few are the passion's captives who succeeded in bringing it to their awareness, in bringing it to light, as far as it is humanly possible. Petrović is, without a shadow of a doubt, a representative of that small circle. Everything is transient, but the story remains. With his stories about fishing, about his travels and everything else besides, Alas indebted the Serbian literature and culture. With him at our side, we have become quicker and better at "reading water", as well as the world, in which we perish so quickly, barely managing to squeeze in a couple of fishing trips.

And on that note, one more time – there is but one Mihailo Petrović, Alas.

TRAVELS AND TRAVELOGUES*

Milan BOŽIĆ
University of Belgrade, Faculty of Mathematics

Even a brief insight into the biography of Mihailo Petrović Alas suggests he was a man constantly on the move. Even properly written biographies can at times be misleading as biographers, sometimes even unconsciously, delineate events in the person's life which can be interpreted in different ways. On the other hand, what travels are if not events?

Moreover, very few readers would read lengthy descriptions of a hero's everyday life. Occasional anecdotes may make a biography absorbing, but without such details the narrative remains vapid. The biography of Mihailo Petrović Alas doubtless confirms our starting assumption. This is evidenced both in literal and metaphorical sense.

In literal terms, the most important part of his life were his, rather serious journeys. In the epoch in which he grew up, his travel to Paris to continue his education after completing the Great School in Belgrade, was not an insignificant endeavour. As years passed by, he was an unremitting, incessant traveller. He went around present-day Belgrade suburbs and Danube backwaters, where he caught fish and earned money, was striving to stay alive or performed important state duties during the world wars, was visiting Paris to which he would always return, or was travelling to "ordinary" scientific conferences and meetings. In his

* A revised and supplemented version of the paper initially published in the catalogue *Mihailo Petrović Alas: the Founding Father of the Serbian School of Mathematics* (SASA, 2018).

A Laplander, around 1930
(SASA Archive, 14188/13-5)

advanced and working age, he visited a number of world capitals, discharging his scientific and war-state duties. He was also a coder of the Serbian and later Yugoslav Army.

Still, the travels we dwell on in this article are rather special, in terms of time, place and circumstances.

A significant turnabout took place in the fourth decade of the 20th century and the seventh decade of Petrović's life. Between 1931 and 1939, he spent almost every summer on long journeys, often scientific exhibitions. He travelled practically across the entire world, from the North to the South Pole. It is worth reiterating that Petrović embarked on these journeys in his advanced age, when people rarely cross their thresholds. It not usual even today that someone would go above and under the two great polar circles, let alone then, almost one hundred years ago.

Finally and most importantly, after these journeys, Mihailo Petrović became a travel writer. He published a number of books, voluminous travelogues, which make up two comprehensive volumes of *The Collected Works*.

The secrecy imbuing these Petrović's journeys is further deepened by his labelling them "scientific expeditions", without stating the motives (which, ultimately, can be a personal matter) or the sources of funds and connections with explorers that he travelled with.

In his *Letopis života i rada Mihaila Petrovića Alasa (Chronicle of the Life and Work of Mihailo Petrović Alas)*, Petrović's most notable biographer Dragan Trifunović[121] thoroughly covers the fourth decade of the 20th century, but mentions nowhere the motivating factors behind these Petrović's grand journeys. As meticulously noted by Trifunović, Petrović left behind him an enormous number of letters and notes, in which he addressed his friends, relatives, colleagues (either in the fields of science or fishing), government and paragovernment authorities and many others.

However, even a most thorough insight into his legacy does not reveal how he embarked on his journeys. Dragan Trifunović wished to believe that Petrović undertook them for scientific and thus state purposes, in order to decipher the earth's magnetic field and ice movement on the poles so as "to study the possibilities of ship navigation". Of course, the question immediately arises as to

Eskimo shamans, around 1931 (SASA Archive, 14197/II-3-6)

why a country, impoverished and without a significant influence such as Yugoslavia, would have played any role in such expensive research.

Still, it is known that Petrović had numerous friends and acquaintances, whom he gained primarily during his education abroad, notably in Paris, who were, in their advanced age, wealthy enough to undertake such expeditions and invited their friend Mihailo Petrović to join them as well. This hypothesis is supported by the fact that, for instance, a six-member French expedition set off on the first journey to the North Pole from Dunkirk, joining a Norwegian expedition that numbered around twenty persons. For those less familiar with the history of the period, France was a superpower at the time, contrary to Norway. The author of this paper is more inclined to the second theory, but would also like it to turn out that Mihailo Petrović dealt on these journeys with important inventions which were put to use in the oncoming war. Even if it had not been so, he left to future generations voluminous travelogues with numerous comments which we will cover by the end of this paper. If someone thinks this was not a great achievement, they should consider the fact that the Serbs do not seem to be particularly fond of travels.

The first journey (summer 1931)

Northward Ho!

From France to Greenland and almost all the way to the North Pole.

The first expedition started from the historically famous Dunkirk. This journey is well-documented, which attests to Petrović's degree of diligence, typical of natural scientists and less common among mathematicians. He also shows, in a way, his second nature, reflected in his scientifically grounded curiosity towards the sensory world around him. Before setting

Personal possessions of Mihailo Petrović, his loyal traveling companions: a leather suitcase with initials, a metal cup in a leather casing with initials, the mechanically powered dynamo flashlight (Luzy), a photo camera (Hutig A:G Dresden, 1908). I. Marković, 2018 ("Mihailo Petrović" Foundation)

off, he took a picture of himself in the uniform of a reserve officer, with binoculars and in boots[122]. They took extremely difficult routes, even measured by present-day standards. They even went beyond modern standards as living people would today be replaced by drones and satellites.

In his travelogue, based on which a precise map of the journey was made, Mihailo Petrović first gives a short overview of the route. He then focuses on the pictures that left on him the greatest impression. He writes about icebergs, polar bears, and outlines the tasks of scientific expeditions. Just like an anthropologist, Petrović analyses Eskimos, their economy and style of life.

> "Of everything I have had the chance to see on this unusual journey, my meeting with the most primitive man living today on the planet excited me the most – it was the first roaming Eskimo that we came across. It was a creature clad in bearskin. When it saw us, this creature suddenly stood up, grabbed its short spear and hesitated for a moment whether to run away or stay. This was the first Eskimo that we came across, a real Eskimo nomad, who for months, until a real polar night sets in, is roaming and hunting, incessantly eating and sleeping on ice wherever he finds himself at the moment. Nomadic families usually roam together. They leave on ice, together with dogs and small children, the sledges pulled by dogs, in which they carry their most necessary things. The men, women and older children go their separate ways looking in ice for a hole that a seal made beforehand with its mouth, and then – the hunt takes place... The hole widens up to half a metre. Lying on ice and holding his head above the hole, the Eskimo waits for a seal to appear, which, swimming under the ice, uses each hole to pull its head through it and breathe.

At the right moment, the spear tied to the hunter's hand pierces the animal, which gets instantly pulled from the water and killed".

Petrović notes down all morphological features of this group of Eskimos in a coherent and systematic way.

"All of them were smaller people, whose height did not exceed 1.50 metres. They have large, elongated sculls, with the upper parts of their foreheads narrowed. Their cheeks are wide and plump, noses flat, eyes small, black and barely open, probably due to severe weather they are constantly exposed to from their childhoods, and due to the sparkling whiteness of the snow and ice in which they spend their lives. Their hands and legs are small commensurately with their bodies, and the upper parts of their bodies are strongly developed; most of them are fat. They are dark red in the face. These are, doubtless, the specimens of the human species that today most resemble a primitive man, at least in terms of the way of life... They rarely live up to 50, perhaps due to their great voracity for meat. An Eskimo, as soon as he catches an animal, seal, walrus, polar bear etc. starts immediately, on the spot, to eat, ingesting even up to seven kilograms of meat per meal. However, such short lives seem not to apply to women, who very often live up to 60–80 years. This can probably be put down to their less arduous lives than pursued by their men. Women entirely resemble men. They are not more beautiful than them, behave in the same way, and therefore cannot be distinguished. Apart from raw meat and raw fish, they do not eat anything else. They know not of salt, they do not particularly need it because the animal food that they exclusively use contains sufficient salt for the needs of their organisms."

Petrović describes the Eskimos' abodes in a picturesque and documentary way.

"When in their settlements, the Eskimos live under tents or in huts made of ice blocks, which they sometimes cover with snow. Their tents are made of sealskin, propped by wooden poles and stones; the cracks are plugged with Greenland moss. There are no windows and the entrance is closed with a curtain made of seal intestines, split and stitched one to another; such curtain does not let in wind and lets in some light. The huts are made of ice blocks cut by knife and assembled into semi-spherical cupolas (called the "igloo" in their language). A narrow tunnel 3–5 metres in length, through which they squeeze on all fours, connects the interior of the igloo with its exterior. When I tried to enter an igloo through such tunnel, I almost fainted halfway because of the indescribable stench coming from the entrance of the igloo and, going on all four backwards I returned to the exit faster than I came in. During long polar nights, a primitive lamp burns in the igloo, chiselled in the soft stone, with the wick made of spun moss. The lamp lets in faint light and a bit of warmth. They very rarely use fire as they have no fuel, and are used to eating only raw food. While sitting in their huts, they are well clad in skin. When they want to go to sleep, they lie one next to another, completely naked, on the spread skin of the polar bear and cover themselves with long skin. Their clothes are made of the skins of seals, deer or the polar bear, or their combination. They are sown by women with needles made of fishbone. Both men and women wear trousers, often decorated, as well as coats with woven or etched images. They usually put on two pairs of clothes; when at the open sea, they put on one more sealskin, which preserves the bearskin underneath from becoming wet. They put on their heads skin hoods,

> which leaves only their eyes, nose and mouth open. They often pull over their eyes little sealskin with two thin horizontal slits to look through; they thus save themselves from snow needles which can cause eye inflammation, and from the reflection of the snow and ice. They have on their legs some sorts of socks, in the form of bags made of skin of polar rabbits, over which they pull sealskin boots."

The Eskimos from eastern Greenland, about whom Petrović wrote, never quarrelled or fought.

> "They have phlegmatic temperaments, they are open and good-natured, but without excitement and exuberance. Once they stop looking upon a foreigner with mistrust and realise they have nothing to be afraid of, only they care about is how not to make him angry or sorrowful. No one has ever heard them complaining of their tough life or anything else. They have a very poor notion of ownership. They believe everything can be shared with others apart from women, clothes and tools. When one of them catches a seal, all those nearby can eat as much as they want. This, however, also implies the duty that each of them must hunt or assist in hunting and preparing tools and gear when it is requested from him."

Understandably, in his descriptions of the Eskimos' lives, Petrović devotes most attention to their hunting practice.

> "As soon as, in early spring, the sky clears up so that it is possible to hunt, and the time allows them to survive outside, the Eskimos go hunting in groups – some of them on sledges, others in boats or dinghies. [...] Sledges are pulled by large and strong Eskimo dogs, wild and sanguinary almost like wolfs, but very obedient to the Eskimos. [...] They use for navigation large boats and light dinghies. [...] Large boats with wide sails of sealskin, which carry 6–12 persons each, are used for entire families with their luggage, or for whale hunting. Much more interesting for them are light dinghies for individuals, called "kayaks", which play a very important role in the Eskimos' lives. They obtain primarily what they need for everyday life and food: meat and fat, skin for clothes, footwear and tents, bones for tools and other. Kayaks are made of tanned sealskin, stretched over wooden bars and tightly tied to the bars with straps and strings of skin or intestines. They are entirely closed, covered and waterproof. The Eskimos get used to handling them from their earliest childhood. They use them to navigate the seas, look for prey and brave strong winds, even storms, because kayaks cannot sink unless pierced by solid objects."

Petrović's overview of the method of hunting and adequate equipment is rather exhaustive and detailed, but he did not stop there. He also devoted particular attention to the spiritual life of the Eskimos, their customs and beliefs.

> "Above all, the Eskimos have their own language which, although poor, has expressions even for things going beyond everyday life. They have their beliefs, superstitions, even their naïve explanations of what is happening around them. Their language is very poor in words and expressions, and can therefore be very quickly and easily mastered. One word often expresses an entire idea, but as the number of ideas is small, the number of such words is also very limited.

Mihailo Petrović's passport. I. Marković, 2018 ("Mihailo Petrović" Foundation)

Sentences are very short, often consisting of one word only. The Eskimos' religious ideas are highly undefined and vague. They believe in monsters and evil spirits that only their sorcerers can influence. Natural phenomena such as: the northern polar light, thunders, very rare in those regions, become a good or evil will of those monsters and spirits. The land of the dead is entirely similar to the land of the living – the Eskimo continues his hunt there and can die once again in that land. The appearance of sunlight after clouds suggests that a young man died somewhere in that moment, and the Great Spirit makes the sky and earth cloudy and shiny so as to solemnly receive his soul. The sky is a spacious earth with holes, and these holes are stars. This land is inhabited by people who left this earth to another world; when they pour down water, the water leaks through those holes to this world – this is rain. When a man or animal die, the moon carries the soul to the land upwards. When the moon is not seen, it carries the souls. A man's soul can be transformed into the soul of any type of animal. Good people become people again, and the evil ones become animals; everything that lives cannot be destroyed."

Finally, in the third layer, Petrović explains in detail various phenomena explored by polar expeditions and gives a highly in-depth overview of the main expeditions to the North Pole from which, at the time he was writing, less than a half a century elapsed. This travelogue – and the author applies the same method, in a highly meticulous way, in other travelogues as well – is, in fact, a true textbook on geography, demography and economy of the regions he visited. It may be possibly called the Encyclopaedia of the Arctic Circle.

The cover page of the book *On the Remote Islands* in which Mihailo Petrović describes the voyage of scientific expedition to the southern polar region in 1934–1935 (Library of SASA, C 7/12;9)

The second journey (summer 1932)

Eastward Ho!

Across the Atlantic to the Sargasso Sea, Caribbean islands, Antilles and Bermuda.

This journey could have the status of "a pleasure trip" both in Petrović's time and today. Mihailo Petrović travelled with his friends to the islands of Central America, the Azores, Haiti, Bermuda... Being a good travel writer, he feels he must intrigue his readers. Apart from picturesque descriptions, the most interesting is his division of sea plunderers into regular and irregular, i.e. state and ordinary pirates. State pirates – this is how I began to read about famous sir Francis Drake who eventually became an admiral and defeated the Spanish Armada which attempted to invade England – looted their enemies, waging a sort of "a trade war", though with somewhat more violent methods than today, but with similar consequences.

Of course, Petrović meticulously describes fishing and the economy of Central American islands, publishing even two travelogues about these journeys.

The third journey (summer 1933)

Northwestward Ho!

To Labrador and Newfoundland.

This route was also directed rather to the north, but already in a warmer place. People lived there, there was an economy mainly based on fishing and whale hunting. Being a passionate fisherman, Petrović did not miss any of the most important products of the region – the cod. While the northern route was coloured with ice, coldness and dangers, Petrović describes this route with many more human colours.

The fourth journey (summer 1934)

Southward Ho!

Towards the Antarctic via the Atlantic route.

If you thought that the South was warm and the North cold, you were wrong. Of course, it depends on how southward you go. On his journey to the Horn of Africa, Petrović saw penguins, visited Saint Helena, the famous island where Napoleon spent in exile his last days. He missed to see the giant tortoise from Napoleon's time. It died aged around 200, a year before the arrival of Petrović's expedition. The British Empire brought tortoises mainly from the Galapagos islands. Petrović wrote they were owned by the sovereign United Kingdom of Great Britain and Northern Ireland and, as the Crown's property, were roaming around numerous parks of the governor's residence, with the governor's children "riding" on their back. This tradition is strong still today with the oldest "movable living being"[123] Jonathan[124] that lives on Saint Helena. In his travelogue "Around Far-Flung Islands", Petrović published a photograph with two tortoises for which he claimed to originate from Napoleon's time. However, the photo was taken in 1886 and contemporary biometric analyses have confirmed it was Jonathan and his "lady"[125] that he outlived by around twenty years. Petrović cannot be blamed for this error – he received the photographs together with the claim about the origin – but he perfectly noted the fauna which is still today a world attraction. Above all, he saw "the future oldest animal"!

Map of the route traveled from the French coast to Madagascar in 1935

The fifth journey (summer 1935)

Westward Ho!

To the Indian Ocean.

The fifth expedition went through the Suez towards the Indian Ocean. Just like the second one, it focused mainly on French colonies, which are today already independent countries. However, they also visited, as reflected in the travelogue name, far-flung islands, which still today fascinate the public in BBC science and nature documentaries. Petrović again showed his directing talent, selecting the flora and fauna that he explored and wrote about. Madagascar understandably takes the main place, which is in fact a smaller continent, with numerous living species that can be found nowhere else on earth[126]. He also examined the indigenous people's way of life, displaying his unusual gift to describe exactly what we consider necessary. We can

In his book *In the Empire of Pirates*, Mihailo Petrović writes about notorious pirates, such as Charles Vane, who robbed the Antilles, the Bahamas and the Bermudas.

only imagine what Petrović would have made had he had, instead of his camera, modern multimedia recording devices!

The sixth and last journey (summer 1939)

Warward Ho!

The break in his journeys lasted for four years. We do not know the reasons. Tiredness, finance, the crisis in Europe on the eve of the war. This journey was somewhat shorter. It ended on the Azores, 1500 km northwest from Portugal. Petrović set off on this journey with the same French team with whom he had travelled across the world. As this journey was also "on the route of eels", we can assume with certainty that it, along with the second journey, served as the basis and inspiration for the *Eel Novel*. As Petrović mentions at almost each his journey the sea flora, particularly fish, his narrative is imbued with his fishing sentiments as well. The journey routes suggest that other expedition participants were also highly interested in fish, notably eel movement. As the names or functions of members are rarely mentioned, we cannot claim with certainty that Petrović's fellow travellers were interested concretely in eel. Still, it is not disputable that eels, passing through the entire Atlantic and the Baltic Sea to spawn and close their life cycle are still today intriguing to experts and us, laymen, alike. Mihailo Petrović returns to his country after this journey and goes to the war, approaching his final days.

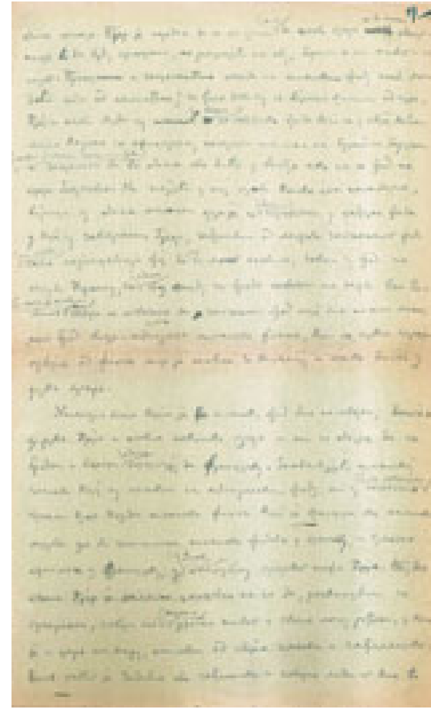
Travelogues

All descriptions of the grand journeys that Petrović undertook in the fourth decade of the 20th century are in fact his own. They are a part of voluminous manuscripts which, unlike his mathematical work, were created very suddenly.

Already after his first journey, in summer 1931, Petrović addressed[127] his friend Pavle Popović[128], president of Srpska književna zadruga (Serbian Literary Cooperative), proposing that his travelogues be printed "to the amusement of the Serbian people". Being a long-lasting Petrović's friend, Pavle did not hesitate and published the first travelogue in the literary edition of Zadruga, regular circle, in the *Savremenik (Contemporary)* library, thus immortalising Petrović in another way, unexpected not only for us, but for Petrović as well. Thus, along with being a travel writer, Petrović became a literary author. The most important information linking Popović and Petrović is the fact that Popović was elected a professor of the Great School in 1904, along with Simo Lozanić and a few others. The following year, he was elected a professor at the newly founded University. The purge carried out by members of the Karađorđević Dynasty after the coup d'état was rather comprehensive. At most a third of those who were professors of the Great School in 1904 and later of the University survived. A similar age and favour they enjoyed with the new Dynasty created strong mutual connections among this small group of people. The Dynasty's importance declined after the Great War, but personal connections remained, perhaps even becoming stronger, as the political background sustaining them disappeared.

Given the above, the assessments that Petrović imposed with his travelogues should be taken with reservations. Pavle Popović was most probably well informed about Petrović's journeys, seeing in them not only a potential literary text, but also a connecting transversal of his generation. Such approach can also explain how the first travelogue of Mihailo Petrović, who was after all an amateur and beginner in literary profession, was printed in the *Savremenik* edition, where the works of the most eminent Serbian writers were published.

This cooperation continued. As many as five Petrović's books were published by Srpska književna zadruga: *Kroz polarnu*



Manuscript of Mihailo Petrović for preparation of his book *In the Empire of Pirates* ("Adligat" Society)

In his book *With the Ocean Fishermen*, Mihailo Petrović writes about endurance, persistence and sobriety of the oceanic fishermen.

oblast (Through the Polar Region) (1932)[129], *U carstvu gusara (In the Empire of Pirates)* (1933)[130], *Sa okeanskim ribarima (With Ocean Fishermen)* (1936)[131], *Po zabačenim ostrvima (Around Far-Flung Islands)* (1936)[132] and *Roman jegulje (Eel Novel)* (1940)[133]. Apart from the *Eel Novel*, the rest are travelogues[134], – the first from the first journey, the story about pirates and fishermen from the second, and far-flung islands from the third.

The *Politika* daily published "abbreviated editions" of Petrović's travelogues, with his notes and reflections. *Politika* may have exerted a stronger influence on Mihailo Petrović becoming a paradigm of the travelogue genre among the Serbs than all publications of Pavle Popović.

It is also true, or at least it again seems so to the author of this article, that Mihailo Petrović understood well the power of the media, using his self-promotion gift to the utmost in his epoch. Others, however, believe this is not befitting for such a great man. The exact explanation probably lies somewhere "in the middle", which makes it both tedious and irrelevant.

Of the fifteen volumes of *The Collected Works*, *Travelogues* make up the two largest. Slobodanka Petković (the foreword to book 13, i.e. second volume of *Travels*) wrote an excellent article where those who like curiosities can find a lot of information missing in this overview.

Being a descendant of men and women who built their reading careers on Petrović's travelogues, I read all his travel writings while still a child. Colleague Mihailo Pantić, who contributed to the exhibition catalogue and this monograph with an outline of Petrović's fishing activity and literary work, proposed that Mika's literary work be subjected to a serious analysis. The timeframe for such endeavour being too short for this occasion, I am confident that Pantić is right and that a separate "project", as it is said today, should be devoted to such task.

Petrović's travelogues and other works can be accessed in digital form in the Virtual Library of the Faculty of Mathematics[135].

MIHAILO PETROVIĆ'S FISHING – ONE VIEW

Nenad TEOFANOV
University of Novi Sad,
Faculty of Sciences

I

Mathematical phenomenology is an original theory by Mihailo Petrović, which, as a mathematical theory of the activity of causes, was briefly presented to the public in his accession academic speech when he became a full member of the Serbian Royal Academy. In less than 10 years, Mihailo Petrović grew from a graduate of the Belgrade Faculty of Philosophy into a world-class mathematician and became a full member of the Serbian Royal Academy. After completing his studies at the Belgrade Faculty of Philosophy's Department of Natural and Mathematical Sciences, Mihailo Petrović moved to Paris for further education. The year was 1889, late September, and the World Exhibition (Exposition Universelle) was on, an ambitiously organized event marking the centenary of the French revolution. The Eiffel tower dominated the exhibition, which was built for this occasion. In this period, later named the Belle Epoque[136], Paris was the center of significant social changes. In addition to scientific, technological and cultural innovations, the emergence of entertainment industry and consumer society took place. Educational, scientific and medical institutions, alongside casinos, cabarets and restaurants in Paris were the leading ones in Europe at the time.

Studying at the prestigious l'École normale supérieure enabled Mihailo Petrović to form himself as a scientist under the influence of the top-notch professors among whom the most important one was probably

Henri Poincaré, who, together with the other committee members, approved the subject of Petrović's future doctoral dissertation. The committee before which Petrović defended his doctoral dissertation at Sorbonne in 1894 consisted of equally famous mathematicians Hermite, Picard and Painlevé.[137]

The Serbian Royal Academy in Belgrade recognized the quality of Petrović's scientific discussions from that period and consequently he became a corresponding member in 1897. That same year, the first International Congress of Mathematicians was held in Zurich, and Poincaré was invited to participate as one of the four plenary lecturers.[138] At the opening ceremony, Adolf Hurwitz highlighted the importance of aligning mutual cooperation and individual act of mathematical research: "No science, unless perhaps philosophy, has such a brooding and solitary character as mathematics. Nevertheless, in the breast of a mathematician subsists the need for communication and understanding with colleagues."[139]

The subsequent world congress of mathematicians, held in Paris in 1900, was marked by the rivalry between the two approaches to nature and the essence of mathematical research, a formalistic one and an intuitionistic one, as they were termed later. Poincaré was classified into the intuitionistic group, whereas David Hilbert was the main proponent of formalism, who delivered his famous lecture on mathematical challenges heading into the 20th century.[140]

Mihailo Petrović took part in the congress and it is very likely that he attended Hilbert's lecture.[141] Henri Poincaré, Petrović's professor and one of the four plenary lecturers in Paris, was also in the audience. One of Hilbert's problems, the so-called Poincaré Conjecture, was included in the Clay Mathematic Institute's Millennium Problems list, which was published in 2000. The list contained seven mathematical problems for the solving of which an attractive reward of one million dollars per problem was offered. The Poincaré Conjecture has been the only solved problem from the list up to now.[142]

A few months prior to Hilbert's lecture, on January 9, 1900, Mihailo Petrović delivered his accession speech titled *On Mathematical Theory of Activity* at the ceremony held on the occasion of his appointment as a full member of the Serbian Royal Academy. Petrović's theory, later termed mathematical phenomenology[143], has been original, ambitious and promising, in accord with the spirit of the most important mathematical achievements of that time.

II

Mihailo Petrović delivered his accession speech at the Serbian Royal Academy almost one year after he was elected its full member on February 4, 1898. In his speech he expounded the "first draft of a theory" and announced the direction of his future research. The theory was about *the causes of phenomena*, with an emphasis on disparate, different phenomena and *about possibility of discerning analogies among those phenomena*. For this purpose, the term *activity of causes* was introduced, representing the dynamic side of the cause, embodied in the *aspiration* of the cause, which is defined by its meaning and intensity. The discussion aims to show the following:

Mihailo Petrović's memory from fishing (SASA Archive, 14197/II-23)

"That it is possible to introduce the term of activity as a pure, general, precise term, which possesses all characteristics of terms in mathematical analysis;

That it is possible to develop a general, purely mathematical theory the subject of which would be the following: researching various activities in terms of their dynamic nature, looking into their various combinations and determining the effects that result from the impacts of those activities on a certain phenomenon;

That the theory developed in such a way can be applied to seeking quantitative laws of all phenomena for which the activities of the cause are known, regardless of their nature."[144]

In order to clarify this abstractly formulated goal, Petrović states a more specific idea: "Instead of gravitational, electrical, magnetic, chemical, etc. forces, we would deal with a general term of *causes* and their *activities*; instead of a mechanical, physical, chemical, etc. phenomenon which is being observed, we would have an abstract concept of *effects*; instead of special laws based on which the mentioned specific causes act, we would deal with the laws applicable to the aspirations of causes according to which they attempt to affect the phenomena. Knowing the way in which a cause or a group of causes aspires to strengthen or weaken a phenomenon, we can calculate their effect and determine the law based on which the phenomenon will change when the strength of the cause with such aspirations changes."

Finally, various phenomena, observed with the proposed method, could be unified under a more comprehensive theory. According to Petrović, all separate theories obtained in such a way would be then observed from a *higher point*, from which they would look as parts of one and the same whole. Thus, Petrović used his accession speech to expound the program of his future scientific work, shortly stating the idea concerning the way in which his theory of activity will be treated with the help of mathematical analysis, and the general aspect this theory will have once it is fully developed.

Petrović further observes historical phenomena, environmental impact on the development of various biological species, science of language development, etc, in line with the idea of developing a theory which would combine and bring to a common basis a huge number of disparate theories, which otherwise would not share any interdisciplinary ties whatsoever. Undoubtedly, such an approach was inspired by the success of the mathematical description of physical phenomena. As an illustration of analogy among disparate phenomena, we mention examples of proportionality of some physical quantities. That is the simplest possible relation between the two physical quantities in nature, and it is described by a linear function.[145]

The continuous flow of electric charge (direct current, without the change of the velocity of a charge carrier) determines current intensity. Such current intensity, defined by a change in charge quantity which passes through a certain point of the wire in a unit of time, is proportional to the difference of potentials (voltage) at the ends of the wire:

$$I = \frac{dQ}{dt} = -\frac{V_2 - V_1}{R},$$

where R stands for resistance, and V_2 and V_1 stand for potentials. The minus sign shows the direction, i.e. that the flow goes from the point with higher potential towards the point with lower potential. If G denotes conductivity, then we obtain a connection which is also known as the Ohm's law:

$$I = \frac{dQ}{dt} = -G\,(V_2 - V_1).$$

The next example is related to a thermal phenomenon. If a metal bar the length of which is denoted by l is heated at one end, and constant temperature is maintained at the other end, then the quantity of heat passing through the bars' cross section in a unit of time is proportional to the difference in temperatures at the bar's ends:

$$\frac{dH}{dt} = -\frac{kA}{l}(T_2 - T_1),$$

where A denotes an area of the bar's cross section, and k is thermal conductivity of the material. The minus sign says that the heat flows in the direction opposite to the temperature increase. With the notation

$$K = -\frac{\kappa A}{l}$$

for the bar's thermal conductivity, we get the equation

$$\frac{dH}{dt} = -K(T_2 - T_1).$$

We can notice the analogy with the Ohm's law. However, instead of the flow of electric charge, here we are dealing with the flow of heat, caused by the difference in temperatures, not in potentials.

The third example looks into the flow of liquid through a pipe (without turbulence) which occurs due to the difference in pressures at the ends of the pipe. If r is the pipe's radius, l is the pipe's length, and η is its viscosity (measure of its inner friction), then the volume of liquid which passes through the pipe's cross section in a unit of time is shown with

$$\frac{dV}{dt} = -\frac{\pi r^4}{8\eta l}(p_2 - p_1),$$

where p_1 and p_2 are pressures at the ends of the pipe. The minus sign marks the direction of the flow of the liquid from the end with higher pressure towards the end with lower pressure. Such a relation is named Poiseuille equation. With the notation F for flow conductivity through the pipe, we get

$$\frac{dV}{dt} = -F(p_2 - p_1).$$

There is a similar law, for instance, related to diffusion (change in the concentration of a solution) which relates a concentration gradient with the difference in concentrations at the ends (of sump)

$$\frac{dn}{dt} = -C(n_2 - n_1),$$

where C denotes diffusional conductivity (of the sump).

Therefore, the flow phenomenon is characterized by a general relation that is evident in the following table:

Current intensity	$\frac{dQ}{dt} = -G(V_2 - V_1)$
Heat	$\frac{dH}{dt} = -K(T_2 - T_1)$
Flux	$\frac{dV}{dt} = -F(p_2 - p_1)$
Diffusion	$\frac{dn}{dt} = -C(n_2 - n_1)$

Mihailo Petrović wanted to generalize the analogies of this type of phenomena in sociology, economy, linguistics, and to describe them with mathematical formulas, by expanding the existing terminology. Petrović dealt with mathematical phenomenology for the rest of his life,

Mihailo Petrović – Mika Alas on the Danube, Belgrade 1911 (*Collected Works*, Book 14) (Digital Legacy of Mihailo Petrović)

developing his theory of causes, activities and analogies in a number of scientific discussions and books: *Analogies among Disparate Phenomena* (1902), *An Attempt at a General Mechanics of Causes* (1905, translated into French in 1906), *Elements of Mathematical Phenomenology* (1911), *On Mihailo Petrović's Works* (1921), *Phenomenological Mapping* (1933), *Mathematical Analysis and Oceanographical-Biological Problems* (1939) etc.

It is impossible to briefly present Petrović's theory, nevertheless, one may caught a glimpse of the goal of his approach in the following quotes:

> "If activities of all the factors that either actively or passively participate in causing or maintaining a certain phenomenon would be known, the phenomenon would be fully understandable and its state would be known in advance at any moment, the same way as in rational mechanics, where the state of a movement is known at any moment, provided that the forces that cause it, resistances that hinders it, and material relations that are maintained during the movement are known. A form of mathematical laws concerning phenomena primarily depends on the roles and activities of causes that participate in them: in two phenomena, no matter how mutually disparate they may be in terms of their specific nature, *mathematical laws will be the same by their form if the corresponding factors in them play the same roles and if they have the same dynamic nature of activity.....*
>
> The common characteristics of the flow, imposed by a certain type of a mechanism on various disparate phenomena, clad in specific meanings contained in the observed specific natural phenomenon are also externally expressed by specific properties that are immeasurably versatile, depending on the specific nature of the phenomenon. Thus, the growth of an element will be manifested either as an increase of translation or rotation in movement, or as a gradual change of some color from red to purple, or as warming of an object, or as an electrical current that is becoming stronger and stronger, speeding up of a chemical reaction, worsening condition of a disease, etc."[146]

Given the previously said, one can easily perceive the similarities with the idea of the Grand Unified Theory the physicists have been dealing with, in particular since the mid– twentieth century, under the influence of Einstein's research. In the last decades of his life, Einstein was preparing himself to develop a unified field theory, i.e. a theory that would unify all natural laws owing to a unified mathematical language. Instead of separate groups of laws for individual physical phenomena, Einstein wanted to discover the whole, which would unite all the laws. More precisely, it was a search for a "mathematical sentence" that would include gravity and electromagnetism. Gravity was described by Einstein's Theory of General Relativity, whereas the other force, electromagnetism, was described by Maxwell's equations from 1864. Maxwell's unification of electrical and magnetic forces in the 19th century has been one of the invaluable scientific contributions. "Before Maxwell, the electricity flowing through a wire, the force generated by a child's magnet, and the light streaming to earth from the sun were viewed as three separate, unrelated phenomena. Maxwell revealed that, in actuality, they formed an intertwined scientific trinity. Electric currents produce magnetic fields; magnets moving in the vicinity of a wire produce electric currents; and wavelike disturbances rippling through electric and magnetic fields produce light.[147] Einstein intended to generalize Maxwell's program by formulating a theory of unified description of natural law that would include electromagnetism and gravity.

In the meantime, two more forces were experimentally discovered: strong nuclear force and weak nuclear force, and eventually the unified theory was to include all four forces. During the late 1960's and 1970's physicists managed to describe the weak nuclear force and the strong nuclear force by applying the methods of the quantum field theory to electromagnetic force. Therefore, all three forces that are not related to gravity can be described by using the same mathematical language. Nevertheless, when those methods are applied to the fourth force of nature, gravity, mathematical description becomes useless, unusable.[148]

Among numerous examples that Petrović used to corroborate the ideas of his phenomenology there is a detailed description of a research conducted by professor Umberto D'Ancona in the field of biocenosis.[149] In this research from 1926, empirical data from fish markets in Rijeka, Trieste and Venice were observed. D'Ancona studied a natural balance between predator and prey fish species and eventually found an optimum related to fishing activity. When this activity drops below a certain level, predators are favorized, whereas when fishing increases above this optimum level, the effect is opposite. Petrović states that this result fits into the mathematical model that was previously formulated by the Italian mathematician Vito Volterra[150] in the form of differential equations, as well as some known fluctuations of fish species that were observed and explained by Volterra's equations.

III

According to Mihajlo Pantić, among all his interests and preoccupations, Mihailo Petrović – Mika Alas attached undeniable importance to his primordial passion – fishing.[151] In similar manner, Dragan Trifunović writes that the life of Mihailo Petrović was fulfilled with fishing and science, two natures constantly complementing one another, thus creating a unity and harmony, making Petrović the fisherman always present in Petrović the scientist. To his colleague Milutin Milanković, who was not at all interested in fishing, Mika Alas used to say that the best combination was: "A little bit of fish, a little bit of book!"

Petrović developed a passion for fishing prior to his passion for science. After his father Nikodim Petrović passed away (in 1875), his grandfather Novica Lazarević introduced him to the world of fishermen on the Danube and the Sava rivers. "Fishermen's life, charm of many nights spent on the Sava and the Danube rivers, passion and experience when fish is caught, all those things attracted the young Petrović to this sort of life."[152]

Even in the formal sense, Petrović was a fisherman by vocation: in 1882, at the age of fifteen, he became an apprentice (with fisherman Čuklja), then in 1888 he became a fishing journeyman, and upon his return from his doctoral studies in Paris, he became a master fisherman. In the meantime, during his stay in Paris, Mika was dreaming about his master's diploma, yearning for it as much as he was yearning for his doctorate degree in mathematics:

> "I used to wander and walk along the river Seine watching how they caught fish, dreaming of becoming a professional fisherman after I finish my studies and return to Belgrade. It was when I was a student back in Belgrade that I learned the fisherman trade and got my certificate of craftsmanship; I prefer it to my doctoral diploma."[153]

As a professional fisherman, Mika Alas took part in a number of important social activities. Thus, for instance, he participated in drafting the Law on Freshwater Fishing (1900), as well as in resolving the disputes with the neighboring countries, Austria-Hungary and Romania, and in developing the relevant conventions (1905–1908), in London he organized a well-noted exhibition of Serbian fishing (1907), whereas in Belgrade he organized the first exhibition of Belgrade fishing where he displayed the 83 kilo catfish that he caught in the Sava (1908), he founded a stock company with the purpose of organized exploitation of fish harvested from Ohridsko and Prespansko lakes (1921).

Still, he preferred life with fishermen on the Sava and the Danube to all these activities. There are many testimonies about this, especially about the allure of the fishermen's way of life, their joint work, code of conduct, mutual understanding and solidarity. Thus, for instance, M. Milanković wrote that, on one occasion, Mika Alas announced he would go to the river, but he unexpectedly came to his office in the Institute, because his fellow fishermen, after a good catch and good income, got drunk, got into a fight and finally ended up in custody. Academician Petrović was obviously fond of these "common" and "uneducated" people who were, plainly speaking, "honest and good-natured".[154]

Academician Stevan Sremac also spent some time with Petrović on the Sava and the Danube, in an environment totally new to him. On the occasion of publishing Stevan Sremac's Collected works in 1938[155], Petrović recalled their past adventures and retold the content of a story which Sremac was preparing, and eventually published his own story about the life and adventures of fishermen in the prewar period.

Prince Đorđe Karađorđević had similar experience, writing in his autobiography that, during the mathematics class, he complained to professor Petrović that he got bored in the palace, and it was then when the professor proposed to take him to the Sava river and teach him the craft of fishing: "Fishermen greeted the professor as an old friend. He knew everybody, and everybody knew him. Nobody stood up to pay him respect, everybody was going about their business, and greetings were expressed by shouts. My presence did not bother anyone, and it did not raise any major curiosity. Even if someone had recognized me, it could not have been noticed."[156] However, these activities did not sit well with his father, king Petar: "I am not against fishing. It is a good sport, and fishermen are honest people. I even used to fish myself, but this was during my leisure time, and I was not an heir to the throne. You are in a different situation. Your actions are in the limelight, and everything you do is commented among people. You do not belong only to yourself anymore, remember this, and take care not to cause any unwanted reaction by your behavior."[157]

Immediately after his abdication in 1909, prince Đorđe spent some time on the Danube, and in his memoirs he gave a touching description of this period: "I am running away from my home, I am running away from my family – I am running away from myself. I am spending days and days on the Danube. Together with professor Petrović I am camping in fishermen's cottages, sharing life with common fishermen. I am dressed just like them – I have one cap which I never take off. And the professor is wearing always the same straw hat. His face has gotten dark and rough from wind and sun. We keep fishing constantly, day and night, and old fishermen, my friends, keep shaking their heads worryingly, watching me depressed... The professor keeps quiet... On the lapel of his coat, even here, on the water, the pen which I used to sign my abdication sticks out..."[158]

Mihailo's life was full of curiosities. The same was with his transoceanic voyages. He crossed the Atlantic for the first time in 1924, to attend the mathematicians' congress in Toronto. He set out on his next transoceanic journey seven years after that. He was 63 at the time. During five consecutive summers from 1931, he cruised all over the Atlantic Ocean, getting into its polar areas and finally sailed deep into the Indian Ocean. He converted memories from those voyages into interesting travelogues, among which the most known one is *The Novel of an Eel,* published in 1940.

Regarding his first stay at the Sargasso Sea (1932), a few years later he complained to Milanković that he was walking around like an idle tourist, and that he could have solved the interesting, centuries-old problem of the breeding process of an eel instead. This "centuries-old problem" Mihailo Petrović tried to solve during his next two missions heading toward the Sargasso Sea in 1938 and 1939. A fishing net, which was specially prepared for this second mission, unhooked during the lifting and stayed at the bottom of the Sargasso Sea for good. Upon his return, Petrović was making design for the improved fishing net for his next trip that was planned for 1940, but the trip never happened because World War II broke out. The secret of the eels has not been fully resolved up to the present day.

In his essay "The Secret of Fish" from 1985[159] Živojin Pavlović wrote: "Thus he named his book on eel a *novel* (although it was written as a mixture of a travelogue and a scientific discussion), because the life of eels contains two characteristics of this literary genre: *adventure* and *mystery*. But, unlike fictional stories (the plot of which, in the final instance, must also have an outcome), this story, at least during Mika's life, in spite of his three attempts to resolve it by himself, remains deprived of its outcome and the resolution of a mystery."

IV

The fish is a symbol of Christianity, from its very beginning.[160] However, the carved carp that decorates the entrance door of Petrović's house in Kosančićev venac and testifies of his love for fishing, hardly symbolizes the house owner's faith.

In the voluminous written legacy of Mihailo Petrović, there are almost no clues that would shed light on his views concerning Christianity. His worldview must have been formed under the influence of his father Nikodim, who was a priest and professor at seminary, and, after his death, of his grandfather Novica Lazarević, archpriest at the Cathedral church. In such an environment, Mihailo Petrović adopted patriarchal principles and respect for traditional values, above all in the customary sense. Mihailo Petrović was dealing with scientific facts, tangible phenomena, and therefore religious topics were not the subject of his numerous discussions, because he obviously deemed them scientifically irrelevant. Since he thought very highly of Napoleon,[161] it is appropriate to mention here an anecdote familiar to mathematicians. The story has it that Napoleon, after Laplace gave him a copy of his new book titled *Celestial Mechanics*, asked the author how it was possible that in the book on the structure of the universe there was no mention of its creator, God. Laplace replied: "Your Highness, I did not need such a hypothesis". The same goes for the entire work of Mihailo Petrović, in which there is no place for contemplating and speculating about extrasensory phenomena, because he was focused on visible phenomena and the rationalistic explanation of this world's phenomena.

Still, as mentioned before, Mihailo Petrović cherished customs, so, for instance, he set St. Apostle Philemon's Day, December 5, as patron saint's day for his musical troupe "Suz". The troupe's patron saint's day was celebrated in line with all the customary rules, and there were usually more than 500 guests. Since this was the period of Nativity fast, it was necessary to catch sufficient quantities of fish on time, so 10 to 15 days prior to the patron saint's day Mika Alas went fishing everyday with his fellow fishermen. J. Mihailović testifies:

> "We were preparing for St. Philemon's Day, 5 December 1913. We had been fishing for several days now. When we went fishing on December 3, we had no luck the entire morning. We hardly provided enough for lunch that day.
>
> Right after noon, we continued fishing. When we were already on the water, before throwing the fishnet, Mika prayed to Saint Philemon, and we all prayed together with him. He uttered these words: – Help us, our patron saint, Saint Philemon, we are doing this for you, not for ourselves!
>
> Then he threw the fishnet into the river, and when we pulled it, a catfish was rolling inside it, almost two meters long. When we lifted it, it weighed 124 kilograms. Beside it, there was a lot of other fish. That afternoon we had a very good harvest."[162]

When, apart from this and some similar examples, he was supposed to state his view on the experience of perceptions outside the worldly reality, Petrović was doing it in the spirit of his worldview and positions of from his phenomenology's point of view. For instance, he concludes his book *Phenomenological Mapping* from 1936 with Chapter XI "Mythology of Facts" in which he wrote:

On the Danube shore with a foreign delegation (Belgrade, 1898: Professor Petrović is wearing a cap)
(SASA Archive, 14197/II-8)

> "The oldest, more voluminous human perceptions of the outer world, myths of various peoples in the childhood stage of awareness and cognition development, are nothing but a special form of mapping. The basis for mythical mapping lies in the mystical anticipation of a primitive mind implying that, beyond the visible, concrete world, there is another one, inaccessible to it, filled with entities whose play behind the scene determines the events that take place in the visible world."

This quote brings us back to the idea of fishing in psychoanalysis. In a dictionary of symbols[163], fishing is the extraction of elements of the unconscious, but not by directed and rational research, but rather by allowing them to emerge spontaneously. Here, the unconscious is compared to a water expanse, a river, a lake or a sea in which many riches are hidden, and which will be lifted to the surface by anamnesis and analysis, just like a fisherman lifts fish in his fishnet.

Based on this symbolism, we introduce the analogy based on which the water expanse of the unconscious is equal to the space of ignorance or hidden knowledge. Thus fisherman becomes the symbol of knowledge bearer or mediator, and fishnet becomes a knowledge medium or a carrier. The fish caught in this mystical mapping become the owners of knowledge that transforms them into knowledge bearers – fishermen.

Mihailo Petrović's memory from fishing (SASA Archive, 14197/II-24-1)

By using this analogy, fishing which Mihailo Petrović took up literally, i.e. "in the visible world" is expanded by fishing where the catch was of different nature. In the mathematical net of professor Petrović, the first such net in Serbia, his doctoral students were caught. 11 doctoral theses in total were defended under his mentorship from 1912 to 1938. Petrović's students then taught their students to fish in mathematical sense, so the tree of mathematical fishermen grew wider, and most of our professional mathematicians can be found in it, whereas Mihailo Petrović is in the root of this tree. About eight hundred mathematicians, out of which more than five hundred are Serbian, are connected by mentorship of their doctoral dissertations, as successors of the "arch-mentor" professor Petrović.[164]

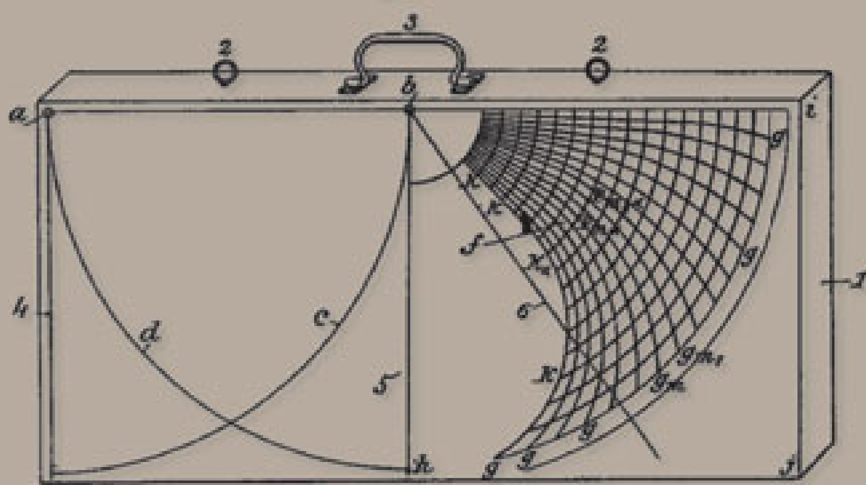
Professor Vladeta Janković writes that, in a sense, it could be said that the Christian church was born at the moment when Jesus invited four fishermen to follow him.[165] By analogy among disparate phenomena, we could say that mathematical sciences in Serbia were born when Mihailo Petrović returned from Paris, and owing to his enormous efforts invested in the field of education, scientific work, seminars, founding of the Academy's journal titled *Publications de l'Institut Mathématique*, and his mentorship of the first generation of professional mathematicians in Serbia.

Finally, near the end of his life filled with fishing, when setting out on long journeys, Mika Alas used to say: "I am already in my late years, what I have done, I have done. If I return from this journey alive and manage to do something else, it is pure benefit. And if I die, it does not matter. I will be buried there where the death finds me. It would be best if I died on a ship and they throw me into the sea so that fish can eat me and have their revenge because I was catching and eating them a lot."[166]

Something similar was said by Serbian Patriarch Pavle, when he was flying over the sea and the airplane got into the zone of turbulence. Asked what he thought about the possible airplane crash, the Patriarch replied: "In relation to myself, I will see it as an act of justice, because during my lifetime I ate so much fish that it would be strange if they would not eat me now."[167]

These quotes bring us back to Mihailo Petrović's thought which says "it is not rare in science that a certain phenomenon, with its appearance, more or less has point of semblance to other phenomenon, totally different from it and with no real relations with it, but similar to it in a way."[168]

Fig. 1.



MIHAILO PETROVIĆ: INVENTIONS AND PATENTS

THE HYDROINTEGRATOR OF MIHAILO PETROVIĆ ALAS

Radomir S. STANKOVIĆ
Mathematical Institute of SASA

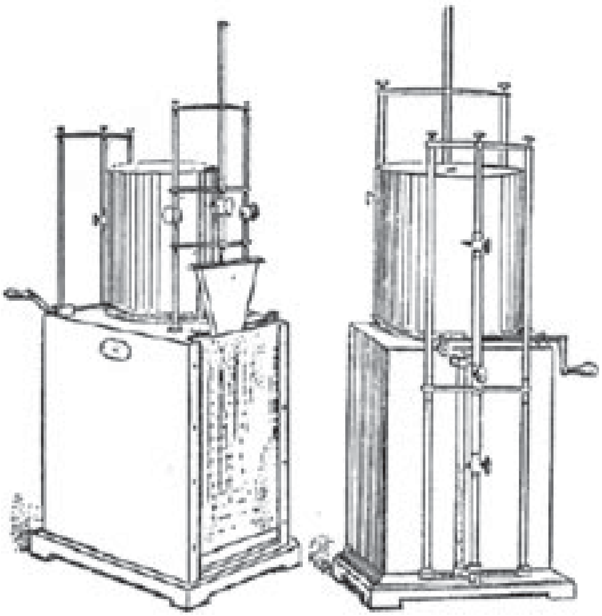
Building computing machines and other technical devices for the acceleration and automatization of calculating procedures has been a subject of scientific interest for centuries, and even millenniums, to which, for instance, *Antikythera mechanism*[169] bears testimony.

Taking into account the writings of Mihailo Petrović Alas on the equipment used for fishing and the detailed guidelines for its usage, it is hardly surprising that this eclectic research scientist turned his practical character towards building devices for solving specific mathematical problems. Having in mind the topic of his doctoral dissertation (Petrović 1894), it is completely understandable that he had chosen differential equations as the subject of his research within that field.

As his nickname – Alas ("a river fisherman"), clearly indicates, Mihailo Petrović spent a lot of time fishing on the Danube and Sava rivers, and it is only natural that he had chosen water as the main element of his device for solving differential equations, which he simply dubbed the "hydrointegrator".

In this paper we are going to describe the basic parts of the hydrointegrator, as well as its working principles, give comments on one specific realization, as well as on the potential improvements suggested by Petrović, and with a view to all that was previously said, name the contributions of Mihailo Petrović Alas to building analogue computers.

HYDROINTEGRATOR



Petrović's sketch of the hydrointegrator
(Digital Legacy of Mihailo Petrović)

The originality and effectiveness of a solution is probably more important than emphasizing the primacy of an invention, but it should still be noted that the hydrointegrator of Mihailo Petrović Alas is the world's first analogue computing machine for integrating differential equations based on hidraylic principles. Up until then the devices based on the principle of the law of hydraulics had been suggested only for solving algebraic equations, such as Veltmann's machine of 1889.

The working principle of the hydrointegrator is based on noticing the equivalence of laws of changes to water levels when a body is immersed into it and the procedure for integrating certain types of differential equations, including Riccati's equation[170], in whose solving Petrović was particularly interested. With a view to that, the hydrointegrator can be considered to be what Petrović calls the materialization of mathematics as the ultimate goal of mathematical phenomenology, seen as drawing analogies between certain mathematical problems and physical, often disparate, phenomena, with a view to resolving the former, as noted in Petrović's publications on the said topic (Петровић 1911; Petrović 1921).

The hydrointegrator represents the result of Petrović's considerations and research of many years, to which testify his notes from the lectures on mechanics given by professor König at *Collège de France* in Paris, which Petrović attended in 1892. These notes contain elements of the input–output system that Petrović called integraph. It represents the devices for calculating the surface of irregular geometrical figures circumscribed by closed contours. The corresponding page from the notes of Mihailo Petrović is reproduced in Trifunović (see Трифуновић 1968a: 459, Picture 112).

Petrović later applied these considerations and other original solutions to a practical realization of the hydrointegrator. Let us mention that Petrović's work on that problem had been announced as early as 1896 by his senior colleague Ljubomir Klerić, a professor of mechanics at the Faculty of Philosophy of Belgrade Grand School, placing a special emphasis on the originality of his approach.

PETROVIĆ'S PUBLICATIONS ON THE SUBJECT OF THE HYDROINTEGRATOR

In accordance with his principles of properly documenting proposed solutions, Petrović published four papers containing a detailed description of the working principle, building process and practical applications of the hydrointegrator. It was first presented at the French Academy of Sciences, by professor Paul Émile Appell, on 17 May, 1897. It is important to note that, due to its significance, this paper was reprinted in *Journal de Physique* (Petrović 1897: 476–479), departing from its usual practice.

Petrović published this paper in Serbian, with certain additions, in 1898, in *Srpski tehnički list (Serbian Technical Journal)*, a bulletin of the Association of Serbian Engineers and Architects, as noted in the subtitle of this magazine (Петровић 1898). The front page of this magazine is shown in Figure 1. The following two publications dedicated to the hydrointegrator were published by Petrović in French in the *American Journal of Mathematics,* in 1898 and 1899, respectively (Petrović 1899a; 1899b).

СРПСКИ
ТЕХНИЧКИ ЛИСТ
ОРГАН УДРУЖЕЊА СРПСКИХ ИНЖЕЊЕРА И АРХИТЕКТА

УПРАВНИ ОДБОР УДРУЖЕЊА

ИЗ НАУКЕ И ПРАКСЕ

Figure 1. The cover page of *The Serbian Technical Journal* containing the paper *On hydraulic integration* by Mihailo Petrović Alas and a schematic diagram of the hydrointegrator's working principle

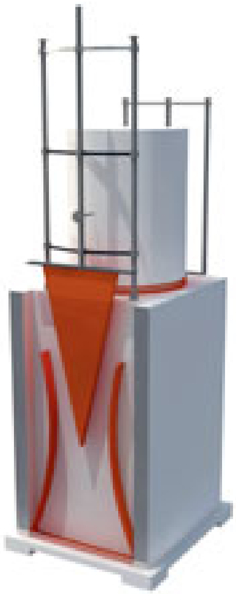


Figure 2 (left): 3D model of the hydrointegrator, author of 3D model of the hydrointegrator Petar Vranić (Mathematical Institute of SASA)

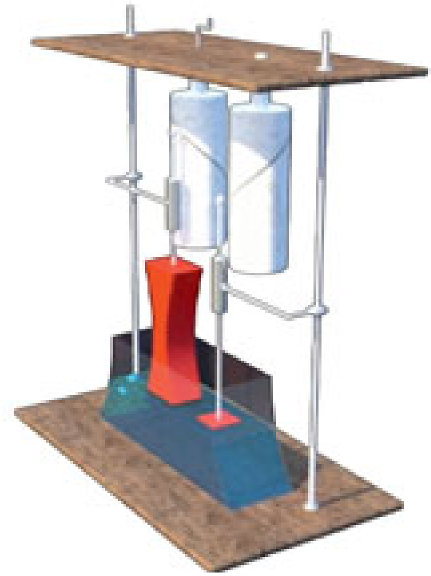


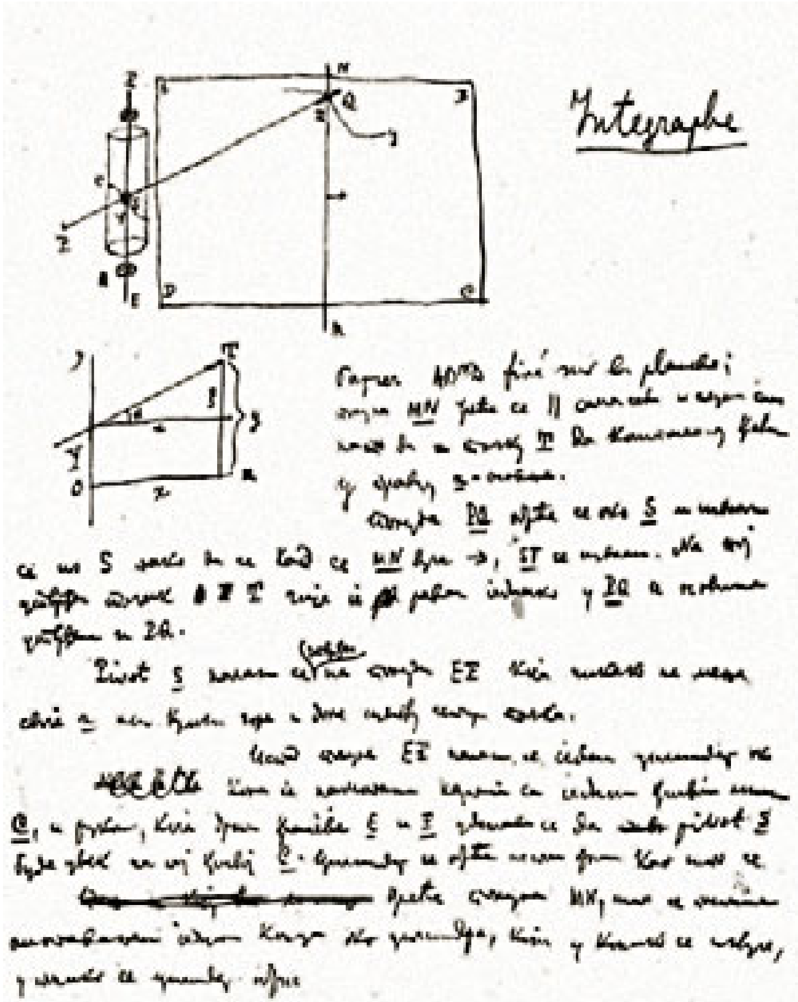
Figure 3. 3D model of basic elements of a hydrointegrator with a rotating input roller, author of 3D model Petar Vranić (Mathematical Institute of SASA)

ELEMENTS OF THE HYDROINTEGRATOR'S CONSTRUCTION

The hydrointegrator, viewed as a computing device, consists of the following functional units:

1. *Input unit*, which is in essence a body of a certain shape with or without a rotating cylinder. The function of the body shape represents the input data for the hydrointegrator. In the case of there being added a rotating cylinder, the additional input data represents the function determining the way in which the bodies are immersed in a water-filled tank. This function is realized by a groove in the rotating cylinder which corresponds to the graph of the selected function. The shaft of the body being immersed in the water slides along the groove.
2. *Arithmetical unit*, which comprises a tank of a certain shape filled with water, a body of the precisely defined shape, belonging to the input unit, which is immersed in the tank, and a floating buoy, which is used for determining the water level in the tank.
3. *Output unit*, which is in the shape of a rotating cylinder, with a pencil for tracing solutions on the paper fastened onto the surface of the cylinder.

As previously mentioned, using the rotating cylinder for tracing solutions, which can be said to represent an original contribution to building analogue computers, had been considered by Petrović as early as during his stay in Paris in 1892. He presented this idea as a technical solution in the first paper devoted to the hydrointegrator of 1897 (Petrović 1897).



Sketch of a rotating cylinder as an element of the hydrointegrator (Library of SASA, A40/120)

In the article published in *Srpski tehnički list* (Петровић 1898), as well as in the following two papers on this subject (Petrović 1899a, 1899b), Mihailo Petrović examines several versions of hydrointegrators and proposes certain improvements.

Figure 2 shows a 3D model of the basic version of the hydrointegrator, and Figure 3 an improved version of the device with a rotating input cylinder. The improved version has never been practically realized.

Apart from adding the rotating cylinder as an element of the input unit, in (Петровић 1898) Petrović proposes that at the bottom of the tank in which the body is immersed an orifice be made through which water would continually flow out. The diameter of the orifice would be defined so as to ensure that the contraction of the current of water flowing out be complete. Thereby an additional parameter in defining the functionality of the hydrointegrator would be set up, using the quantity of the displaced water in observing changes to the water level in the tank, as a result of immersing in it a body of a certain shape from the input unit. This would enable integrating different types of differential equations, depending on defining the said parameters related to the input and arithmetical unit of the hydrointegrator.

In (Петровић 1899б) Petrović proposes the application of a clock mechanism for providing constant angular velocity and synchronization of the turning of the input and output rotating cylinders joined together by a kinematic connection. The clock mechanism would ensure that the floating buoy on the input cylinder, which is used to determine the manner of immersing the body into the water tank, always crosses the same distance in a set period of time, which improves the precision of the final result.

Owing to the kinematic connection between the input and output rotating cylinders, the hydrointegrator of Mihailo Petrović can be considered as a combination of hydraulic and kinematic analogue computing machine that is used for integrating one specific differential equation, based on the selected parameters. Changing the parameters allows for integrating a broader class of differential equations.

Bearing in mind some aspects of a practical nature, more specifically, the simplicity of replacing the tank and the immersed body, Petrović proposes the construction of a hydrointegrator in which the tank and the shaft of the body are placed sideways and turned facing outwards, so that they could easily be replaced by a tank and body of another appropriately chosen shape. Thereby the hydrointegrator can easily be reprogrammed for integrating another type of differential equations. In that sense, the pair (*body*, *tank*) can be taken to be a subprogramme in terms of modern computing terminology. The parameters used for making a selection of an equation that the hydrointegrator is to solve are the following:

1. The shape of an immersed body;
2. The shape of a tank;
3. The existence of an orifice of a pre-selected and appropriate diameter at the bottom of the tank;
4. The manner of immersing the body in the tank, that is, the choice of function according to whose graph the shaft of the immersed body is moving; and
5. The diameter of the rotating cylinders in the input and output unit.

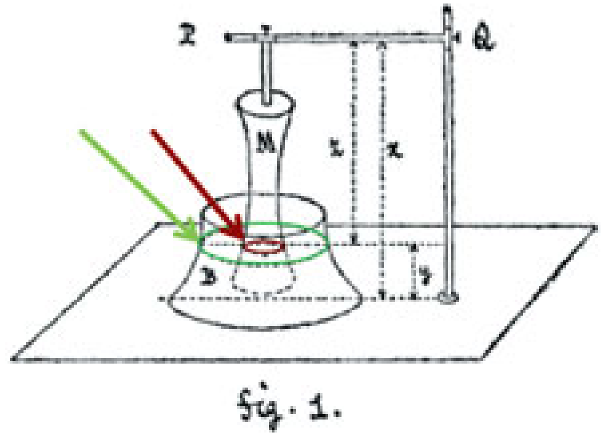


Figure 4. An illustration of hydrointegrator's working principle from the first page of *The Serbian Technical Journal*, January and February 1898, Notebook 1 and 2, figure 1.

WORKING PRINCIPLES OF THE HYDROINTEGRATOR

The basis for building the hydrointegrator represents the observation within the field of mathematical phenomenology that by an adequate choice of the shape of the body *M* and the tank *B* in which the body is immersed, they can be so adjusted that the law of changes to the water level in a tank is correlated with the procedure for integrating a previously selected differential equation.

Let us label the surfaces of the horizontal cross section of the tank *B* and body *M* as $\phi(y)$ and $F(z)$ on a plane at a distance *x* from the bottom of the tank, see Figure 4. Due to the immersing of the body *M* into the tank *B*, the distance of the body from the bottom of the tank changes from *x* to $x - dx$, while the level of fluid is raised from *y* to $y+dy$, so that the volume of the fluid that shifted from *y* to $y+dy$ equals $(\phi(y)-F(z))dy$. The volume of the displaced fluid is equivalent to the volume of the fluid taken up by the body *M* submerged at the value of *dz*, that is, $F(z)dz$, which yields

$$(\phi(y)-F(z))dy = F(z)dz.$$

As the relation $z = x - y$, is satisfied, it results in the following

$$\frac{dy}{dx} = \frac{F(x-y)}{\phi(y)} \,. \tag{1}$$

The aforementioned improvement of the construction presented in (Petrović 1897) requires adding an orifice at the bottom of the tank, through which it is continually drained, and thus in that case the change to the water level in the tank is equivalent to the volume of the displaced water upon submerging the body *M* and the volume of water flowing out of the orifice at the bottom of the tank during the period of time *dt*.

As a man of practical bent, Petrović, taking into account the materials and the technology at his disposal at the time, proposed that the immersed body and tank have two flat sides parallel to each other, as well as a flat bottom part, whereas the other two sides should be arbitrarily but adequately selected curved surfaces as in Figure 5, where the breath of the tank $\phi(y)$ and the body $\theta(z)$ at the height y and z functions are freely chosen, as are the distance α and β of the parallel sides of the tank B and the body M. In that case $\Phi(y)=\alpha\phi(y)$ and $F(z) = \beta\theta(z)$, so that the equation (1) represents

$$\alpha\phi(y)\frac{dy}{dx} = \beta\Theta(x-y).$$

If the shapes of the body and tank are selected in that order, as in

$$\phi(y) = \frac{1}{\alpha f(y)} \quad \text{and} \quad \theta(z) = \frac{1}{\beta}\psi(z),$$

the hydrointegrator integrates the equation of the following form

$$\frac{dy}{dx} = f(y)\psi(x-y).$$

In the improved model of the hydrointegrator proposed by Petrović, the manner of immersing the body M would be determined by a grooved rotating input cylinder, as shown in Figure 6. Provided that the diameters of the input and output cylinders D and E are mutually different, a more general equation follows

$$\frac{dy}{dx} = f(y)\psi(ax-y).$$

If the body M is in the shape of a prism, then it holds that $\theta(z) = const. = \beta'$, so that the hydrointegrator works as an integraph for calculating the surface of the cross section of the tank in the plane of a picture

$$x = \frac{\alpha}{\beta\beta'}\int \phi(y)dy.$$

As shown in (Петровић 1898), the hydrointegrator works as an integraph even in the case when the body B is in the shape of a prysm.

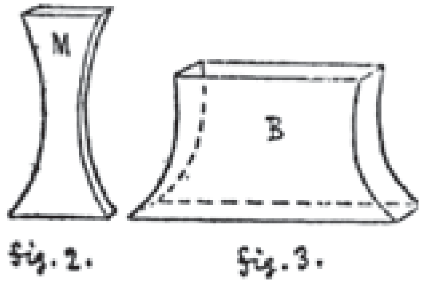


Figure 5. The realization of a submersible body and a tank with parallel sides according to Petrović's initial suggestion, figure taken from [Петровић 1898]

In case the input rotating cylinder D is grooved with the function $\eta=f(\zeta)$ according to which a body is immersed, then $x=\eta=f(\zeta)$ the level of the water y as a function of ζ brings the solution of the equation as follows

$$\alpha\phi(y)\frac{dy}{dx} = \beta\theta(f(\zeta)-y)f'(\zeta).$$

The solution is traced with a pencil as a graph on the output cylinder E.

Petrović asserts in (Петровић 1898) that the same device could integrate different equations by changing the shape of the tank and the immersed body and by adding several different tanks and integral cylinders, which resulted in the construction as the

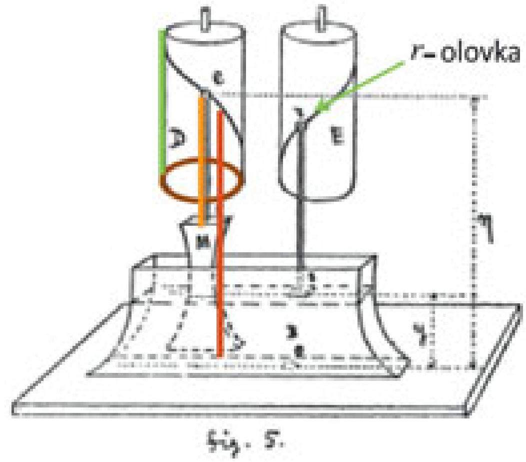


Figure 6. The principle of work of a perfected version of hydrointegrator, by adding the rotating input cylinder, figure taken from [Петровић 1898]

one shown in Figure 2, with the aim of enabling a simple replacement of a tank and body as the basic elements of the arithmetical unit of the device, placed on the outside in order to provide easy access.

As Petrović showed in (Петровић 1898), by selecting the body and tank of different shapes, as well as setting up other parameters, such as for example, the manner of immersing the body, equation (1) becomes a general model that incorporates five specific differential equations, including Riccati's equations

$$\frac{dy}{dx} = X(t) - \lambda^2 y^2$$

in whose methods of solving Petrović took a special interest, as well as the equation

$$\frac{dy}{dx} + F(y) = F(y)\psi(x),$$

$$\Phi(y)\frac{dy}{dx} + \lambda\sqrt{y} - af'(x) = 0,$$

$$\Phi(y)\frac{dy}{dx} = k(f(x) - y)f'(x),$$

$$\frac{dy}{dx} = f(y)\psi(ax + y).$$

Moreover, for another set of parameters and entry data, the hydrointegrator could also solve the equations such as the following

$$P(x,y)dx + Q(x,y)dy = 0. \tag{2}$$

Class (2) contains equations such as

$$\psi(x-y+\lambda)dx+(\varphi(y)-\psi(x-y+\lambda))dy=0,$$

$$f(y)dy+\psi(z)dz=0.$$

A detailed account of the aforementioned assertions related to the types of equations which could be solved by the hydrointegrator through selecting the manner of immersing, the shape of the body and tank and adding an orifice at the bottom of the tank, are presented by Petrović in (Петровић 1898; Petrović 1899a; 1899b).

At the time when Petrović was working on the hydrointegrator, it was the first machine for integrating differential equations that ran on the principle of hydraulics. A significant advantage of this compared to other devices of similar kind lied in the possibility of solving a greater number of equations by recombining its constructive elements. One of the existing integrators at the time, the Jacob's integrator, is based on some other principle, and could only solve Riccati's equation of the first order.

In (Петровић 1898) Petrović carried out a comparison of the proposed hydrointegrator with the corresponding devices with similar application, although based on different principles, presented in the catalogue of computing machines of 1892–1983 (von Dyck 1892). Figure 7 shows an extract from (Петровић 1898) in which Mihailo Petrović summarizes the advantages of his hydrointegrator.

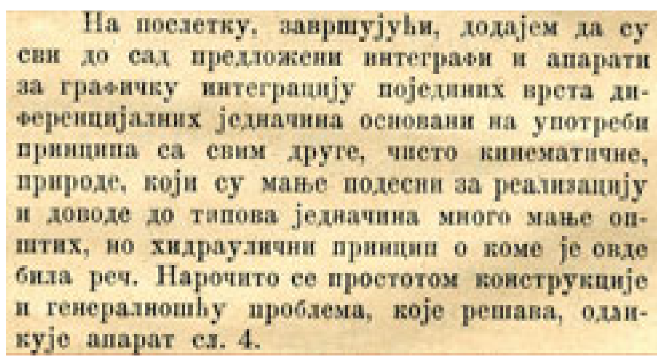

На последку, завршујући, додајем да су сви до сад предложени интеграфи и апарати за графичку интеграцију појединих врста диференцијалних једначина основани на употреби принципа са свим друге, чисто кинематичне, природе, који су мање подесни за реализацију и доводе до типова једначина много мање општих, но хидраулични принцип о коме је овде била реч. Нарочито се простотом конструкције и генералношћу проблема, које решава, одликује апарат сл. 4.

Figure 7. A segment of text from (Петровић 1898) which refers to the comparison of the hydrointegrator with similar devices from that era

SCIENTIFIC PROMOTION OF THE HYDROINTEGRATOR

At the World Exposition of 1900 in Paris, France, at the Serbian Pavilion designed by the architect Milan Kapetanović, professor of geometry at the Technical Faculty of the Belgrade Grand School, Serbia presented the hydrointegrator of Mihailo Petrović among other representative exhibits, as an example of its scientific achievements. A prototype of the hydrointegrator was constructed for that occasion, for which a French constructor of precise mechanisms had been hired. There are no records of the name of that constructor, but he is mentioned in the letter of Mihailo Petrović addressed to the Minister of National Economy, in which he requests financial support for the realization of this device (Трифуновић 1968a). In that letter Petrović explicitly states that the working principle of the device was published in *Comptes rendus de l'Académie des Sciences de Paris*. At the said World Exposition, the hydrointegrator was awarded with a gold medal.

It is significant to note that at the time of the World Exposition, from 6 to 12 August, 1900, the International Congress of Mathematicians was also held in Paris, with Mihailo Petrović participating for the first time at a mathematical congress. As the prototype of the hydrointegrator had been made up to full functionality, it is reasonable to assume that the specialist public was familiar with this unique machine for integrating differential equations. As far as we know, it was the only model of the hydrointegrator that had been physically realized. The reconstruction of the hydrointegrator was carried out by Professor Dragan Trifunović in 1980, with the assistance of the architect Gradimir Bosnić. This reconstruction is kept at the Mathematics Department of the Faculty of Forestry in Belgrade.

The second international acknowledgement for the development of the hydrointegrator that Mihailo Petrović received was by the Mathematical Society of London, which conferred on him an honorary diploma in 1907 (Трифуновић 1968a).

CONTRIBUTIONS OF MIHAILO PETROVIĆ TO BUILDING ANALOGUE COMPUTING MACHINES

It has been widely accepted that the hydrointegrator of Mihailo Petrović is the first analogue computing machine based on the hydraulic principle, with particular emphasis on the fact that the same device had the capacity to integrate several differential equations (de Morin 1913; Митриновић 1955, 1958, 1960; Price 1900; Трифуновић 1968а; Willers 1949). Furthermore, the work on building the hydrointegrator is considered to be one of the most cited mathematical results of Mihailo Petrović.

The original contributions of Mihailo Petrović can be summarized as follows:

1. The application of hydraulic principle for solving differential equations;
2. The application of calculating elements in the form of immersed bodies;
3. The combination of the hydraulic principle and kinematic connection between the input and output rotating cylinders; and
4. Considerations related to the analogies between analytical facts for curved line integrals and geometrical facts from the theory of minimal surfaces with capillary phenomena (Петровић 1911). The analysis of these analogies would serve for building the hydrointegrator with several chambers in the arithmetical unit equipped with capillary tubes.

It should be noted that such a solution as the one with capillary tubes had been applied in the hydrointegrator of 1936 for solving Fourier's partial differential equation. This hydrointegrator designed by Lukyanov works on the principle of analogy between heat conduction and the model of fluid flow in capillary tubes (Lukyanov 1937, 1939).

More details about Petrović's work on the hydrointegrator can be found in (Stanković 2004; Трифуновић 1968а, 1968б, 1982).

REFERENCES

(de Morin 1913) de Morin, H., "Les appareils d'intégration", Paris: Gauthier–Villars, 1913, 194–197.

(Lukyanov 1937) Lukyanov, V. S., *Technical Computing on Hydraulic Devices*, Moscow, 1937.

(Lukyanov 1939) Lukyanov, V. S., "Hydraulic Devices for Technical Computations", accessed at: *AN SSSR, Otd. Tech. nauk*, Vol. 2, No. 58, Moscow, 1939, 53–67 (in Russian).

(Митриновић 1955) Митриновић, Д. С., „Белешка о делатности Михаила Петровића у области диференцијалних једначина", *Весник,* VII/1–2, 1955, 125–127.

(Митриновић 1958) Митриновић, Д. С., „Белешка о Михаилу Петровићу", *Математичка библиотека*, Београд, 1958, бр. 7, 5–8.

(Митриновић 1960) Митриновић, Д. С., „Прилози за биографију Михаила Петровића", *Весник друштва математичара и физичара Народне републике Србије*, XII/1–2, 1960, 143–175.

(Petrović 1894) Petrović, M., *Sur les zéros et les infinis des integrales des équations différentielles algébriques,* Thèses présentés à la Faculté des Sciences de Paris, No. 823, Paris: Gauthier-Villars, 1894, p. 109; 21,3 x 26,6 cm.

(Petrović 1897) Petrović, M., "Sur un procédé d'integration graphique des equations différentielles", *Comptes rendus Hebdomadaires des Séances de l'Académie des Sciences*, Paris, 1897, CXXIV/20, 1081–1084, presented in the French Academy of Sciences by professor P. Appell, on 17 May, 1897; reprinted in *Journal de Physique*, Paris, 1897, 476–479.

(Петровић 1898) Петровић, М., „Хидраулична интеграција", *Срйски шехнички лисш*, Београд, 1898.

(Petrović 1899a) Petrović, M., "Sur l'intégration hydraulique des équations différentielles", *American Journal of Mathematics*, Baltimore, 1899, XX/4, 293–300.

(Petrović 1899b) Petrović, M., "Appareil à liquid pour l'intégration graphique de certains types d'équations différentielles", *American Journal of Mathematics*, Baltimore, 1899, XXII/1, 1–12.

(Петровић 1911) Петровић, М., *Елеменши машемашичке феноменолоīије*, Српска краљевска академија, Београд, 1911, XIII + 774 стране.

(Petrović 1921) Petrović, M., "Méchanismes communs aux phénomènes disparates", *Nouvelle Collection scientifique*, Directeur Émile Borel, Librairie Félix Alcan, Paris, 1921, 279 pages. (Shortened account in (Петровић 1911)).

(Price 1900) Price, A.W., "Petrovich's apparatus for integrating differential equations of the first order", *Philosophical Magazine*, Vol. 49, 1900, 487–490.

(Stanković 2004) Stanković, M., R. S. Stanković, "Hydrointegrator by Mihailo Petrović Alas: a contribution to the design of computing machines", in: *Proceedings EMCSR 2004*, Austrian Society for Cybernetics Studies, Vienna, 169/5, 2004, 211–215.

(Трифуновић 1968a) Трифуновић, Д. В., *Лешойис живоша и рада Михаила Пешровића*, (*Chronique de la vie et les œuvres de Michel Petrovitch*), l'Académie Serbe des Sciences et des Arts, Classe des sciences mathématiques et naturelles, 1969, VIII, présenté à la I[re] séance de la Classe des Sciences mathématiques et naturelles, le 16 février 1968, par MM. Radivoje Kašanin et Vojislav V. Mišković.

(Трифуновић 1968б) Трифуновић, Д., (уредник), *Михаило Пешровић 1868–1943*, Републички одбор за прославу стогодишњице рођења Михаила Петровића, Београд, 1968.

(Трифуновић 1982) Трифуновић, Д., *Михаило Пешровић Алас*, Дечије новине, Горњи Милановац, 1982.

(von Dyck 1892) von Dyck, W., *Katalog mathematischer und mathematisch-physikalischer Modelle, Apparate und Instrumente*, Deutsche Mathematiker–Vereinigung, München: Wolf & Sohn, 1892–1893.

(Willers 1949) Willers, F. A., *Mathematische Instrumente*, München/Berlin: Verlag von R. Oldenburg, 1943, S. III + 305, Russian translation, *Машемашические инсшрументи,* Москва, 1949, 302 pages.

MECHANICS AND ENGINEERING IN MIHAILO PETROVIĆ'S WORK*

Katica R. (STEVANOVIĆ) HEDRIH
Mathematical Institute of SASA
University of Niš, Faculty of Mechanical Engineering

„Not only do true poetry and true science have common points, but they also share deep common characteristics. One such characteristic, so much so that it is sometimes difficult to differentiate science from poetry, is *discovering and utilizing similarities among disparate elements and facts*."

Mihailo Petrović, 1925.

It is evident that Mihailo Petrović, as the founder of the Serbian School of Mathematics and having been an inspiration to numerous Serbian mathematicians of his "first generation of doctoral students", was spiritually hugely influential man of his time. It is for this reason that Mihailo Petrović has primarily been celebrated as a mathematician, whereas the promotion of his numerous ideas and achievements in other various fields, which are equally important, has been neglected. It is only now, when we mark 150 years since the birth of Mihailo Petrović, that the SASA Gallery is hosting an exhibition offering a major retrospective

* A revised and supplemented version of the paper initially published in the catalogue *Mihailo Petrović Alas: the Founding Father of the Serbian School of Mathematics* (SASA, 2018)

of his achievements spanning a significant number of subject areas, which is made available to modern generations [11, 12].

Mechanics, as a fundamental science in mechanical engineering, and in other technical sciences, is a very complex science, but also a basis for application in many mathematical and technical sciences, as well as in many multidisciplinary researches. This points to the fact that one who wants to deal with mechanics and mechanical engineering needs to possess expertise spanning a significant number of fundamental subject areas, as well as to be able to link a multitude of multidisciplinary ideas and materialize them into realistic and useful engineering systems of practical interest.

Mihailo Petrović was a unique and brilliant Serbian mathematician, an inspirer and a scientist with broad scientific culture and possessing knowledge ranging through all the fields of theoretical and applied disciplines and capable of materializing his ideas into the constructions in the field of mechatronics and mechanical engineering.

Petrović's theory that was expounded in his books titled *Elements of Mathematical Phenomenology* [5, 6] and *Phenomenological Mapping* [3, 10], is interpreted by many as "the mathematical foundations of natural philosophy", reminiscent of Newton's work *Philosophiae Naturalis Principia Mathematica* (*Mathematical Principles of Natural Philosophy*), which was published in May 1687. This comparison is acceptable and highly significant for cognitive sciences studies. However, in my view, Petrović's theory rises in importance when it comes to the application and identification of the same non-linear dynamics models in physically totally disparate areas of sciences, through knowledge transfer from one field of natural and technical sciences to another, including humanities as well [1, 2, 13].

Figure 1. The depth measurer of the land artillery: “Télémètre à sextant”, patent no. 413.730

MECHATRONICS AND MECHANICAL ENGINEERING IN MIHAILO PETROVIĆ’S PATENTS

It is evident that for devising mechatronic devices, machine constructions and the invention of hydro integrator, besides mathematical knowledge, one undoubtedly has to apply knowledge of non-linear dynamics and fluid mechanics, which are fundamental sciences in mechanical engineering.

Mihailo Petrović often stressed the phrase – *materialization of differential equations*. Engineers love the phrase, because if an idea can contribute to science, it can make a difference in everyday life only if it is realized into a specific device, mechanism or machine. Mihailo Petrović was a theorist and a scientist, but also an experimenter and a constructor who knew how to apply his theoretical and mathematical knowledge to such a degree that through the desired dynamics model, it could be handed over to engineers for realization. Some examples of such practically oriented discussions and realized ideas are: *work*

RÉPUBLIQUE FRANÇAISE.

OFFICE NATIONAL DE LA PROPRIÉTÉ INDUSTRIELLE.

BREVET D'INVENTION.

XII. — Instruments de précision, électricité.

3. — Poids et mesures, instruments de mathématiques, compteurs et procédés d'essai.

N° 413.730

Télémètre à sextant.

Michel PETROVITCH et Miloche TERZITCH résidant en Serbie.

Demandé le 11 février 1910.

Délivré le 2 juin 1910. — Publié le 17 août 1910.

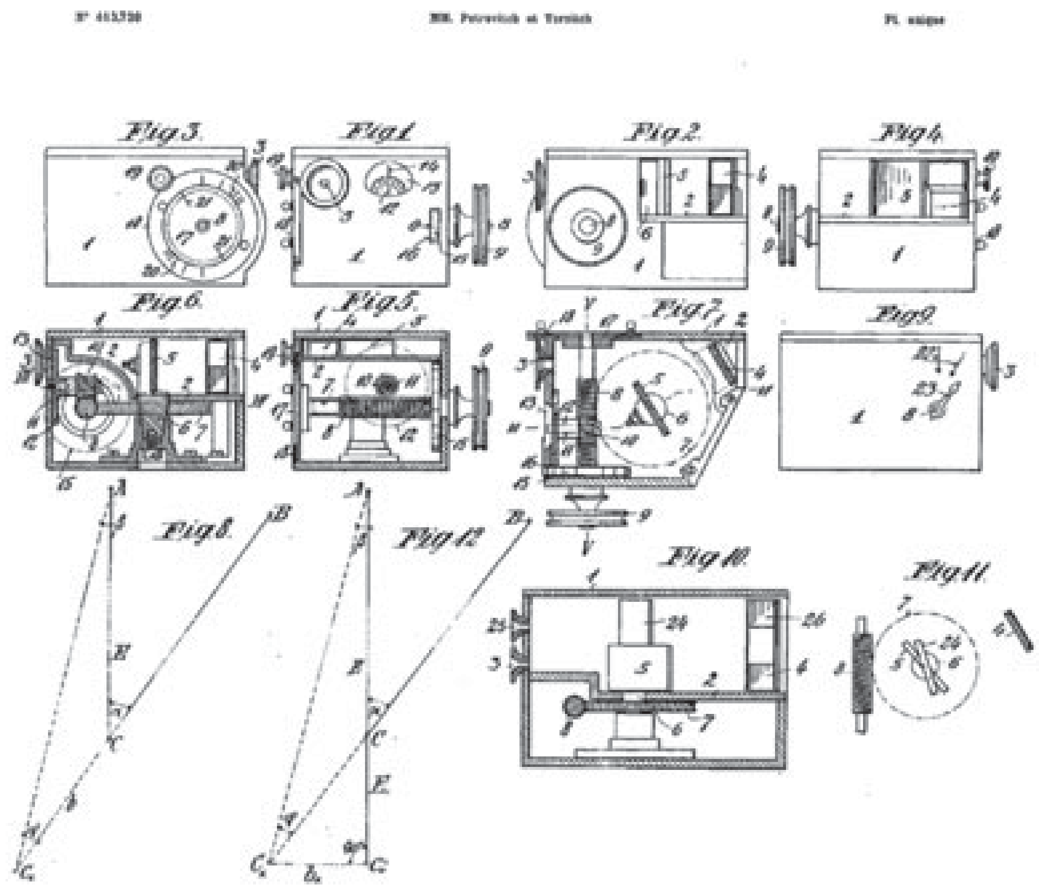


Figure 2. A sketch of the part of the mechanism of the depth measurer – named "Télémètre à sextant", patent no. 413.730

on mathematical modelling and analogue calculating machines, on *the hydraulic integrator – hydro integrator,* and on the *range indicator* made for the needs of the Military-Technical Institute in Kragujevac.

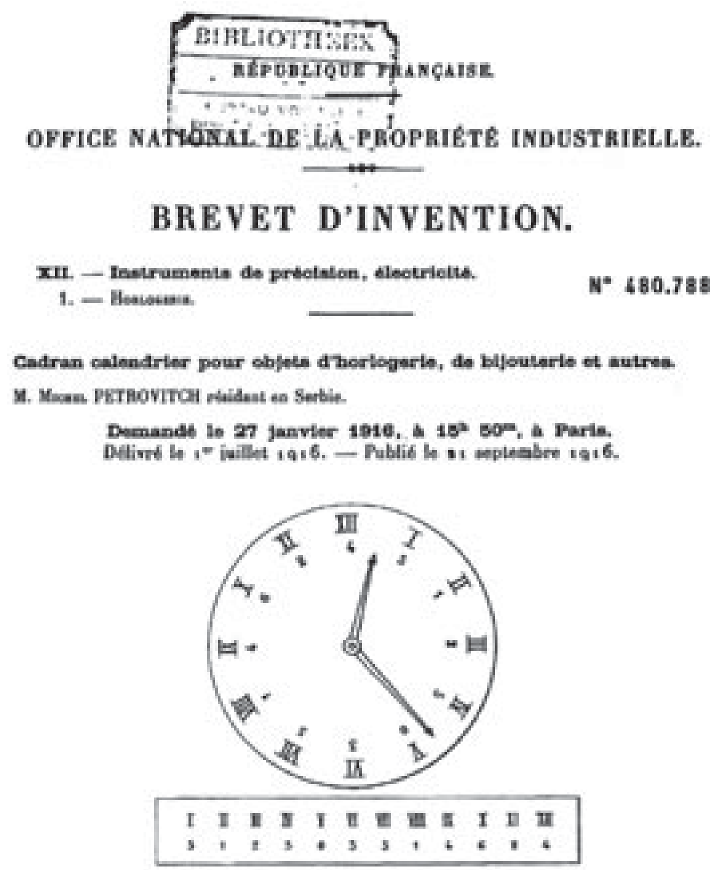
Here we will present several mechatronics inventions and machine constructions through the presentation of ten patents that had interesting and important applications. Each of these inventions and devices illustrate a high level of Petrović's creativity and his capability of transforming abstract ideas and sketches into very usable devices and inventions in the field of mechatronics and mechanical engineering.

From the bibliography of his works and archive documents of the Patent Offices in France and Great Britain, we can learn that Mihailo Petrović patented ten inventions. For nine of them he was granted a patent in France, and for one in Great Britain.

The first patent is *the range indicator*, construed for the needs of the Military-Technical Institute in Kragujevac. He made this device together with the general of the Serbian Army Milorad Terzić. The patent was bought and realized in Serbia and in Russia (Paris-1910; No. 413730.). Petrović submitted an application for this patent to the French Patent Office on February 11, 1910, and the patent with number 413.730 was granted to him on August 17, the same year (Figures 1 and 2).

Petrović's *Eternal calendar* was recorded in his bibliography as an original author's contribution, and made official under the patent number 480.788, granted to him on September 21, 1916 by the French Patent Office, based on the documents and applications submitted on January 27, 1916 (Figure 3).

His next invention is *the construction of the cogwheel transmitter* (Paris-1913; No. 463082), in the field of machine constructions and mechanical engineering. This patented machine construction, according to D. Trifunović, PhD, is a precursor of automatic gearboxes for cars and it enables a constant transmission of the number of revolutions per minute. Petrović applied for this patent together with Svetolik Popović, a machine


RÉPUBLIQUE FRANÇAISE

OFFICE NATIONAL DE LA PROPRIÉTÉ INDUSTRIELLE.

BREVET D'INVENTION.

XII. — Instruments de précision, électricité.
1. — Horlogerie.

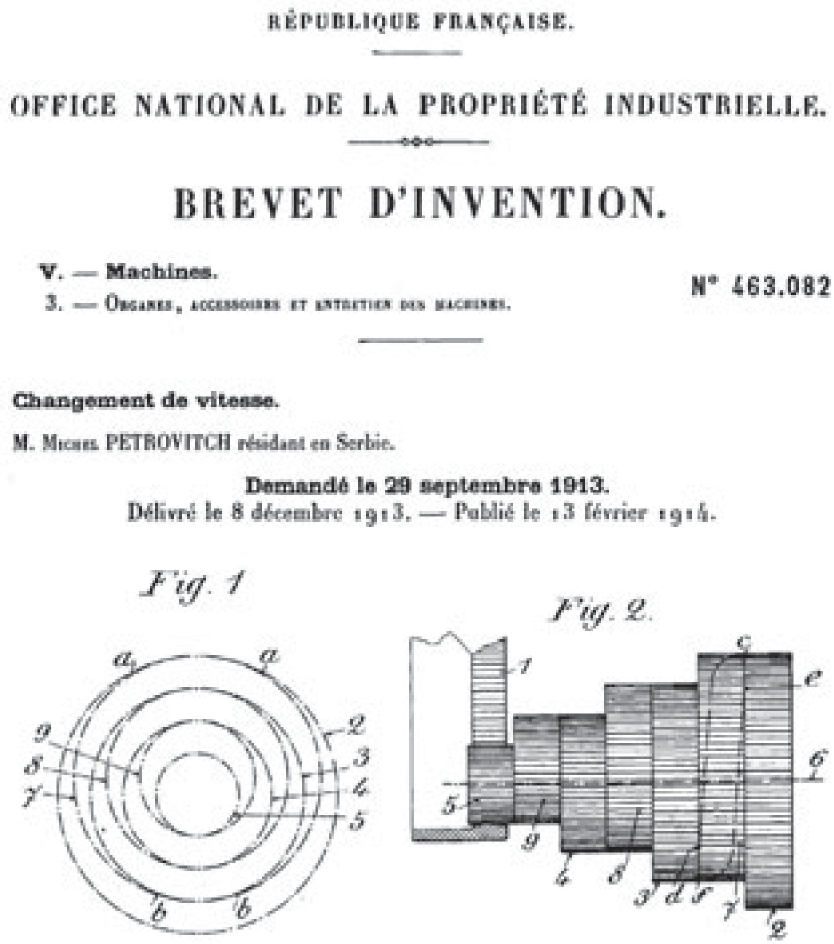
N° 480.788

Cadran calendrier pour objets d'horlogerie, de bijouterie et autres.

M. PETROVITCH résidant en Serbie.

Demandé le 27 janvier 1916, à 15h 50m, à Paris.
Délivré le 1er juillet 1916. — Publié le 21 septembre 1916.

Figure 3. Sketches of Petrović's Eternal calendar



RÉPUBLIQUE FRANÇAISE.

OFFICE NATIONAL DE LA PROPRIÉTÉ INDUSTRIELLE.

BREVET D'INVENTION.

V. — Machines.
3. — Organes, accessoires et entretien des machines.

N° 463.082

Changement de vitesse.

M. Michel PETROVITCH résidant en Serbie.

Demandé le 29 septembre 1913.
Délivré le 8 décembre 1913. — Publié le 13 février 1914.

Figure 4. A sketch of the *construction of the cogwheel transmitter* model (Paris 1913; no. 463082) patent no. 463.082, issued on 13 February 1914

nautical engineer from the Serbian Nautical Society. The application was submitted to the French Patent Office on September 29, 1913, and patent with number 463.082 was issued to him on February 13, the following year (Figure 4).

The invention named *multiple compression ration cogwheel transmitter with bending conical cogwheels* was submitted on August 31, 1912, and it was approved by the French Patent Office on January 17, 1913 with number 447.861. He applied for this patent together with Svetolik Popović, nicknamed Suljim, a machine-boat engineer from the Serbian Nautical Society. This patent model has similar structure as the patented model named ***the construction of the cogwheel transmitter*** (Paris-1913; No. 463082), with number 463.082, which was issued on February 13,

RÉPUBLIQUE FRANÇAISE.

OFFICE NATIONAL DE LA PROPRIÉTÉ INDUSTRIELLE.

BREVET D'INVENTION.

V. — Machines.

3. — Organes, accessoires et entretien des machines.

N° 447.861

Changement de vitesse avec pignons étagés raccordés par des engrenages en hélice conique.

MM. Svetozar POPOVITCH et Michel PETROVITCH résidant en Serbie.

Demandé le 31 août 1912.

Délivré le 9 novembre 1912. — Publié le 17 janvier 1913.

FIG. 1.

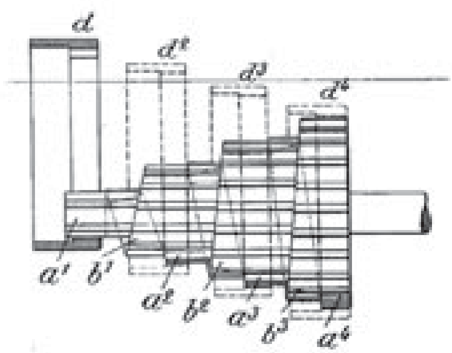


FIG. 2.

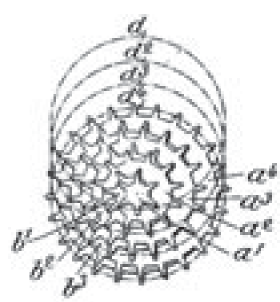


Figure 5. Discovery named *Multiple compression ration cogwheel transmitter with bending conical cogwheels* was submitted on 31 August 1912, and the patent was granted on 17 January 1913, with the no. 447.861, by the French Institute for Patents.

1914 based on application dated September 29, 1913, and represents new generation of helical gear for automobile transmissions (Figures 4 and 5).

The next invention by Petrović also belongs to this group of mechanical constructions in the form of cogwheel transmitters of the number of revolutions per minute. It was patented under the name *automobile gearbox* under the patent number 476.320, based on application dated October 17, 1914, and the patent was granted on June 27, the following year, in 1915 (Figure 6).

A device for quick launching and ejecting of missiles, intended for the use on the old type of cannons, in the air and water, as well as on land and at sea, was patented by the French

RÉPUBLIQUE FRANÇAISE.

OFFICE NATIONAL DE LA PROPRIÉTÉ INDUSTRIELLE.

BREVET D'INVENTION.

X. — Transport sur routes.
4. — Automobilisme.

N° 476.320

Changement de vitesse.

M. Michel PETROVITCH résidant en Serbie.

Demandé le 17 octobre 1914, à 14h 55m, à Paris.
Délivré le 4 mai 1915. — Publié le 27 juillet 1915.

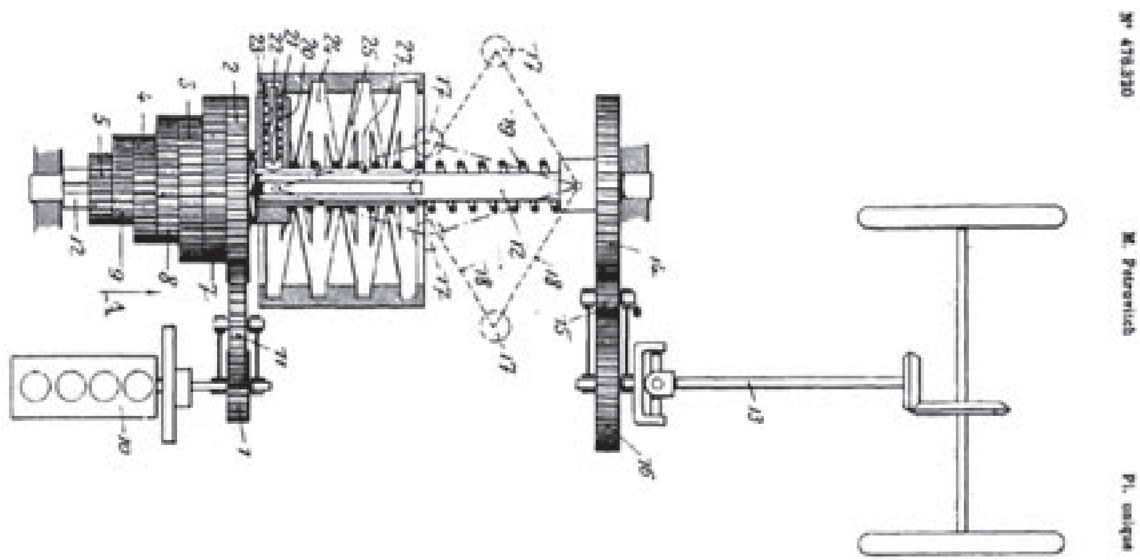
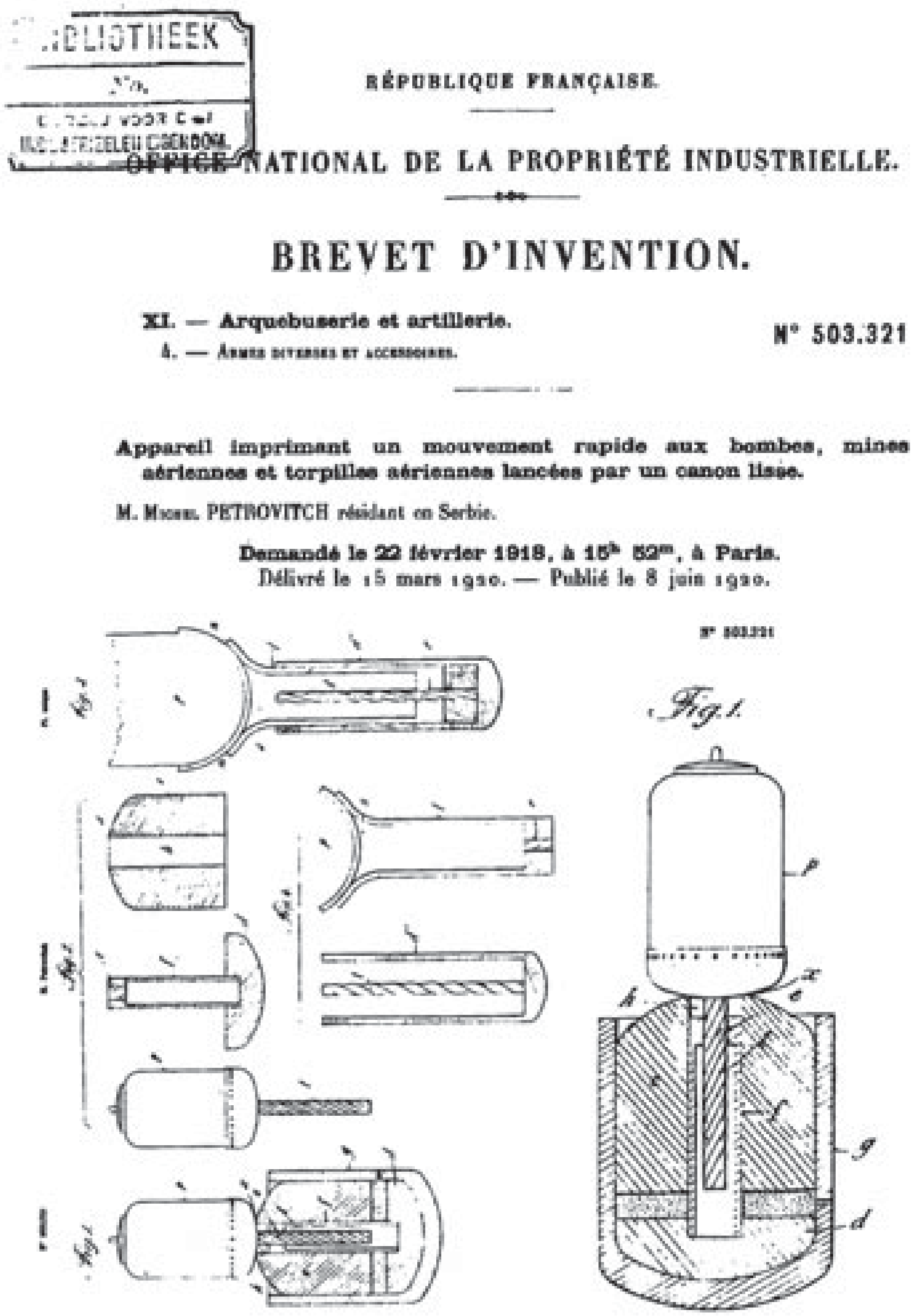


Figure 6. A sketch of an automobile gear, patent no. 476.320

patent number 503.321. Petrović submitted the application for this invention on February 22, 1918 to the French Patent Office, and the French patent under the number 503.321 was granted to him on June 8, 1920 (Figure 7). With this device, the initial axial-rotation of the missile fired from old-type smoothbore cannon was achieved. This initial axial-rotation of the missile resulted in the decrease of the resistance of the environment (air or water).



RÉPUBLIQUE FRANÇAISE.

OFFICE NATIONAL DE LA PROPRIÉTÉ INDUSTRIELLE.

BREVET D'INVENTION.

XI. — Arquebuserie et artillerie.
4. — Armes diverses et accessoires.

N° 503.321

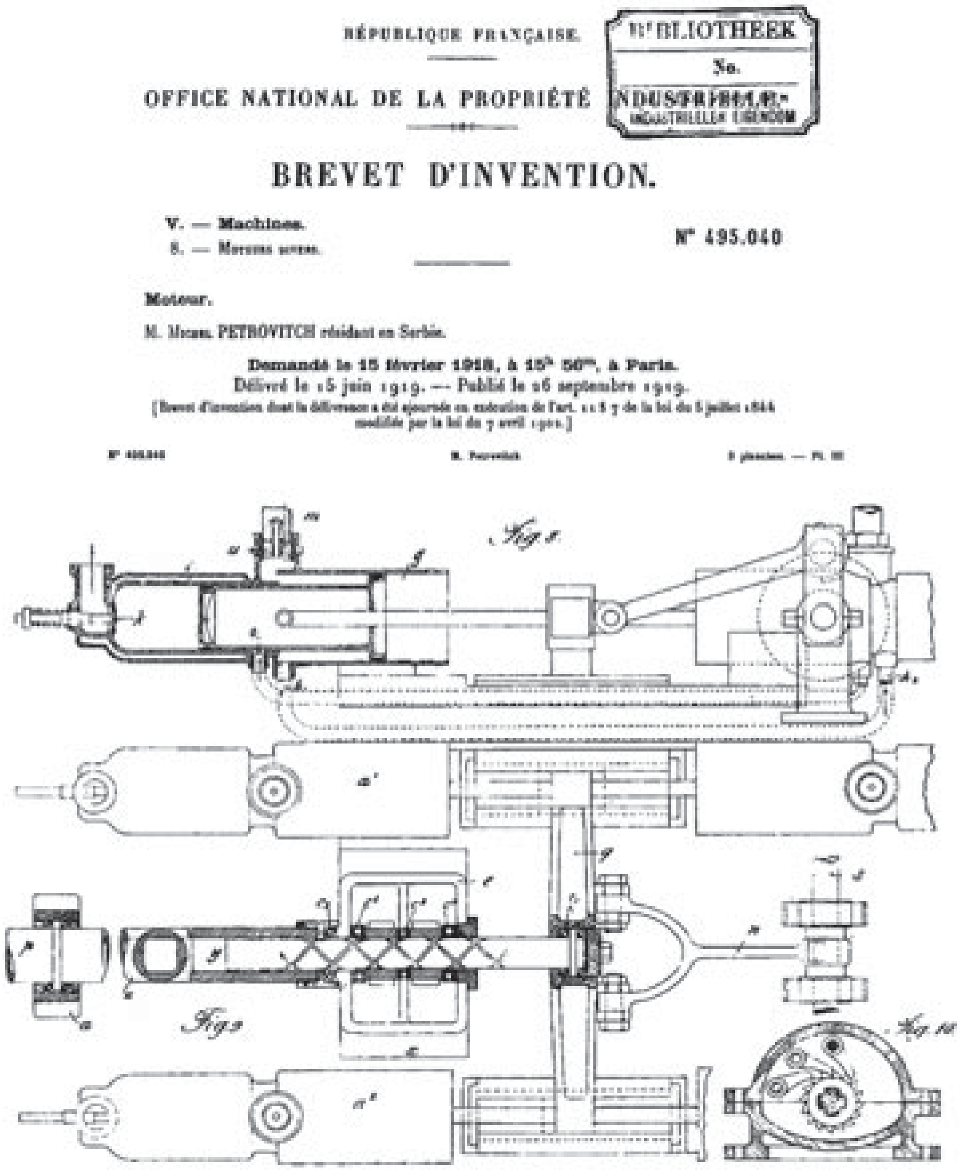
Appareil imprimant un mouvement rapide aux bombes, mines aériennes et torpilles aériennes lancées par un canon lisse.

M. Michel PETROVITCH résidant en Serbie.

Demandé le 22 février 1918, à 15h 52m, à Paris.
Délivré le 15 mars 1920. — Publié le 8 juin 1920.

Figure 7. Sketch of the device for ejecting and launching missiles, meant for usage both in water and air, as well as on ground and on sea. Protected by French patent no. 503.321

The last patented invention, which we find recorded in Petrović's bibliography, is a *model of the motor with a piston of alternating impact*, the main spindle of which was made with the coil for transmission of the piston movement. Petrović submitted an application for patenting this invention on February 15, 1918, and the patent under the number 495.040 was granted to him on September 26, the following year (Figure 8).



RÉPUBLIQUE FRANÇAISE.

OFFICE NATIONAL DE LA PROPRIÉTÉ INDUSTRIELLE.

BREVET D'INVENTION.

V. — Machines.

N° 495.040

Moteur.

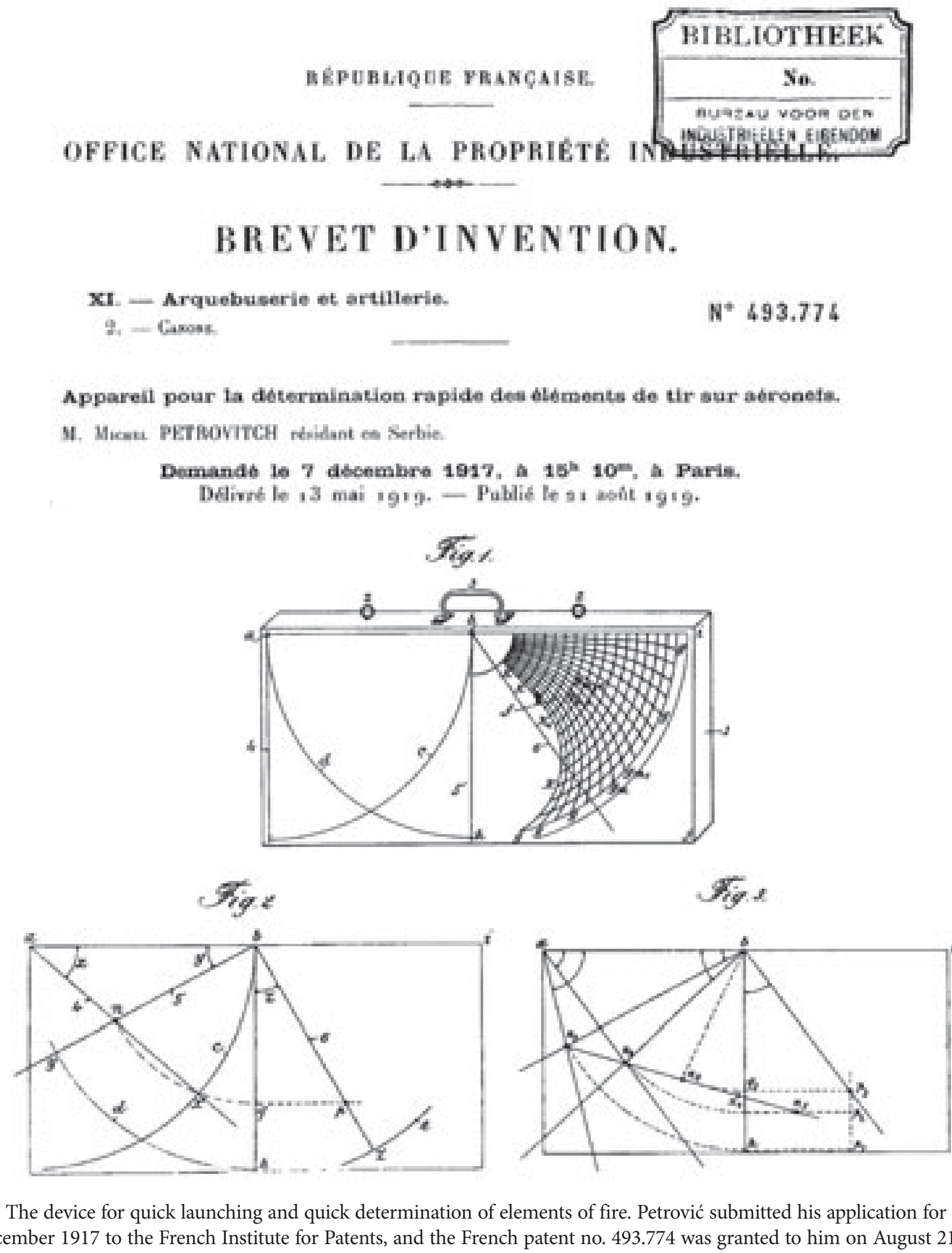
M. Milan PETROVITCH résidant en Serbie.

Demandé le 15 février 1918, à 15h 50m, à Paris.

Délivré le 15 juin 1919. — Publié le 26 septembre 1919.

Figure 8. Sketch of a *model of a motor with the piston of alternating impact*, patent no. 495.040

The *device for rapid launching and rapid determination of fire* was an important invention of Petrović for military use. Petrović submitted the application for this invention on December 7, 1917 to the French Patent Office, and the French patent number 493.774 was granted to him on August 21, 1919 (Figure 9).



RÉPUBLIQUE FRANÇAISE.

OFFICE NATIONAL DE LA PROPRIÉTÉ INDUSTRIELLE.

BREVET D'INVENTION.

XI. — Arquebuserie et artillerie.
2. — Canons.

N° 493.774

Appareil pour la détermination rapide des éléments de tir sur aéronefs.

M. Michel PETROVITCH résidant en Serbie.

Demandé le 7 décembre 1917, à 15h 10m, à Paris.
Délivré le 13 mai 1919. — Publié le 21 août 1919.

Figure 9. The device for quick launching and quick determination of elements of fire. Petrović submitted his application for this device on 7 December 1917 to the French Institute for Patents, and the French patent no. 493.774 was granted to him on August 21st 1919.

To these patents one should also add the model of *the efficient avoidance of a minefield* (Mémoire No. 120, 1920). Petrovć also protected with a patent in France his device named *the means for assuring the buoyance of ships following damages caused by collision, mine, torpedo or stranding*. This device consists of a large number of bodies, in the form of a balloon, with appropriate sources of gases under pressure for inflation, which can be handled manually, remotely or automatically, with an appropriate electromagnetic device. The inflation of the elements of this device prevents or slows down sinking of a vessel (a ship). Petrović submitted the application for this device to the French Patent Office on November 24, 1917, and patent number 515.072 was issued to him only on March 24, 1921 (Figure 10). He also patented this invention in Great Britain in 1918 under the patent number 121.279 (Figures 10, 11 and 12).

In addition to the patented technical devices in the area of mechatronics and mechanical engineering, Mihailo Petrović is also an author of a series of inventions for which there are no records that they had been protected by a patent. One of those devices is the hydraulic integrator, to which a separate chapter in this monograph is dedicated. Here we described a certain number of devices based on the original patent elaborates available in electronic form.

The depth measurer is Petrović's invention intended for measuring depth at which an object is submerged into water – partially (e.g. a boat) or fully (e.g. a submarine). Available information show that number 96371 is associated with this patent from 1918, and that *English Admiralty gave a positive opinion about it*. According to some sources, Petrović received an invitation from British admirals in relation with this invention, but there are no written records about it. It is possible that the number associated with this invention represents the number of the application submitted to the French Institute for Industrial Property (Institut national de la propriété Industrielle) or to some other patent institution.

* * *

While researching Petrović's patents, Snežana Šarboh, MA, searched available European patent databases, and quoted my article published in the monograph titled *Legende Beogradskog Univerziteta (The Legends of Belgrade University)*. Particularly useful in that regard were the databases of the European Patent Office (ESPACENET) and the German Patent and Trade Mark Office (DEPATISNET). Upon request of the author of this article, Ivana Atanasovska, PhD, continued the search of the *Espacenet* (European Patent Office) database and found patent documentation for all ten Petrović's patents. Based on the search of these databases, a total of 10 (ten) granted patents of Petrović was confirmed, and consequently the table in the appendix was drawn up. Nine patents were registered in France and one in Great Britain. Nevertheless, we cannot in all certainty claim that this is the final number of Mihailo Petrović's patented inventions.

RÉPUBLIQUE FRANÇAISE.

OFFICE NATIONAL DE LA PROPRIÉTÉ INDUSTRIELLE.

BREVET D'INVENTION.

VI. — Marine et navigation.

1. — Construction des navires et engins de guerre.

N° 515.072

Dispositif pour assurer la flottabilité des navires en danger.

M. Mich. PETROVITCH résidant en Serbie.

Demandé le 24 novembre 1917, à 15h 57m, à Paris.

Délivré le 20 novembre 1920. — Publié le 25 mars 1921.

[Brevet d'invention dont la délivrance a été ajournée en exécution de l'art. 11 § 7 de la loi du 5 juillet 1844 modifiée par la loi du 7 avril 1902.]

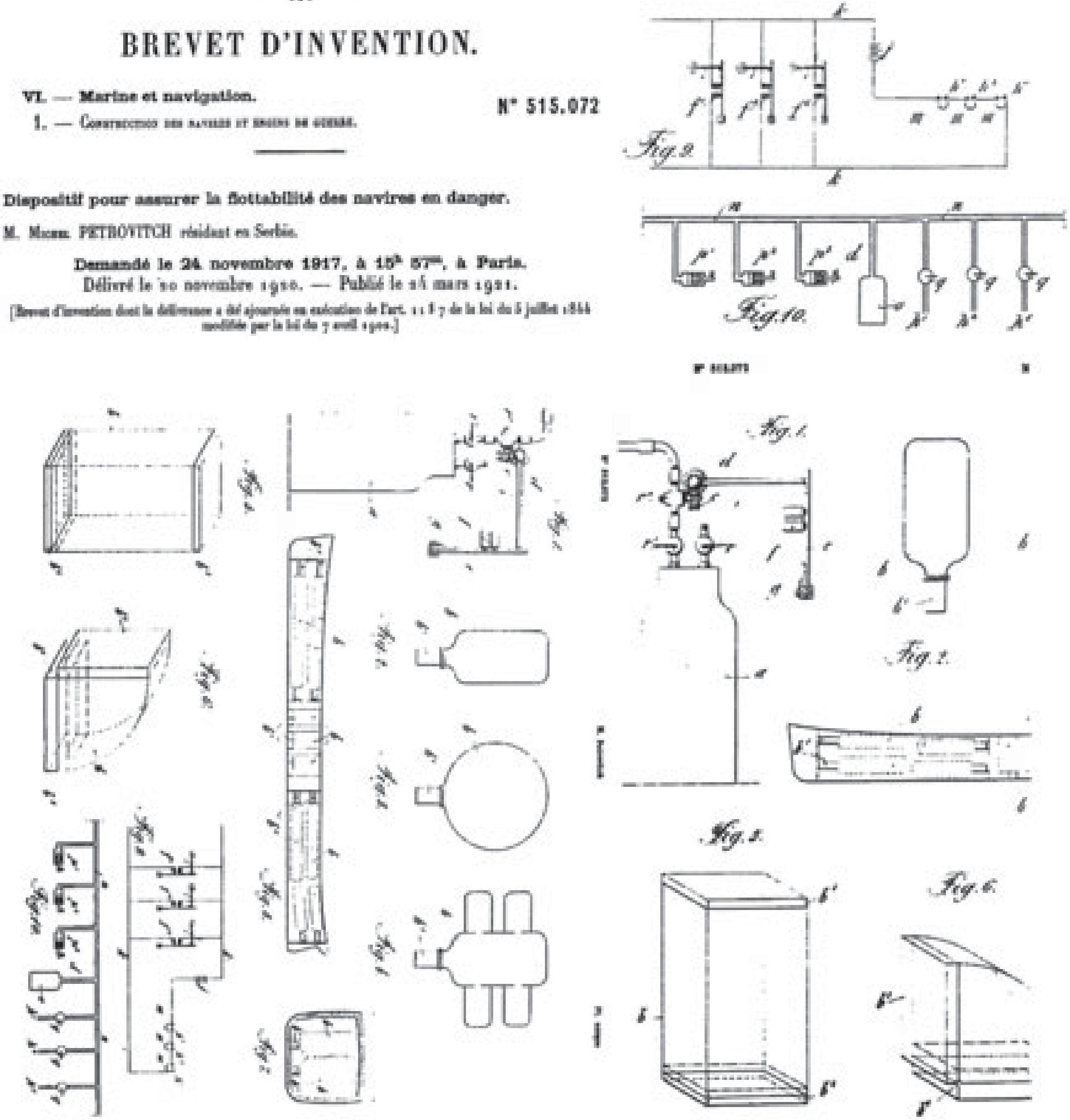


Figure 10. The means for assuring buoyoncy of ships following damage made by crash, mine, torpedo or stranding

NOTE.—*The application for a Patent has become void.*
This print shows the Specification as it became open to public inspection.

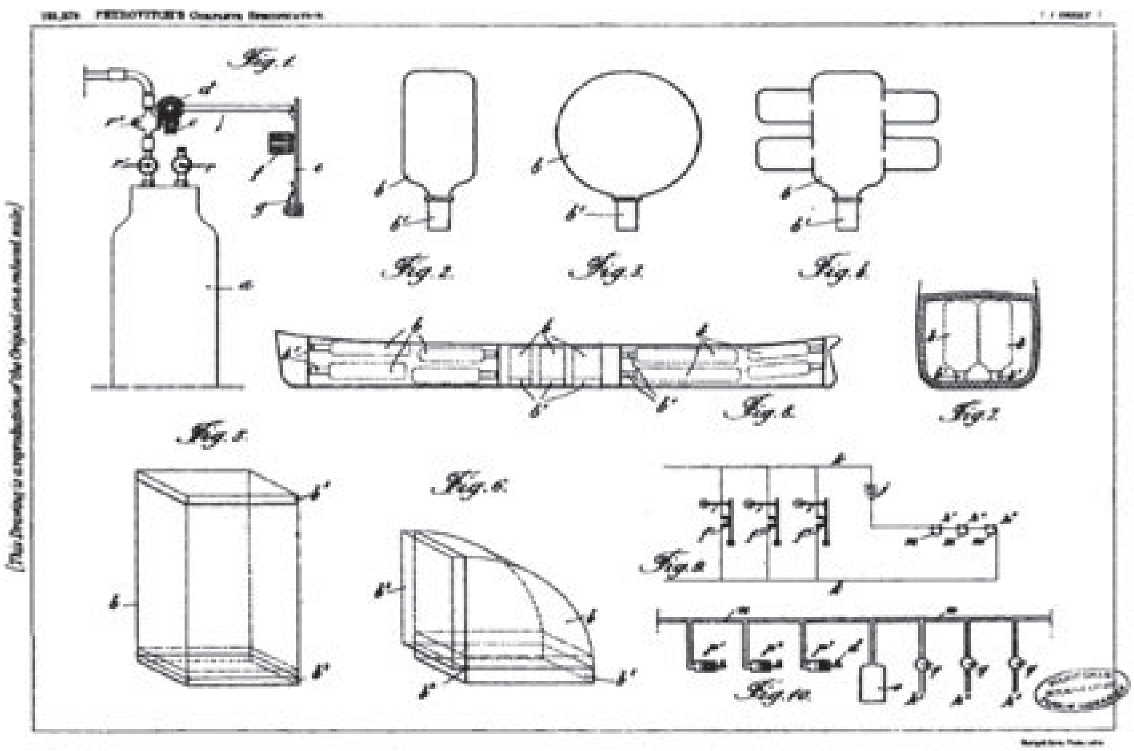
121,279

PATENT SPECIFICATION

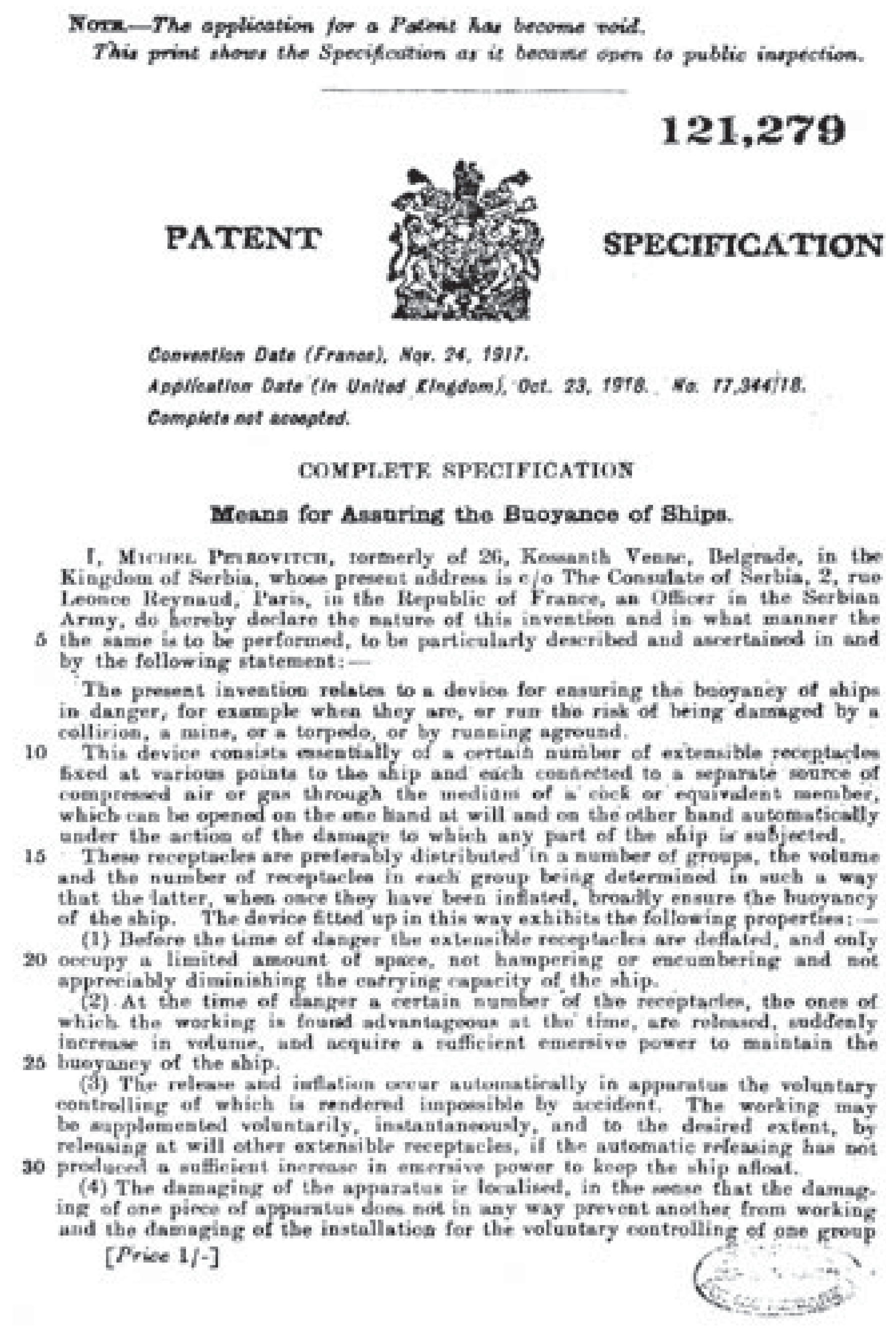
Convention Date (France), Nov. 24, 1917.
Application Date (in United Kingdom), Oct. 23, 1918. No. 17,344/18.
Complete not accepted.

COMPLETE SPECIFICATION.

Means for Assuring the Buoyance of Ships.



Sketch 11. One of the sketches of the patent granted in Great Britain, patent no. 121.279, for the discovery named *The means for assuring buoyoncy of ships following damage made by crash, mine, torpedo or stranding.*



Note.—The application for a Patent has become void.
This print shows the Specification as it became open to public inspection.

121,279

PATENT SPECIFICATION

Convention Date (France), Nov. 24, 1917.
Application Date (in United Kingdom), Oct. 23, 1918. No. 17,344/18.
Complete not accepted.

COMPLETE SPECIFICATION

Means for Assuring the Buoyance of Ships.

I, Michel Petrovitch, formerly of 26, Kossanth Venac, Belgrade, in the Kingdom of Serbia, whose present address is c/o The Consulate of Serbia, 2, rue Leonce Reynaud, Paris, in the Republic of France, an Officer in the Serbian Army, do hereby declare the nature of this invention and in what manner the same is to be performed, to be particularly described and ascertained in and by the following statement:—

The present invention relates to a device for ensuring the buoyancy of ships in danger, for example when they are, or run the risk of being damaged by a collision, a mine, or a torpedo, or by running aground.

This device consists essentially of a certain number of extensible receptacles fixed at various points to the ship and each connected to a separate source of compressed air or gas through the medium of a cock or equivalent member, which can be opened on the one hand at will and on the other hand automatically under the action of the damage to which any part of the ship is subjected.

These receptacles are preferably distributed in a number of groups, the volume and the number of receptacles in each group being determined in such a way that the latter, when once they have been inflated, broadly ensure the buoyancy of the ship. The device fitted up in this way exhibits the following properties:—

(1) Before the time of danger the extensible receptacles are deflated, and only occupy a limited amount of space, not hampering or encumbering and not appreciably diminishing the carrying capacity of the ship.

(2) At the time of danger a certain number of the receptacles, the ones of which the working is found advantageous at the time, are released, suddenly increase in volume, and acquire a sufficient emersive power to maintain the buoyancy of the ship.

(3) The release and inflation occur automatically in apparatus the voluntary controlling of which is rendered impossible by accident. The working may be supplemented voluntarily, instantaneously, and to the desired extent, by releasing at will other extensible receptacles, if the automatic releasing has not produced a sufficient increase in emersive power to keep the ship afloat.

(4) The damaging of the apparatus is localised, in the sense that the damaging of one piece of apparatus does not in any way prevent another from working and the damaging of the installation for the voluntary controlling of one group

[*Price* 1/-]

Figure 12. The first page of the granted patent under patent number 121.279, for the discovery named *The means for assuring buoyoncy of ships following damage made by crash, mine, torpedo or stranding*. This discovery was submitted in Great Britain on 23 October 1918, and granted under patent number 121.279.

THE TABLE OF MIHAILO PETROVIĆ'S PATENTED INVENTIONS

	Patent number	Original title of the patent	English translation of the title of the patent	Application submission date	Date of granting of patent	Patent co-applicant
1	FR 1413.730	Télémetre a sextant	Range indicator	February 11, 1910	August 17, 1910	Terzić Milorad
2	FR 447.861	Changement de vitesse avec pignons étagés reccordés par des engrenages en héllice conique	Multiple compression ration cogwheel transmitterwith bending conical cogwheels	August 31, 1912	January 17, 1919	Popović Svetolik
3	FR 463.082	Changement de vitess	Construction of the cogwheel transmitter	September 29, 1913	February 13, 1914	
4	FR 476.320	Changement de vitess	Automobile gearbox	October17, 1914	July 27, 1915	
5	FR 480.788	Cadran calendrier pour objets d'horlogerie, de bijeouterie et autres	Eternal calendar	January 27, 1916	September 21, 1916	
6	FR 515.072	Dispositif pour assurer la flottabilité des navires en danger	Means for assuring the buoyance of ships following damages caused by collision, mine, torpedo or stranding	November 24, 1917	March 24, 1921	
7	FR 493.774	Appareil pour la détermination rapide des elements de tir sur aéronefs	Device for rapid launching and rapid determination of fire	December 7, 1917	August 21, 1919	
8	FR 495.040	Moteut	Motor with a piston of alternating effect	February 15, 1918	September 26, 1919	
9	FR 503.321	Appareil imprimant un movement rapide aux bombes, mines aériennes et torpilles aériennes lamcées par un canin lisse	Device for launching and ejecting of missiles	February 22, 1918	June 8, 1920	
10	GB 121.279	Means for Assuring the Buoyance of Fhips	Means for assuring buoyance of ships following damages caused by collision, mine, torpedo or stranding	October 23, 1918		

FR and GB represent two-lettered designations of France and Great Britain, in accordance with the appropriate standard of the World Intellectual Property Organization, WIPO.

FINAL COMMENTS ON PETROVIĆ'S INVENTIONS AND PATENTS

Based on the results obtained by various searches of scientific papers and numerous patent databases one may conclude that Mihailo Petrović was an authentic author of at least ten original inventions in the field of mechatronics and machine constructions, and of at least ten patents. In addition to the five French patents, which Dragan Trifunović, PhD, mentioned and elaborated on [7, 8, 9], the latest research has come upon additional four French and one British patent. However, we cannot in all certainty claim that this is the final number of Mihailo Petrović's patented inventions.

With the passage of time, it may be concluded that Mihailo Petrović's work in various fields of science and engineering had stimulated scientific progress in Serbia, not only in mathematical sciences, but also in other natural sciences, humanities and technical sciences as well. Since Petrović was primarily celebrated as a mathematician, this aspect of multidisciplinarity of his life and work was somewhat neglected in historical reviews. In all likelihood, Petrović developed an interest in practical side of science in the period from 1889 to 1894, during his stay in Paris, where he acquired thorough knowledge, not only in mathematics, but also in other sciences, primarily in physics and theoretical and applied mechanics, and when he managed to understand the difference between geometry and dynamics. At Sorbonne, as an excellent student, he earned diplomas in mathematics and physics and a PhD degree in mathematical sciences having been taught by the distinguished French mathematicians Hermite, Painleve and Picard, nevertheless he developed his talents under the influence of Poincaré, as one of the three doctoral students of this unique scientist with numerous multidisciplinary achievements. He came to Belgrade in 1894 and soon showed his exceptional talent through a wide range of scientific achievements and practical discoveries, inventions and patents in the field of mechatronics and mechanical engineering, nevertheless it was with his hydraulic integrator that he became a precursor in the field of computing. As he himself mentioned on several occasions, when it comes to his practical work, he found inspiration in his paper dealing with natural philosophy, titled *Mathematical Phenomenology*. His book titled *Elements of Mathematical Phenomenology* is a *powerful tool* for the application of *philosophy of phenomenological mapping* in other sciences. In this respect, each invention and its patent was an example of materialized idea. The great Serbian scientist Milutin Milanković [3] pointed to the importance of this theory immediately upon its publishing, having said that "it is a pity that it has been published only in the Serbian language", and consequently not available to the global scientific community of that time.

REFERENCES

[1] Хедрих (Стевановић), К., *Беседа о Михаилу Петровићу*, Легенде Београдског Универзитета, Универзитет у Београду, Универзитетска библиотека „Светозар Марковић“ у Београду, 2005, стр. 37–48.

[2] Хедрих (Стевановић) К., *Беседа о Михаилу Петровићу и фасцинантној нелинеарној динамици*, Српски математичари, Зборник предавања одржаних на скупу у оквиру манифестације Мај месец математике 2012, Српска академија наука и уметности, Универзитет у Београду и Завод за издавање уџбеника, Београд 2015, стр. 29–64.

[3] Миланковић М., *Личност Михаила Петровића*, Први део у Миланковић М. и Михаиловић Ј., *Мика Алас*, Београд–Калгари, 2012, стр. 459.

[4] Петровић, М., *Феноменолошко пресликавање* (*Phenomenological map*), Српска краљевска академија, Београд, 1933, стр. 33.

[5] Петровић, М., *Елементи математичке феноменологије* (*Elements of mathematical phenomenology*), Српска краљевска академија, Београд, 1911, стр. 389.
http://elibrary.matf.bg.ac.rs/handle/123456789/476?locale-attribute=sr

[6] Petrović M., *Sur l`équation différentielle de Riccati et applications chimiques.* Sityungsberichte der Kőnigl. – Bőhmischen Gesellschaft der Wissenschaften, Praha, 1896, 39, 1–25.

[7] Трифуновић Д., *Летопис живота и рада Михаила Петровића*, САНУ Београд 1969, VIII+631 стр.; * Д. Трифуновић, *Михаило Петровић Алас – живот и дело*, Дечје новине, Горњи Милановац, 1982, 160 стр.

[8] Трифуновић Д., *Бард српске математике – Михаило Петровић Алас*, Завод за уџбенике и наставна средства, Београд.

[9] Трифуновић Д., *Михаило Петровић Алас – живот и дело*, Дечје новине, Горњи Милановац, 1982, 160 стр.

[10] *Сабрана дела Михаила Петровића*, Том 15, Писма Биографија Летопис, Завод за уџбенике и наставна средства, Београд, 1998.
Историја науке: филм о Михаилу Петровићу Аласу: (6. маја 1868. године у Београду – 8. јуна 1943. у Београду)
https://www.youtube.com/watch?v=yo5mHVlYpxM

[11] Digitalni legat Mihajla Petrovića Alasa
http://alas.matf.bg.ac.rs/~websites/digitalnilegatmpalas/

[12] *Special Issue of International Journal Non-Linear Mechanics*, Vol. 73, Pages 1–128, July 2015 Dedicated to Petrović's theory: Elements of mathematical phenomenology and Phenomenological Mappings. Edited by Katica R. (Stevanović) Hedrih, Ivan Kosenko, Pavel Krasilnikov and Pol D. Spanos.
https://www.sciencedirect.com/journal/international-journal-of-non-linear-mechanics/vol/73/suppl/C

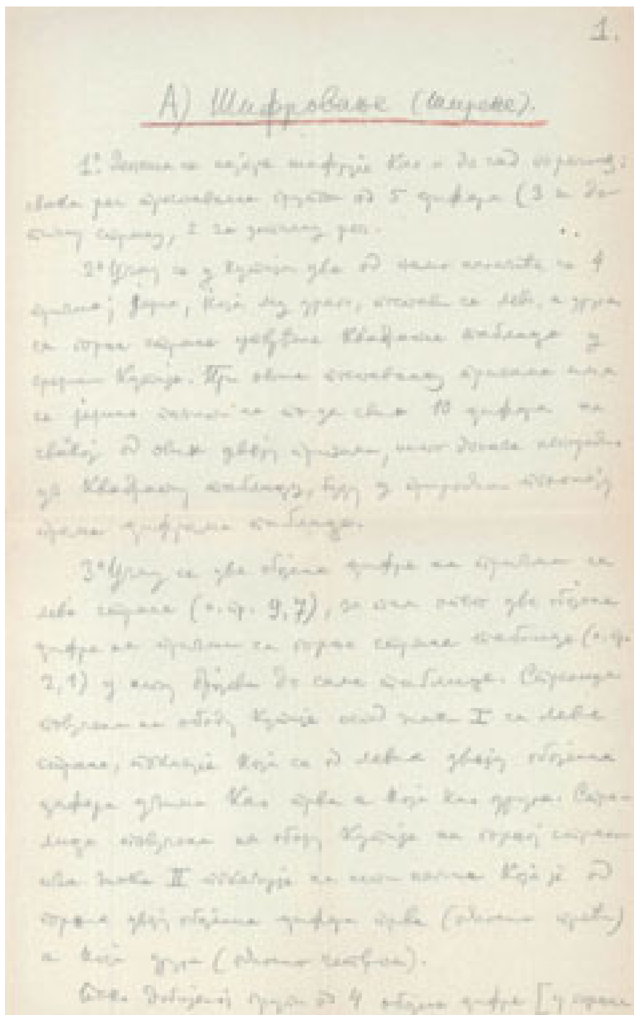
MIHAILO PETROVIĆ ALAS AND THE STATE CRYPTOGRAPHY OF THE INTERWAR PERIOD

Miodrag J. MIHALJEVIĆ
Mathematical Institute of SASA

History acknowledges, and owing to the increasing significance of the field in which he left his mark, history is to place an ever-greater emphasis on the work of Mihailo Petrović Alas in the domain of state cryptography in the period between the two world wars. The results of Petrović's work in the field of cryptography have not stayed on public records, which is not surprising given that research findings in cryptography were in the interwar years classified as military and state secrets. The available documents taken from the Serbian Armed Forces General Staff and the Ministry of the Army and Navy, dating from the period prior to World War II, show that, from the perspective of the general horizon of knowledge at the time, Mihailo Petrović made significant breakthroughs in the design and analysis of coding systems, as well as in the training of staff that operated in the area of cryptography for the purposes of the state.

Petrović's accomplishments in the field of cryptography and coding have been documented in the 15 volumes of the Cipher Bureau of the Intelligence Unit of the Armed Forces General Staff of the Kingdom of Yugoslavia, under the title *Kriptografija – škola za obuku na šifri (Cryptography: Code School),* and in 24 volumes under the title *Sistem (za šifrovanje) (Coding System)*. Based on those documents, the work of Petrović and the ensuing results can be found in the following areas: (a) the methods for encryption; (b) the methods for "breaking" the codes, and (c) educational materials related to the techniques for enciphering and deciphering the encrypted messages.

The basic aim of this chapter is to present the illustrative elements of the source documents accompanied by appropriate commentary. It should especially be noted that an evaluation of cryptographic security of the presented coding systems from modern perspective does not represent the subject of this chapter, because history has shown that nearly all coding systems in operational use prior to World War II are now completely insecure. This fact represents an outcome of the accumulation of knowledge about the techniques that can be used for deciphering the coding systems, as well as of the now available state-of-the-art technological resources.



1.

А) Шифровање

Mihailo Petrović's manuscript on encryption ("Adligat" Society)

CODING IN THE PRESENT DAY AND IN THE INTERWAR PERIOD

Our present-day reality represents leading a “parallel life” in real and digital space, which we use for communication and which contains information of vital importance for our daily life. In the digital space there are no boundaries that separate us one from another and, in order to ensure security and privacy, cipher techniques have been used on a massive scale. The widespread use of ciphers is one of the features that differentiate the present-day coding from that of the time when Mihailo Petrović was concerned with it. Modern cipher techniques are “a product of cryptology”, a mathematics-based scientific discipline: in his time, cryptology had not yet existed as a distinct scientific discipline – the emergence of cryptology as a scientific discipline is tied to reference [1]. Since the mid-twentieth century, cryptology has been established and intensively developed as a foundation for ensuring security and privacy in the digital space, in which the encryption techniques represent one of the key elements. It now includes a set of other elements that are generally classified either within the domain of cryptography or within cryptanalysis. Plainly speaking, cryptography deals with techniques for protection, while cryptanalysis is concerned with techniques for the evaluation of the security of protection or with techniques for “breaking” the cryptographic protection.

A century ago, coding was not developed within a separate scientific discipline, but either as “a specific craft” or, as in Petrović’s case, as “coding designed by a mathematician”.

Cryptography, which once, including his time, was associated only with encryption, has been developed for over two millennia as a skill that enables the protection of secrecy of information, and it is now one of the fundamental approaches for ensuring the security and privacy in the digital space. Over the centuries, a great number of methods have been developed for providing cryptographic protection or coding. Up until the 1950’s, coding was based on a combination of skillfulness and mathematical methods.

The present-day cryptology is based on the pool of knowledge compared to which the one that served as the basis for the design and analytical methods of coding in the 1930’s had been modest to say the least, and thus could not provide the basis for the design characterized by a long-lasting and high-level security. Of this Petrović was well aware: in the introduction to cryptography contained in the volumes in reference [3], he points to the fact that all encryption techniques used in World War I appeared to be insecure, and that it is believed to be necessary to frequently modify the methods of encryption in operational use. This can be exemplified by the following original text from the notebook *Cryptography: Basic Concepts* [reference 3]:

However, few are those who managed to keep the secret of their code intact for long.

It is firmly established that, during the last World War, no method, mode or system of secret correspondence could have been used over a longer period of time.

ALGORITHMS FOR ENCRYPTION

The documents that were at our disposal indicate that the Ministry of the Army and Navy of the Kingdom of Yugoslavia used at least 24 systems for encryption labeled as “System” followed by one of the ensuing numbers: 1, 1a, 2, 2a, 3, 3a, 4, 4a, 5, 6, 6a, 7, 7a, 8, 9, 10, 10a, 11, 12, 13, 14, 15, 16, 17, 18.

The application of the said systems had the following basic requirements: (a) a trained cryptographer, (b) written instructions for work, and (c) only in some cases, certain mechanical devices. The result was to be written on paper and further communicated in the prescribed way, most frequently by telegraph or courier.

If it was included in the system, the basic device used for enciphering was the so-called cipher slide (French *reglette*) – the cited information concerning the slide is presented in the following three figures.

> The cipher slide in question is in its simplest form such that it contains **two fixed well-ordered or jumbled-up letters of the alphabet on a piece of wooden bar**, one on top of another, or only at the top or at the bottom, and in the middle a little movable ruler sliding along the bar, with another set of normal or jumbled-up alphabet. The alphabet lists have to be duplicated on the fixed bar, as well as on the movable ruler.
>
> The alphabet lists have to be duplicated on the fixed bar, as well as on the movable ruler.

Figure 1: Basic information on the module, Notebook 12, reference [3]

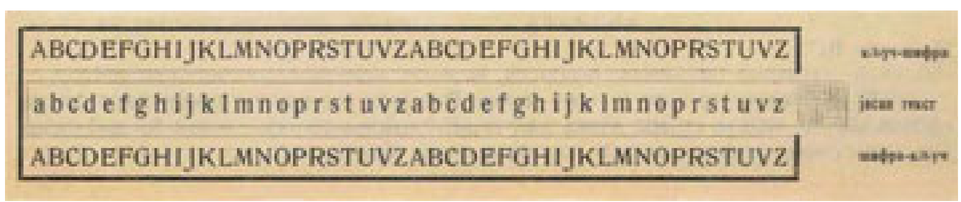


Figure 2: Image of the module, Notebook 12, reference [3]

Volume 12, reference [3] presents the mode of usage of the cipher slide:

> M E T H O D
>
> **OF ENCRYPTING AND DECRYPTING BY USING SPECIAL DEVICES**
>
> A) **Usage of the cipher slide:**
>
> In this volume we are going to introduce a special method of encryption – called a Saint-Cyr method, which in essence is nothing else than a mechanical application of Vigenère’s method in the specified manner.
>
> Moreover, this method also involves complex substitution

From Notebook 12, reference [3]

Starting from the then known methods for compromising (breaking) the methods of encryption, and in order to obtain a higher level of security, the document in Volume 10, reference [3] presents the technique of "double transposition". An essential explanation of the double transposition is given in the following figure.

> Овде ћемо изнети још теже случајеве, јер је у питању не само шифровање јасног текста већ и прешифравање т. ј. једанпут добијена шифра има се још једанпут шифровати.
>
> Овакви системи шифровања могу се назвати још и **шифровање системом дуплог замењивања по таблици**, који може бити са истим или новим кључем.
>
> Другим речима, ако један јасан текст шифрујемо по једном кључу, ми добијену шифру прешифравамо, било тим истим – првобитним кључем или другим новим кључем. Разуме се да је овај други начин много тежи и компликованији.

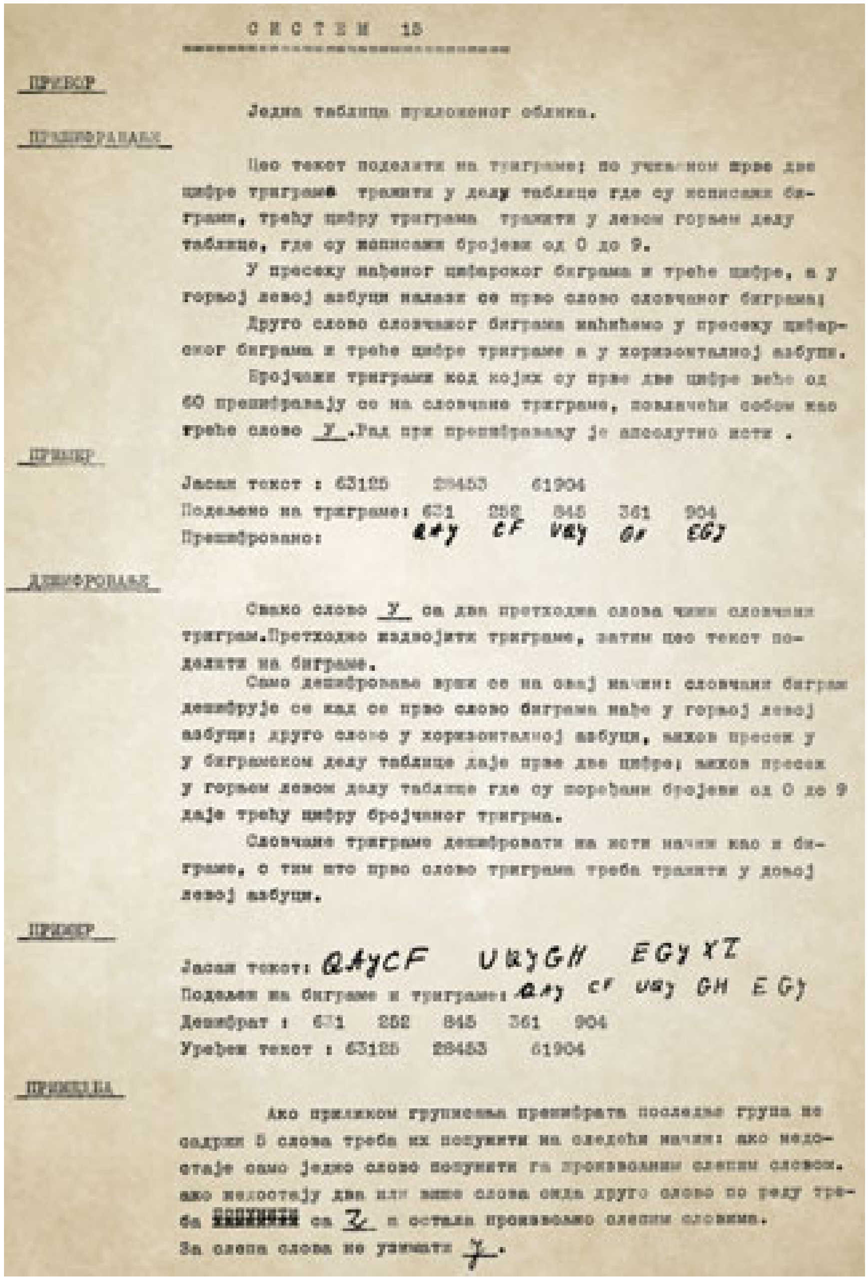
Figure 3: Encryption in accordance with Notebook 10, reference [3]

System 15, reference [2] presents double transposition realized according to the following paradigm:

- by using the chosen cipher algorithm make the first cipher text based on the plain text;
- encrypt the cipher text one more time, in general, using another cipher algorithm.

It is noted that the said approach of improving cryptographic security by iterative encryption represents the basic principle of building the modern block cipher procedures, in which the cipher text is made by repeated encryption using the basic cipher transposition of low-level cryptographic security, thereby providing a high-level cryptographic security after a certain number of iterations.

System 15, in its original form, in compliance with the one given in reference [2], is presented in the following two figures:



С И С Т Е М   15

ПРИБОР

Једна таблица приложеног облика.

ПРЕШИФРАВАЊЕ

Цео текст поделити на триграме; по учињеном прве две цифре триграма тражити у делу таблице где су исписани биграми, трећу цифру триграма тражити у левом горњем делу таблице, где су исписани бројеви од 0 до 9.

У пресеку нађеног цифарског биграма и треће цифре, а у горњој левој азбуци налази се прво слово словчаног биграма;

Друго слово словчаног биграма наћићемо у пресеку цифарског биграма и треће цифре триграме а у хоризонталној азбуци.

Бројчани триграми код којих су прве две цифре веће од 60 прешифрирају се на словчане триграме, повлачећи собом као треће слово У. Рад при прешифровању је апсолутно исти.

ПРИМЕР

Јасан текст : 63125 28453 61904

Подељено на триграме: 631 252 845 361 904

Прешифровано: QAy CF VRy GH EGy

ДЕШИФРОВАЊЕ

Свако слово У са два претходна слова чини словчани триграм. Претходно издвојити триграме, затим цео текст поделити на биграме.

Само дешифровање врши се на овај начин: словчани биграм дешифрује се кад се прво слово биграма нађе у горњој левој азбуци; друго слово у хоризонталној азбуци, њихов пресек у у биграмском делу таблице даје прве две цифре; њихов пресек у горњем левом делу таблице где су поређани бројеви од 0 до 9 даје трећу цифру бројчаног триграма.

Словчане триграме дешифровати на исти начин као и биграме, с тим што прво слово триграма треба тражити у доњој левој азбуци.

ПРИМЕР

Јасан текст: QAyCF VRyGH EGyXZ

Подељен на биграме и триграме: QAy CF VRy GH EGy

Дешифрат : 631 252 845 361 904

Уређен текст : 63125 28453 61904

ПРИМЕДБА

Ако приликом груписања прешифрата последња група не садржи 5 слова треба их попунити на следећи начин: ако недостаје само једно слово попунити га произвољним слепим словом, ако недостају два или више слова онда друго слово по реду треба попунити са Z а остала произвољно слепим словима.
За слепа слова не узимати У.

Figure 4: Encryption system no. 15, reference [2]

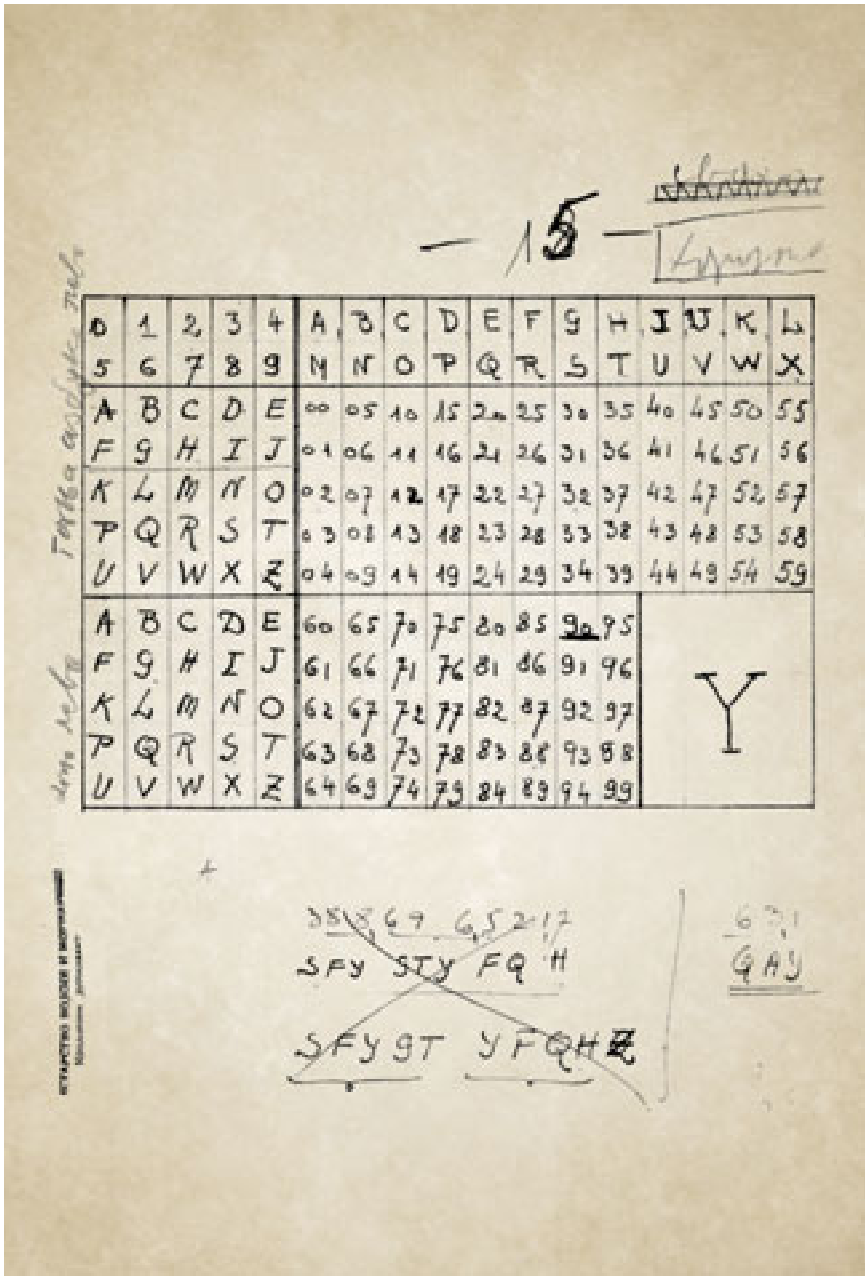
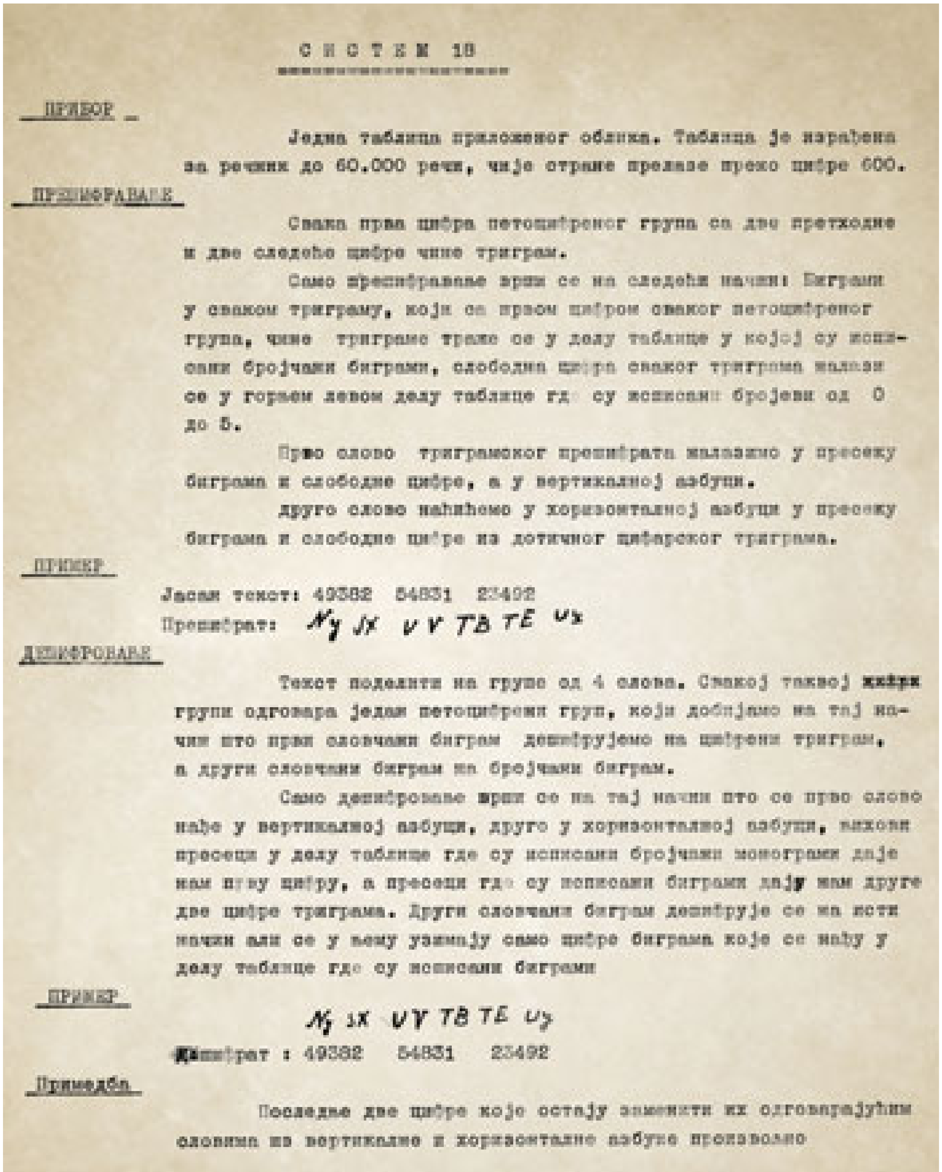


Figure 5: The table used in System no. 15, reference [2]

As the final illustration, the following figure shows coding system 18 in its original form.



С И С Т Е М 18

ПРИБОР

Једна таблица приложеног облика. Таблица је израђена за речник до 60.000 речи, чије стране прелазе преко цифре 600.

ПРЕШИФРОВАЊЕ

Свака прва цифра петоцифреног група са две претходне и две следеће цифре чине триграм.

Само прешифровање врши се на следећи начин: Биграми у сваком триграму, који са првом цифром сваког петоцифреног група, чине триграме траже се у делу таблице у којој су исписани бројчани биграми, слободна цифра сваког триграма налази се у горњем левом делу таблице где су исписани бројеви од 0 до 5.

Прво слово триграмског прешифрата налазимо у пресеку биграма и слободне цифре, а у вертикалној азбуци.

Друго слово наћићемо у хоризонталној азбуци у пресеку биграма и слободне цифре из дотичног цифарског триграма.

ПРИМЕР

Јасан текст: 49382 54831 23492

Прешифрат: Ny JX VV TB TE Uъ

ДЕШИФРОВАЊЕ

Текст поделити на групе од 4 слова. Свакој таквој ~~цифри~~ групи одговара један петоцифрени груп, који добијамо на тај начин што први словчани биграм дешифрујемо на цифрени триграм, а други словчани биграм на бројчани биграм.

Само дешифровање врши се на тај начин што се прво слово нађе у вертикалној азбуци, друго у хоризонталној азбуци, њихови пресеци у делу таблице где су исписани бројчани монограми даје нам прву цифру, а пресеци где су исписани биграми дају нам друге две цифре триграма. Други словчани биграм дешифрује се на исти начин али се у њему узимају само цифре биграма које се нађу у делу таблице где су исписани биграми

ПРИМЕР

Ny JX VY TB TE Uъ

Дешифрат : 49382 54831 23492

Примедба

Последње две цифре које остају заменити их одговарајућим словима из вертикалне и хоризонталне азбуке произвољно

Figure 6: Encryption system no. 18, reference [2], which was used in its improved form in Yugoslavia and for some time after the World War II

ANALYSIS OF THE CRYPTOGRAPHIC SECURITY OF CIPHER ALGORITHMS

The documentation at the Armed Forces General Staff contains information about knowing a set of steps for breaking some, then well-known, coding procedures. The knowledge about those procedures served as the starting point for the design of the coding system resistant to then known attacks.

Volume 10, reference [3] presents an approach to the analysis of the safety of cipher algorithms of "complex transposition with repeated encryption":

Да нисмо знали кључ, ми би смо га морали наћи, али би посао био много компликованији и скопчан са много више времена, јер би имали да решимо три проблема:

Први проблем: Шифру дешифровати системима и начинима објашњеним у свесци бр. 8 и 9. И ако ово изгледа нелогично, ипак се мора приступити прво овом раду па тек онда истраживању кључа и друго. Ако имамо више шифара исте дужине, шифроване овим методом, опет је поступак исти.

Други проблем: Добивши јасан текст, треба одредити за свако слово шифре место које оно заузима у јасном тексту, и

Трећи проблем: Одредивши место за свако слово шифре, које оно заузима у јасном тексту, пронаћи кључ по коме је извршена замена.

Овај последњи проблем дели се на два друга и то:

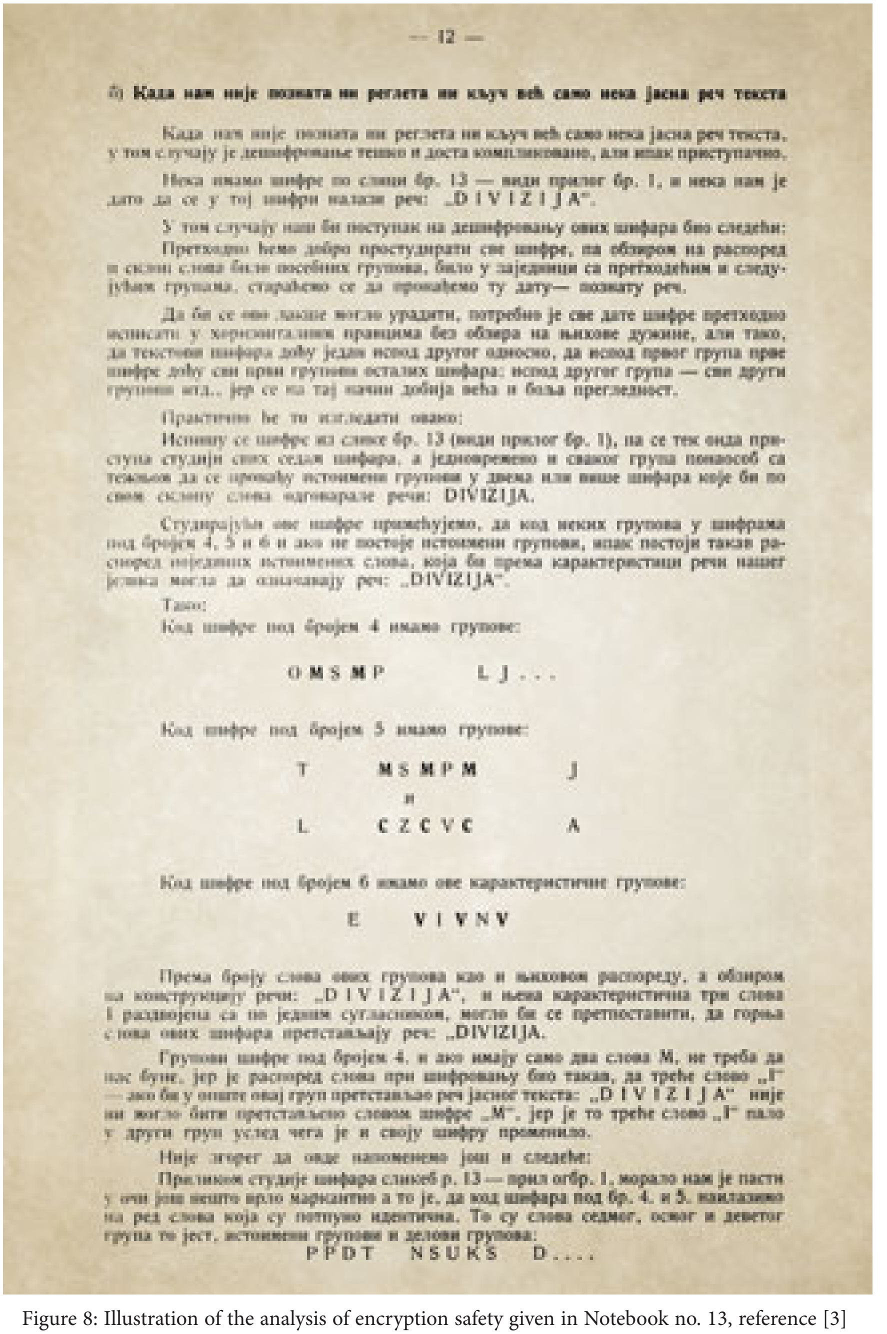
– Одредити дужину кључа, и

– Успоставити кључ онакав какав је узет.

Као што видимо, посао је дуг, тежак и скопчан са много стрпљивости, педантности у раду, воље и методичности. При томе се захтева дубока студија и резоновање, јер се без тога не могу имати резултати, – а природни дар и склоност ка криптографији убрзаће темпо рада и смањити грешке, које ће се неминовно појављивати.

Figure 7: Illustration from the document Notebook no. 10, reference [3], about one of the approaches for "breaking" the code

Volume 13, reference [3], contains the following discussion regarding cryptographic security:



— 12 —

б) Када нам није позната ни реглета ни кључ већ само нека јасна реч текста

Када нам није позната ни реглета ни кључ већ само нека јасна реч текста, у том случају је дешифровање тешко и доста компликовано, али ипак приступачно.

Нека имамо шифре по слици бр. 13 — види прилог бр. 1, и нека нам је дато да се у тој шифри налази реч: „D I V I Z I J A".

У том случају наш би поступак на дешифровању ових шифара био следећи:

Претходно ћемо добро простудирати све шифре, па обзиром на распоред и склоп слова били посебних групова, било у заједници са претходећим и следујућим групама, стараћемо се да пронађемо ту дату— познату реч.

Да би се ово лакше могло урадити, потребно је све дате шифре претходно исписати у хоризонталним правцима без обзира на њихове дужине, али тако, да текстови шифара дођу један испод другог односно, да испод првог група прве шифре дођу сви први групови осталих шифара; испод другог група — сви други групови итд., јер се на тај начин добија већа и боља прегледност.

Практично ће то изгледати овако:

Испишу се шифре из слике бр. 13 (види прилог бр. 1), па се тек онда приступа студији свих седам шифара, а једновремено и сваког група понаособ са тежњом да се пронађу истоимени групови у двема или више шифара које би по свом склопу слова одговарале речи: DIVIZIJA.

Студирајући ове шифре примећујемо, да код неких групова у шифрама под бројем 4, 5 и 6 и ако не постоје истоимени групови, ипак постоји такав распоред поједених истоимених слова, која би према карактеристици речи нашег језика могла да означавају реч: „DIVIZIJA".

Тако:

Код шифре под бројем 4 имамо групове:

O M S M P   L J . . .

Код шифре под бројем 5 имамо групове:

T   M S M P M   J

и

L   C Z C V C   A

Код шифре под бројем 6 имамо ове карактеристичне групове:

E   V I V N V

Према броју слова ових групова као и њиховом распореду, а обзиром на конструкцију речи: „D I V I Z I J A", и њена карактеристична три слова I раздвојена са по једним сугласником, могло би се претпоставити, да горња слова ових шифара претстављају реч: „DIVIZIJA.

Групови шифре под бројем 4. и ако имају само два слова M, не треба да нас буни, јер је распоред слова при шифровању био такав, да треће слово „I" — ако би у опште овај груп претстављао реч јасног текста: „D I V I Z I J A" није ни могло бити претстављено словом шифре „M", јер је то треће слово „I" пало у други груп услед чега је и своју шифру променило.

Није згорег да овде напоменемо још и следеће:

Приликом студије шифара сликеб р. 13 — прил огбр. 1, морало нам је пасти у очи још нешто врло маркантно а то је, да код шифара под бр. 4. и 5. наилазимо на ред слова која су потпуно идентична. То су слова седмог, осмог и деветог група то јест, истоимени групови и делови групова:

P P D T   N S U K S   D . . . .

Figure 8: Illustration of the analysis of encryption safety given in Notebook no. 13, reference [3]

CONTRIBUTIONS TO THE EDUCATION IN CRYPTOGRAPHIC WORK

As noted in section 3, using a coding system required a trained cryptographer and to that end an educational programme was set up for the training in cryptographic work.

Within the programme the focus was not only on introducing the techniques for encryption known at the time, but much attention was also devoted to education in the domain of evaluation of cryptographic security and the methods for compromising it. It is specially noted that in educational documents, reference [3], the volume of text referring to techniques for "attacking" the observed coding system usually far exceeds the volume of text dedicated to the description and usage of the observed coding system.

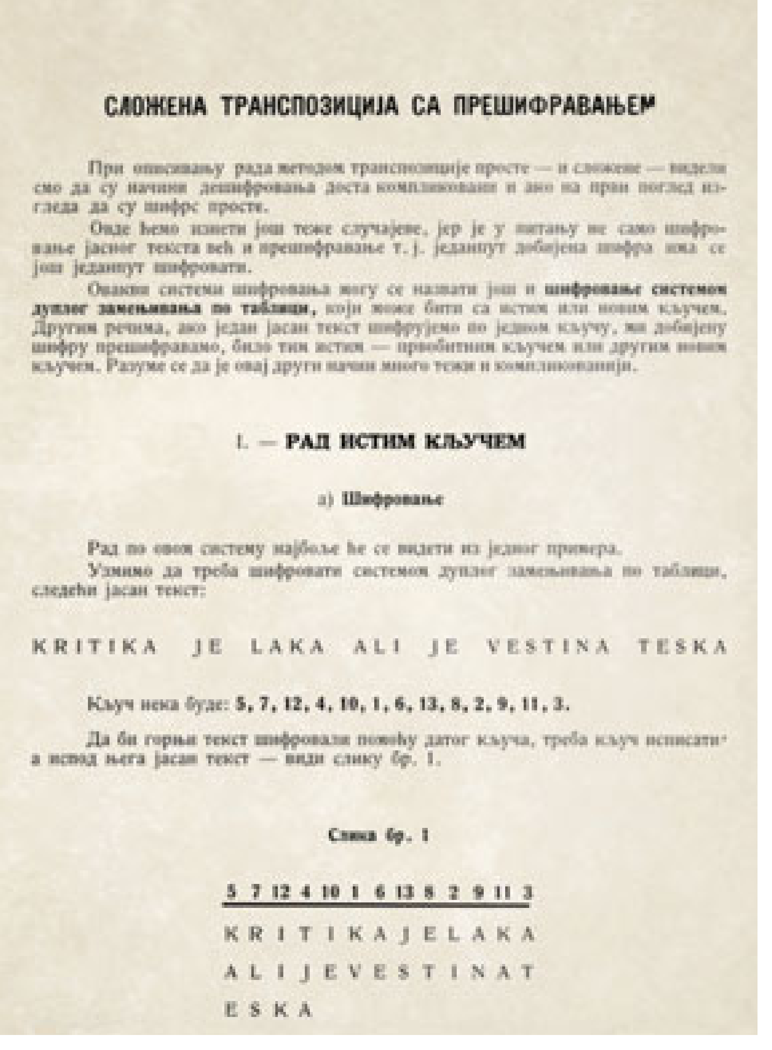
In this section we are going to present some illustrative examples of educational materials for working in the field of cryptography, as well as for techniques for breaking certain codes.

In accordance with all that was previously said, from among the volumes in the series *Cryptography: Code School,* the following is presented as illustrative material:

- Volume: *Basic Concepts*;
- Volume 10: *Complex Transposition with Double Encryption*;
- Volume 14: *An Introduction to the Methods of Encryption Using Key – Dictionary of Secret Correspondence*.

> The invention of secret correspondence is anything but new. Cryptography has its origin from the ancient times, the only difference being that those former methods, systems and modes of usage were completely different from the current ones.
>
> Cryptography, or secret correspondence, is derived from the Greek word *kryptos*, meaning "to hide", and *-graphy*, meaning "to write".
>
> Cryptography or secret correspondence is in its essence, its purpose, and its main objective, a very sensitive and delicate subject.
>
> Sensitive – because the precision of work must be absolutely and fully guaranteed, and delicate, because the very content it conveys is of the most confidential nature, whose disclosure in most cases may have grave and fatal consequences. Greatest care must be taken of the organization, work and secrecy of such correspondence.
>
> Secret correspondence is regularly used by military institutions in times of peace and war alike.
>
> Diplomatic representatives must daily report to their government about the particularly important and confidential matters they found out in the states they are accredited in, which they regularly do by using a secret or code name.

Figure 9: Content illustration of "Cryptography – general terms" notebook, from the School for Encryption Training, reference [3]



СЛОЖЕНА ТРАНСПОЗИЦИЈА СА ПРЕШИФРАВАЊЕМ

При описивању рада методом транспозиције просте — и сложене — видели смо да су начини дешифровања доста компликовани и ако на први поглед изгледа да су шифре просте.

Овде ћемо изнети још теже случајеве, јер је у питању не само шифровање јасног текста већ и прешифравање т. ј. једанпут добијена шифра има се још једанпут шифровати.

Овакви системи шифровања могу се назвати још и **шифровање системом дуплог замењивања по таблици,** који може бити са истим или новим кључем. Другим речима, ако један јасан текст шифрујемо по једном кључу, ми добијену шифру прешифравамо, било тим истим — првобитним кључем или другим новим кључем. Разуме се да је овај други начин много тежи и компликованији.

1. — РАД ИСТИМ КЉУЧЕМ

а) Шифровање

Рад по овом систему најбоље ће се видети из једног примера.

Узмимо да треба шифровати системом дуплог замењивања по таблици, следећи јасан текст:

K R I T I K A J E L A K A A L I J E V E S T I N A T E S K A

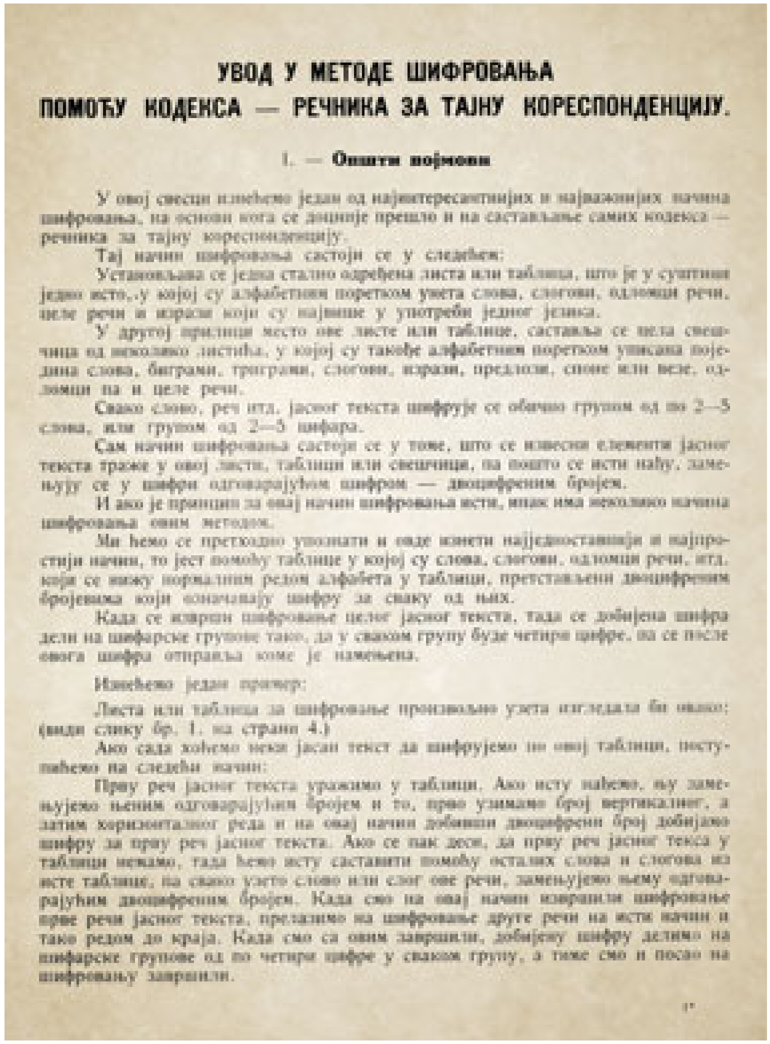
Кључ нека буде: **5, 7, 12, 4, 10, 1, 6, 13, 8, 2, 9, 11, 3.**

Да би горњи текст шифровали помоћу датог кључа, треба кључ исписати а испод њега јасан текст — види слику бр. 1.

Слика бр. 1

5	7	12	4	10	1	6	13	8	2	9	11	3
K	R	I	T	I	K	A	J	E	L	A	K	A
A	L	I	J	E	V	E	S	T	I	N	A	T
E	S	K	A									

Figure 10: Content illustration of Notebook no. 10, from School for Encryption Training, reference [3]



УВОД У МЕТОДЕ ШИФРОВАЊА
ПОМОЋУ КОДЕКСА — РЕЧНИКА ЗА ТАЈНУ КОРЕСПОНДЕНЦИЈУ.

I. — **Општи појмови**

У овој свесци изнећемо један од најинтересантнијих и најважнијих начина шифровања, на основи кога се доцније прешло и на састављање самих кодекса — речника за тајну кореспонденцију.

Тај начин шифровања састоји се у следећем:

Установљава се једна стално одређена листа или таблица, што је у суштини једно исто, у којој су алфабетним поретком унета слова, слогови, одломци речи, целе речи и изрази који су највише у употреби једног језика.

У другој прилици место ове листе или таблице, саставља се цела свешчица од неколико листића, у којој су такође алфабетним поретком уписана поједина слова, биграми, триграми, слогови, изрази, предлози, споне или везе, одломци па и целе речи.

Свако слово, реч итд. јасног текста шифрује се обично групом од по 2—5 слова, или групом од 2—5 шифара.

Сам начин шифровања састоји се у томе, што се извесни елементи јасног текста траже у овој листи, таблици или свешчици, па пошто се исти нађу, замењују се у шифри одговарајућом шифром — двоцифреним бројем.

И ако је принцип за овај начин шифровања исти, ипак има неколико начина шифровања овим методом.

Ми ћемо се претходно упознати и овде изнети најједноставнији и најпростији начин, то јест помоћу таблице у којој су слова, слогови, одломци речи, итд. који се нижу нормалним редом алфабета у таблици, претстављени двоцифреним бројевима који означавају шифру за сваку од њих.

Када се изврши шифровање целог јасног текста, тада се добијена шифра дели на шифарске групове тако, да у сваком групу буде четири цифре, па се после овога шифра отправља коме је намењена.

Изнећемо један пример:

Листа или таблица за шифровање произвољно узета изгледала би овако: (види слику бр. 1. на страни 4.)

Ако сада хоћемо неки јасан текст да шифрујемо по овој таблици, поступићемо на следећи начин:

Прву реч јасног текста тражимо у таблици. Ако исту нађемо, њу замењујемо њеним одговарајућим бројем и то, прво узимамо број вертикалног, а затим хоризонталног реда и на овај начин добивши двоцифрени број добијамо шифру за прву реч јасног текста. Ако се пак деси, да прву реч јасног текса у таблици немамо, тада ћемо исту саставити помоћу осталих слова и слогова из исте таблице, па свако узето слово или слог ове речи, замењујемо њему одговарајућим двоцифреним бројем. Када смо на овај начин извршили шифровање прве речи јасног текста, прелазимо на шифровање друге речи на исти начин и тако редом до краја. Када смо са овим завршили, добијену шифру делимо на шифарске групове од по четири цифре у сваком групу, а тиме смо и посао на шифровању завршили.

1*

Figure 11: Content illustration of Notebook no. 14, from School for Encryption Training, reference [3]

REFERENCES

[1] Shannon, C.E. “Communication Theory of Secrecy Systems”, *Bell System Technical Journal*, 1949, 28, 656–715.

[2] Sistem (za šifrovanje). Sveske 1, 1a, 2, 2a, 3, 3a, 4, 4a, 5, 6, 6a, 7, 7a, 8, 9, 10, 10a, 11, 12, 13, 14, 15, 16, 17, 18, Ministarstvo vojske i mornarice Kraljevine Jugoslavije, (ca. 1930–1940).

[3] Kriptografija – škola za obuku na šifri, sveske 1–15, Otsek za šifru, Obaveštajno odeljenje, Đeneralštab Kraljevine Jugoslavije, (ca. 1930–1940).

MATHEMATICAL LEGACY OF MIHAILO PETROVIĆ, APPENDICES

TADIJA PEJOVIĆ AND THE LOGICAL BRANCH OF MIHAILO PETROVIĆ ALAS' SUCCESSORS

Zoran OGNJANOVIĆ
Mathematical Institut of SASA

"It is important to highlight a significant role Mihailo Petrović played during his work at the Serbian Grand School and later at the University in Serbia and Yugoslavia. Immediately upon his arrival from Paris to Belgrade and his appointment as Grand School professor in 1894, he set out to raise the quality of teaching, to develop scientific work and create scholars. He believed that no serious science, and in particular no development of science in our country, was possible without scholars. Thus, though faced with numerous difficulties, he managed to develop and form the Belgrade School of Mathematics that was at a par with foreign universities. This was particularly evident between the two wars. To illustrate, I shall list the names of doctors of mathematical sciences from this period alone. They were: Mladen Berić (1912), Sima Marković (1913), Tadija Pejović (1923), Radivoj Kašanin (1924), Jovan Karamata (1926), Miloš Radojčić (1928), Dragoslav Mitrinović (1933), Danilo Mihaljević (1934), Konstantin Orlov (1934), Petar Muzen (1937), Dragoljub Marković (1938) and Vojislav Avakumović[171] (1939). As you can see, twelve doctors of mathematical sciences in a short time period is indeed an impressive number, especially given the strict criterion for obtaining the doctoral degree. It should also be noted that all of the above listed scholars became solid scientists and later also university professors." [TP1992, 260]

This is an excerpt from the writing of Tadija Pejović (1892–1982), professor at the Faculty of Sciences and Mathematics, University of Belgrade, and one of the survivors from the celebrated squad of 1,300 corporals in World War I. He was the third doctor of mathematics at the University of Belgrade and, at the same time, the first doctoral student of Mihailo Petrović after the Great War. In 1923, he defended his dissertation on differential equations entitled *New Cases of Integrability of a Significant Differential Equation.* For the remaining part of his career, in addition to scientific research, Pejović also engaged intensively in pedagogical work and in developing high level educational institutions. One of the doctoral students of Tadija Pejović was Slaviša Prešić (1933–2008) [SP2018]. In 1963, Prešić, later a renowned professor at the University of Belgrade, defended his dissertation entitled *A Contribution to the Theory of Algebraic Structures* at the Belgrade Faculty of Sciences and Mathematics, and soon became the key originator of our school of mathematical logic. He was the doctoral supervisor of many of our logicians: Koriolan Gilezan (*Some Generalizations of Pseudo-Boolean Programming,* 1971), Janez Ušan (*On a Class of Quasigroups*, 1971), Svetozar Milić (*A Contribution to the Theory of Quasigroups*, 1972), Nataša Božović (*Unsolvable Problems in Group Theory*, 1975), Žarko Mijajlović (*A Contribution to Model Theory and Boolean Algebras*, 1977), Gradimir Vojvodić (*A Contribution to Research of Mixed-Valued Predicate Calculus*, 1979), Dragić Banković (*Reproductive Solutions of Equations*, 1980), Milan Božić (*A Contribution to Semantic of Relevant Logics*, 1983), Branislav Boričić (*A Contribution to the Theory of Intermediate Propositional Logics*, 1984, co-supervisor Kosta Došen), Miodrag Kapetanović (*A Method of Semantic Tables*, 1996), etc. who set up research groups not only in Belgrade, but also in Novi Sad, Niš and Kragujevac. In Niš, Dragoslav Mitrinović (1908–1995), professor at the School of Electrical Engineering in Belgrade, was supervisor to Živko Tošić (*Analytical Representations of m-Valued Logical Functions over the Ring of Integers Modulo m*, 1972) and Lazar Đorđević (*On a New Class of Cubature Formulas,* 1978), who were later professors at the Faculty of Electronic Engineering and headed research in the area of application of mathematical logic in electronics. In this way, though not directly but more in a methodological sense, Petrović and Pejović contributed to the development of one of the leading scientific disciplines in Serbia.

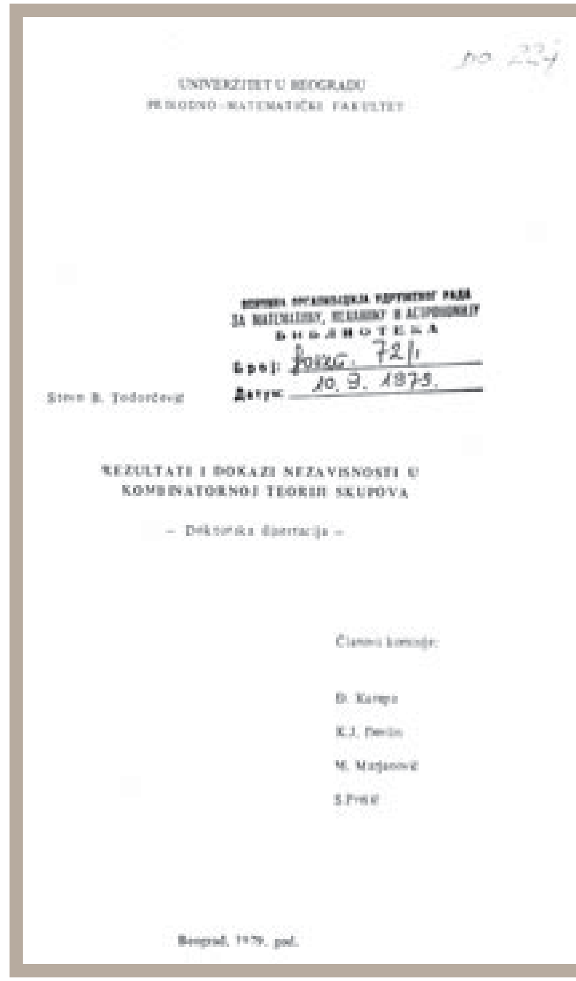
The development of mathematical logic in Serbia was the subject of a conference held at the Faculty of Mathematics in Belgrade in 2010 [ILS2010], while a comprehensive study of the same topic is given in the text by a group of authors [LiS2013]. Although as far back as in 1885 Ljubomir Nedić (1858–1902) defended in Leipzig the doctoral dissertation entitled *Die Lehre von der Quantifikation des Prädikats in der neueren Englischen Logic,* for more than half a century thereafter there was no serious study of mathematical logic in Serbia. It is only in the latter half of the 1950s that Vladeta Vučković (1923–2012), after obtaining his PhD under the supervision of Vojislav Avakumović (1910–1990), embarked upon research on the theory of recursive functions; in early 1960s, he published his first papers in magazines *Zeitschrift für mathematische Logik und Grundlagen der Mathematik* and *Publications de l' Institut Methématique*. At the time, professor Mihajlo Marković (1923–2010) and Svetlana Knjazeva, a then teaching assistant who went on to become a professor, held the first course in mathematical logic at the Faculty of Philosophy, University of Belgrade. Still, the setting up of the Seminar for Algebra and Logic in mid-1960s represented the most significant step forward in the development of this area. At the initiative of Slaviša Prešić, a group of mathematicians and philosophers (Svetozar Milić, Dušan Adamović, Dragica Krgović, Marica Prešić, Branka Alimpić, Aleksandar Kron and Svetlana Knjazeva) formed the Seminar within the Department of Mathematics at the Faculty of Sciences and Mathematics in Belgrade. As of 1970, the Seminar was moved permanently to the Mathematical Institute SASA and its name was changed to Seminar for Mathematical Logic. In this period, Slaviša Prešić also published a book entitled *Elements of Mathematical Logic* [SP1968] which became a compulsory read at faculties where curricula included mathematical logic. It can definitely be said that all domestic researchers in this area were at one point or another members of the Seminar; as a result, more than seventy doctoral dissertations in the area of mathematical logic were defended [MB2012]. After Prešić, the Seminar was chaired by Žarko Mijajlović, Zoran Marković, Aleksandar Kron, Kosta Došen, Đorđe Vukomanović and Predrag Tanović. At regular weekly meetings, members delivered lectures presenting their results, reviews, overviews of important monographs and longer specialised courses. Meetings very often featured guest lecturers, including leading world scholars in this area such as L. Henkin, J. Keisler, J. Burgess, J. van Benthem, H. Barendregt, etc. Over time, the Seminar for Mathematical Logic became the sprouting ground for a number of specialized seminars, some of which are still active today, i.e. the Seminar for Probability Logic (chaired by Miodrag Rašković) and the Seminar for General Proof Theory (chaired by Kosta Došen) in Belgrade, and the Seminar of the Centre for Mathematics and Statistics (chaired by Silvia Gilezan) in Novi Sad. Another important impulse to the development of mathematical logic in Serbia undoubtedly came from the postdoctoral studies of Žarko Mijajlović, professor at the Belgrade Faculty of Mathematics and doctoral student of Slaviša Prešić, in the USA where, at Wisconsin University, he worked for a period of time with one of the greatest living logicians, J.H. Keisler. After his return to Belgrade, Mijajlović, inspired by his freshly acquired knowledge, organized a large number of specialized courses, initiated research in the area of model theory, non-standard analysis, generalized quantifiers etc., and opened the door for a number of domestic logicians to make study visits throughout the USA, Canada and Europe. Žarko Mijajlović was the supervisor to the next generation of logicians, including: Aleksandar Jovanović (*A Contribution to the Theory of Ultraproducts*, 1982), Slobodan Vujošević (*A Contribution to the Theory of Heyting Algebras*, 1982), Miodrag Rašković (*Logics with Measure in Leibniz's Universe*, 1983), Rade Živaljević (*Ten Etudes about*

Mihailo Marković
(1923–2010)

Đuro Kurepa (1907–1993)
(*Pregled NCD*, 2012)

Hyperfiniteness, 1983), Milan Grulović (*Forcing in Model Theory*, 1984), Milenko Mosurović (*On the Complexity of Description Logics with Modal Operators*, 2000), Predrag Janičić (*Building Decision Procedures into Systems for Automated Reasoning*, 2001) etc. who then developed the logical school further in different directions. Thus, the group studying probability logics, set up by Miodrag Rašković first in Kragujevac and later in Belgrade, at the Mathematical Institute SASA (Radosav Đorđević, Zoran Ognjanović, Zoran Marković, Nebojša Ikodinović, Aleksandar Perović, Dragan Doder, Angelina Stepić-Ilić) tops global research in this area, while Predrag Janičić chairs a group that deals with automated reasoning at the Belgrade Faculty of Mathematics (Filip Marić, Mladen Nikolić, etc.).

In addition to direct scientific successors of Mihailo Petrović, Tadija Pejović and Slaviša Prešić, an important role in the development of our school of mathematical logic was also played by academicians Đuro Kurepa (1907–1993) and Mihailo Marković (1923–2010), as well as by Aleksandar Kron (1937–2000), professor at the Faculty of Philosophy in Belgrade, Kosta Došen (1954–2017), member of the Mathematical Institute SASA and professor at the Faculty of Philosophy in Belgrade, and Zoran Marković, long-time director of the Mathematical Institute SASA. Đuro Kurepa achieved important results in the set theory already in his dissertation *Ensembles ordonnées et ramifiés*, which he defended at Sorbonne in 1935. Through his work on the set theory and the foundations of mathematics, Kurepa achieved global fame and made an immense impact in the territory of former Yugoslavia in particular. A number of mathematical notions, such as Kurepa tree and Kurepa's function, were named after him. Kurepa was one of few participants from Eastern Europe at the *International Symposium on the Theory of Switching* organised in 1957 at Harvard University by the renowned H. Aiken, where in a paper entitled *Sets-Logics-Machines* he discussed multiple-valued logics. Under Kurepa's supervision, academician Stevo Todorčević, who today is one of our most highly appreciated mathematicians globally, defended his dissertation entitled *Results and Independence Proofs in Combinatorial Set Theory* in 1979. The contribution of scholars from the Faculty of Philosophy to the development of mathematical logic began with the work of academician Mihailo Marković who defended his dissertation entitled *Formalism in Contemporary Logic* in Belgrade in 1955 and, a year later, received his PhD in London as well, where his thesis, *The Concept of Logic,* was promoted by A. Ayer. From 1962 until 1975, he headed the Department of Logic and Methodology. Aleksandar Kron who, together with Prešić, formed the school of modern mathematical logic in Serbia, studied intuitionism at the University of Amsterdam in 1963–1964 under the supervision of E. W. Beth and A. Heyting. Before an examination board consisting of Mihailo Marković, Bogdan Šešić, Svetlana Knjazeva and Slaviša Prešić, Kron defended his dissertation entitled *Relation between Polyvalent Logics and Probability Theory* in 1965 at the Faculty of Philosophy in Belgrade, where he soon became assistant professor and later also professor of logic. From early 1970s, Kron began studying relevance logics, cooperating with A. R. Anderson and N. D. Belnap in Pittsburgh, while later he also studied modal and quantum logics, proof theory, etc. He organised specialized courses in these areas and helped the development of the next generation of logicians. Under the guidance of Aleksandar Kron, Kosta Došen graduated from the Faculty of Philosophy in 1977 in the area of relevance logics, and obtained his PhD in 1981 at the University of Oxford, with a doctoral thesis on the proof theory entitled *Logical Constants: An Essay in Proof Theory*, supervised by M. Dummett and D. Scott. In 1972–1973, Kron, together with Zoran Marković, held a series of courses on semantics for intuitionistic logic at the Seminar for Mathematical Logic. The result of this cooperation was embodied in Zoran Marković's master thesis in 1974, supervised by Aleksandar Kron. Continuing his work in this area, in 1979 Zoran Marković defended his doctoral dissertation, *Model Theory of Intuitionistic Logic,* at the University of Pennsylvania, supervised by S. Weinstein. Kosta Došen achieved globally recognised results in proof theory, category theory and substructural logics, which is why he was selected for entry in the International Directory of Logicians [IDL2009]. The following dissertations in Belgrade and Novi Sad were supervised by Došen: Branislav Boričić (*A Contribution to the Theory of Intermediate Propositional Logics*, 1984, co-supervisor Slaviša Prešić), Silvia Gilezan (*Intersection Types in Lambda Calculus and Logic*, 1993, co-supervisor H. P. Barendregt), Zoran Petrić (*Equalities of Derivations in Categorial Proof Theory*, 1997), Mirjana Borisavljević (*Sequents, Natural Deduction and Multicategories*, 1997) and Miloš Adžić (*Gödel on Axiomatization of Set Theory*, 2014).


UNIVERZITET U BEOGRADU
PRIRODNO-MATEMATIČKI FAKULTET

Stevo B. Todorčević

REZULTATI I DOKAZI NEZAVISNOSTI U KOMBINATORNOJ TEORIJI SKUPOVA

– Doktorska disertacija –

Članovi komisije:

Đ. Kurepa

K.J. Devlin

M. Marjanović

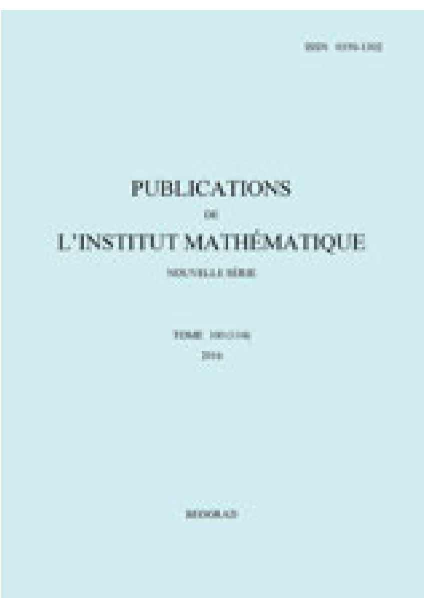
S. Prešić

Beograd, 1979. god.

The cover page of Stevo Todorčević's doctoral dissertation *Results and Evidence of Independence in Combinatorial Set Theory*, Faculty of Natural Sciences and Mathematics, University of Belgrade, board: Đ. Kurepa, K. Devlin, M. Marjanović, S. Prešić, Belgrade 1979.

Meeting of the Logical seminar in hall 2 in SASA.
First row from the left: Đorđe Vukomanović, Aleksandar Kron, Miodrag Rašković.
Second row: Miloš Laban, Radoš Bakić, Aleksandar Krapež, Zoran Marković, Žarko Mijajlović.
Third row: Dragi Radojević, Slaviša Pešić

In this way, mathematical logic in Serbia developed as a product of interaction of two currents – one mathematical and the other philosophical. Research originating from doctoral dissertations and topics presented at the Seminar covers more or less the full scope of mathematical logic. Researchers have worked in the areas of model theory, proof theory, category theory, formal computability theory, set theory, Boolean algebras, non-standard analysis, intuitionistic and intermediate logics, modal logics, probability logics, lambda calculus and computational logics, switching theory and multiple-valued logics as applied in computer engineering, reversible logics, automated theorem proving, etc. As a result of internationally recognised results, and papers and monographs published in the above areas, Serbia is highly ranked in the *Scimago* country rankings published since 1995 by the University of Granada. Hence, in the area of mathematical logic, Serbia is mostly ranked 30th –35th in the world, while in 2015 it even shared the 21st –22nd position on the list, which is possibly the best result ever achieved by any of our sciences. This success is all the greater as *Scimago* rankings only take into account the researchers working in Serbia, while a large number of logicians educated in our country (Boban Veličković, Valentina Harizanov, Ilias Farah, Đorđe Čubrić, Željko Sokolović, Sava Krstić, Žikica Perović, etc.) now permanently occupy prestigious positions as professors worldwide. International cooperation is also taking place through projects, starting from "Types for Proofs and Programs," part of the European Sixth Framework Programme



Cover page of *Publications de l' Institut Methematique*

Kosta Došen and Peter Schroeder-Heister, 2016 (Family legacy of Kosta Došen)

(2002–2003), and *TEMPUS* project "Doctoral School towards European Knowledge Society" (2006–2008), through to "Computational Logics and Higher Algebra" (2016–2017), a bilateral project of Serbia and France, and a COST-action project "The European research network on types for programming and verification" (2016–2020) where research groups from Serbia were coordinated by Silvia Gilezan.

[TP1992] Тадија Пејовић, *Моје успомене и доживљаји 1892–1945*, Београд, 1992.

[IDL2009] D. Gabbay and J. Woods, edts., *The International Directory of Logicians*, College Publications, London, 2009.

[ILS2010] Жарко Мијајловић, едитор, Зборник конференције *History of Logic in Serbia, Faculty of Mathematics*, Универзитет у Београду, 14–15. 6. 2010, Београд, Србија, Преглед НЦД-а 20, 2012, http://elib.mi.sanu.ac.rs/pages/browse_issue.php?db=ncd&rbr=20&start=0

[MB2012] Мирјана Борисављевић, *Doctoral Dissertations in Logic*, Преглед НЦД-а 20, 95–104, 2012, http://elib.mi.sanu.ac.rs/files/journals/ncd/20/ncd20095.pdf

[LiS2013] Mirjana Borisavljević, Silvia Ghilezan, Predrag Janičić, Aleksandar Krapež, Miloš Kurilić, Žarko Mijajlović, Zoran Marković, Zoran Ognjanović, Jovanka Pantović, Zoran Petrić, Miomir S. Stanković, Radomir S. Stanković, Ivan Stojmenović, Đorđe Vukomanović, *History of Mathematical Logic in Serbia* in: Logic in Central and Eastern Europe History, Science, and Discourse, Andrew Schumann (edt), University Press of America, 470–495, 2013.

[SP2018] Жарко Мијајловић, Дигитални легат професора Славише Прешића, Математички факултет Универзитета у Београду, http://alas.matf.bg.ac.rs/~websites/digitalnilegatpresic/

TADIJA PEJOVIĆ AND THE ROLE OF MIHAILO PETROVIĆ ALAS AND THE MATHEMATICAL CLUB IN THE FORMING OF THE MISASA

In his book of memories, Tadija Pejović [TP1992] highlighted the role that Mihailo Petrović played in the forming of the Mathematical Institute SASA, the formal setting up of which in 1946 the latter did not, however, live to see:

> "The scientific work developed on a collective – individual basis, as each of us worked in his own area of expertise, but we all met once a month to discuss the scientific results we had achieved. In these meetings, the delivered lectures were discussed and views were expressed regarding the publication of a given lecture. The group before which individuals expounded their results comprised professors of Theoretical and Applied Mathematics and Astronomy at the University of Belgrade. They were: Mihailo Petrović, Nikola Saltikov, Tadija Pejović, Jovan Karamata, and later also Miloš Radojčić – academic staff of Theoretical Mathematics at the Faculty of Philosophy; Bogdan Gavrilović, Radivoje Kašanin, Petar Zajončovski and later Gojko Vujaklija – academic staff of Theoretical Mechanics at the Technical Faculty; Milutin Milanković, Anton Bilimović and Vjačeslav Žardecki – academic staff of Applied Mathematics at the Faculty of Philosophy; Ivan Arnovljević and Jakob Hlitčijev – professors of Mechanics at the Technical Faculty; Vojislav Mišković – professor of Astronomy at the Faculty of Philosophy. As of 1926, this group came to comprise the Mathematical Club of the University of Belgrade... Though the club had no written rules, it held regular monthly meetings, followed by joint dinners... The club was headed by Anton Bilimović... Over dinner, we shared academic jokes and quips. We organised dinners at taverns offering grilled meat specialties or fish and cheese pie. Very often dinners were also organised at Petrović's vineyard on the Topčider Hill" [TP1992, 235–237]
>
> "In the period from 1930 until 1941, the Mathematical Club of the University of Belgrade represented a very serious group, with no written rules whatsoever... With its magazine, *Publications Mathématiques de l'Université de Belgrade,* the Mathematical Club laid the foundations of today's Mathematical Institute. Hence, the period of work and development of the Mathematical Club from 1932 until 1941 can be considered to represent the first stage of work and development of today's Mathematical Institute in Belgrade." [TP1992, 265–257]

[TP1992] Тадија Пејовић, *Моје успомене и доживљаји 1892–1945*, Београд, 1992.

MIHAILO PETROVIĆ, ALGEBRAIC GEOMETRY AND DIFFERENTIAL EQUATIONS*

Vladimir DRAGOVIĆ
Mathematical Institute of SASA, Belgrade
The University of Texas at Dallas

This paper is a short excerpt from the lecture the author delivered at the first joint session of all three departments of Mathematical Institute of SASA on 22 May 2018, and within the Round table on the scientific achievements of Mihailo Petrović that was held on the same day in the SASA Gallery. Besides the author of this paper, academicians Stevan Pilipović and Gradimir Milovanović, co-presidents of the ceremony marking the 150th anniversary of the birth of Mika Alas, were also speaking.

It would be difficult to find another example of a man and a city with an elementary school, a gymnasium and a tavern named after that man. In this city, a real PhD also became a master fisherman. What do we know about the man and his achievements?

We shall elaborate on a series of Mihailo Petrović's achievements incorporating algebraic geometry and analytical theory of differential equations. Those results were achieved within the time frame of almost half a century, from the first half of the 1890s till the end of the 1930s,

---

* The paper was drafted within the project 174020 "Geometry and Topology of Manifolds, Classical Mechanics and Integrable Dynamical Systems" of the Ministry of Education, Science and Technological Development of the Republic of Serbia. It was supported by the University of Texas at Dallas.

under the strong influence of Petrović's professors Emile Picard and Charles Hermite. Some of them are noted in well-known textbooks and monographs published in France and Russia. Mihailo Petrović passed away in 1943 in Belgrade. In the same city, half a century after Petrović's death, Mathematical Methods of Mechanics Seminar was founded, which primarily focuses on the relation of algebraic geometry and differential equations. Today, twenty five years after the Seminar's foundation, we have just left the door of Petrović's scientific treasury ajar, while wiping the dust off the old manuscripts, thus getting an insight into the work of the great master. We cannot help but wonder how ideas and scientific discoveries have been disseminated and how they travel through space and time.

We live under the well nurtured illusion that the passage of time brings nothing but progress and that scientific progress is a continuous process. Without denying the progress of both science and society, one has to bear in mind that, with the passage of time, many scientific facts, techniques and theories are likely to be forgotten, upstaged, to lose their place in curricula and give way to the latest and more modern scientific discoveries. In science, just like in other areas of creation, there is a trend, which comes and goes, to let, in an inexplicable and unjustified way, valuable achievements fade into oblivion and obscurity. It is our impression that a similar destiny befell the scientific legacy of Mihailo Petrović, regardless of his popularity during his life and after his death, regardless of the number of his students and his students' students, and in spite of his brilliant achievements. We are looking at an impressive list of Petrović's eleven doctoral students and almost nine hundred followers, according to *Mathscinet* data (see detailed overview in professor Pilipović's paper [Pil2018]). This allows the conclusion that the past one century and a half has seen the domination of Mihailo Petrović over Serbian mathematics in the first 75 years, whereas Mika's students and his students' students dominated Serbian mathematics in the remaining 75 years.


A bust of Mihailo Petrović Alas in front of his birthplace, the work of Aleksandar Zarin, 1969

Regardless of this, I can testify that during my student days back in the 1980s, it was only at the lectures within regular course in differential equations held by professor Ljubomir Protić that I could get an insight into the specific scholarly endeavor of Mihailo Petrović and its place in modern science. In September 1988, at Moscow State University, professor Dubrovin asked me to tell him about the significant achievements of Serbian mathematics during my postgraduate interview. With relief, I remembered professor Protić's lectures and talked about the Petrović's method [Pet1896, Pet1899Annalen], which emerged two decades before the similar ideas of great Russian mathematician and mechanic Chaplygin [Chap1919]. (Professor Milorad

Panel at “Mihailo Petrović Alas” Elementary School

Photograph and personal belongings of Mihailo Petrović from “Mihailo Petrović Alas” Elementary School

Bertolino had explored Petrović’s precedence in this matter for more than 15 years, see [Ber1957, Ber1967, BerTrif1971], [Prot2015], [Pea1886]).

Therefore, it is worth commending the projects that entailed publication of the collected works of Mihailo Petrović in 1999 [Pet1999] and this year’s ceremony marking the 150th birth anniversary of our distinguished mathematician, as our collective effort against oblivion and our care for preserving national, but also world scientific legacy.

When preparing myself for my lectures, I avoided consulting *Collected Works* on purpose, prior to choosing the material. For the purpose of this overview, I selected three achievements of Mihailo Petrović in the field of algebraic differential equations. Conditionally, we will name them Theorems A, B and C. As it is well known, Petrović’s scope of work and his scientific interests were very broad, nevertheless his theory of algebraic differential equations stands out as his biggest achievement. It turned out that Theorems B and C are from his papers in *Acta mathematica* from 1899 and *Publications* from 1938, which were not included in the *Collected Works* [Petr1999]. Theorem A resulted from a PhD dissertation [Petr1894] that Petrović defended in 1894 at Sorbonne, under the tutelage of Charles Hermite and Emile Picard, with Paul Painleve as the third committee member.

Theorem A deals with the nature of the solution of a generalization of the Riccati differential equation.

An old French proverb says: *anyone can differentiate, but only those who know can integrate*. It was great French mathematician Liouville who, back in the 1830s, brought up the question of the nature of functions that are integrals of some given functions.The idea of studying the fixed and movable singularities of differential equations is dating back to Fuchs. Fixed are those singularities that depend on the differential equation itself and are common for all its solutions,

Cover page of the Book of notes from Petrović's lectures about simple differential equations, around 1910–1914 (Library of MISASA, 3262)

Paul Painlevé (1863–1933), French mathematician

while the movable ones are the ones which change ("move") depending on the initial conditions and which depend on various solutions. The local solutions of differential equations can be analytically extended, but this procedure can lead to multivalued function as a solution of the given equation. Multivaluedness can be tamed by introducing the appropriate Riemann surface as the function domain. If an equation has no *movable* critical singularities in the neighbourhood of which multivaluedness is generated, then such Riemann surface can be chosen as common for all solutions. During the years preceding Petrović's arrival to Paris in 1889, the following fundamental theorems concerning first order differential equations were proved. Henri Poincare and Lazarus Fuchs proved in 1884/85 that among first order equations linear ones stand out, Riccati and Weierstrass functions, as the only ones without movable critical points. Paul Painleve, in his brilliant PhD dissertation in 1887, showed that first order equations cannot have solutions with movable essential singularities. In the year when Petrović arrived in Paris, two very important papers were published [Kow1889, Picard1899]. Both papers are as relevant today as ever and have served as an inspiration to our Seminar and also inspired some of our recent papers. The paper authored by Kowalevski directed analytical theory of differential equations towards applications in mechanics and created foundations of the so-called Kowalevski-Painleve analysis: to describe classes of equations the general solutions of which have certain given analytical characteristics. Kowalevski herself was dealing with the systems of Euler-Poisson equations that describe the dynamics of movement of a heavy firm object around an immovable point under the influence of the force of gravity. She regarded time as a complex variable for the first time in history in a mechanical problem. By examining conditions under which general solutions would have only poles as movable singularities, Kowalevski discovered a new integrable case which is named after her. For this paper, Kowalevski received the Bordin Prize from the French Academy.

In the mentioned Picard's paper, among other things, an algebraic-geometrical solution of the equation, which will be a special case of the future so-called Painleve equations, was found. These second order equations with characteristic that general solutions only have poles as movable singularities were introduced by Painleve at the beginning of the next century [Pain1902], thus marking the beginning of the new era of systematic studying of second order equations with the so-called Painleve characteristic. Painleve's students continued further research concerning second order Painleve eqations, see for instance [Fuchs1906, Gamb1910]. These papers are as relevant today as ever, and have served as a great inspiration for our recent papers.

THEOREM A

Before we formulate the first Petrović's result which we want to show, let us be reminded of the basic results on the Riccati equations. Those are first order equations, where their right side is a quadratic polynomial P per the unknown function w, with coefficients which are meromorphic functions of the independent variable z:

$$w'=P(w,z)$$

If one particular solution of the Riccati equations is known, the equation is reduced to a linear first order equation. Otherwise it is reduced to linear second order equations and has no movable points. If three particular solutions are known w_1, w_2, w_3, then along any of solutions w cross-ratio is constant $w(z):w_1(z):w_2(z):w_3(z)$.

Petrović looked into the generalization of the Riccati equation

$$w' = \frac{P(w,z)}{Q(w,z)}$$

where P, Q are polynomials in w, the coefficients of which are algebraic functions of z.

First he showed that without losing generality, it can be reduced to equations with the form

$$w' = \frac{P_{n+2}(w,z)}{Q_n(w,z)},$$

where P_{n+2}, Q_n are polynomials in w of degree n+2 and n, respectively. Then he decomposed the problem into four sub-cases:

1. case: polynomial equation Q=0 allows more than two different roots $w_i=f_i(z)$;
2. case: polynomial equation Q=0 allows exactly two different roots;

3. case: polynomial equation $Q=0$ allows exactly one root;
4. case: polynomial Q does not contain w.

Petrović Theorem A (1894): *In case 1 all single-valued solutions are rational. In case 2 all single-valued solutions are reduced to one transcendent function. In case 3 all single-valued solutions are reduced to at most two transcendent functions. Case 4 is in line with the Riccati equation and all solutions are reduced to three transcendent functions.*

In his proof, Petrović skillfully used the known Picard theorems from a complex analysis and the so-called Lemma on critical points:

If the function $f(z,w)$ reaches infinite value at a certain point (z_0,w_0) and if $1/f$ is holomorphic in a neighborhood of the point, the z_0 is a movable critical algebraic point of the equation $w'=f(z,w)$.

Under the condition that P,Q are polynomials in w, the coefficients of which have finitely many isolated singularities, Golubev proved the following theorem:

Golubev Theorem (1911): *If the generalized Riccati equation has three rational solutions, then each single-valued solution is rational.*

Applying the subtle theory of function growth, Malmquist rounded Petrović's research with the following result:

Malmquist Theorem (1914): *If the generalized Riccati equation is not reduced to a Riccati equation, then each single-valued solution is rational function.*

This improves Petrović's achievements in cases 2 and 3. Except in the works of Golubev [Gol1911] and Malmquist [Mal1914], the mentioned Petrović's Theorem A occupies a significant place in the famous monographs authored by Picard and Golubev [Pic1908, Gol1950].

THEOREM B

Before we state the basic result of the only Petrović's paper published in *Acta mathematica*, let us mention two relatively similar results which preceded it, concerning the equations which do not explicitly depend on an independent variable. The aforementioned Weierstrass equation has this form:

$$(z')^2=P_3(z).$$

Its solutions are given in terms of the so-called Weierstrass P-function, which is a double periodic (i.e. elliptic) meromorphic function. The other result comes from one of Petrović's mentors, Hermite:

> **Theorem (Hermite):** *If the solutions of the equation with the form $Q(w,w')=0$, where Q is a polynomial in no w,w' do not have any movable critical points, then they set the curve of genus 0 or 1. The solutions are either rational or depend rationally on exponential or elliptic functions.*

Petrović deals with the problem when equation $Q(w,w',\ \dots,\ w^{(n)})=0$, where Q is a polynomial, has elliptic, i.e. double periodic solutions. He found the necessary geometric conditions, using the polygon technology that he developed in his PhD dissertation [Petr1894]. Based on a polynomial Q polygon Π is constructed. The vertices of the polygon can be simple or multiple. To each multiple vertex D a characteristic polynomial k_D is assigned. The polynomial Q can be presented in the form of the sum s of addends with the form of

$$S_i\, y^{m_{0i}} y'^{m_{1i}} \dots y^{(p)m_{pi}},$$

where S_i are constants.

The vertices of the polygon are determined by formulas $(M_i,\ N_i)\, i=1,\ \dots,\ s.$

$$M_i=m_{0i}+m_{1i}+\cdots+m_{pi};$$

$$N_i=m_{1i}+2m_{2i}+\cdots+pm_{pi}.$$

It may happen that for various $i,\ j$ pairs coincide $(M_i,N_i)=(M_j,N_j)$.

Then the corresponding vertex is *multiple*. Petrović was developing theory of such polygons, which generalize the Newton polygons and are adjusted to algebraic differential equations, and made substantial contributions to the theory in his dissertation (see also [Stan1999]). "Characteristic B" which exists in the theorem, also stems from the mentioned thesis.

> **Theorem B (Petrović 1899):** *If the equation $Q=0$ has an elliptic solution, then the polygon has "characteristic B": either it has an edge with a negative integer angle coefficient or it has a multiple vertex D, so that the characteristic polynomial of vertex k_D has at least one whole-number root that is located between the angle coefficients of the edges that meet in the vertex D.*

Example: let us apply the Theorem to the equation $P_m(y'')=Q_n(y)$, where $P_m,\ Q_n$ are given polynomials of a variable of degree m,n respectively. Polygon Π is a triangle $\triangle ABC$ the vertices of which are $A(0,0),\ B(n,0),\ C(m,\ 2m)$.

Condition B here implies that the triangle △ABC has to be an acute triangle. The only edge with negative coefficient is BC, if $n>m$. Angle coefficient is $2m/(m-n)\in Z$. So, $n>m$, $2m/(m-n)\in Z$. Examples of the pairs are: $(m, n) \in \{(1, 2), (1, 3), (2, 4), (2, 6)$.

THEOREM C

In his paper [Pet1938], Petrović showed this very general and elegant theorem:

Theorem C [Petrović 1938] *There is a series of functions $u_1(t)$, $u_2(t)$,... so that for each algebraic differential equation by $x, y, y', \ldots y^{(p)}$ an independent variable x and a solution y can be expressed as the function of parameter t via the finite number of functions $u_i(t)$ in quadrature or without quadrature.*

Functions are determined as the solutions of series of systems, for each natural number p there are $2^{(p^2)}$ systems

$$\frac{du_i}{dt} = \sum_{j=1}^{p} f_i^j e^{u_j}, i, j, = 1, \ldots p, \text{ where } f_i^j \in \{0,1\}.$$

At the very end of his paper, Petrović noticed that theorems of similar type concerning the reduction of algebraic differential equations were proven by Appelrot and Lagutinski. Since he had no opportunity to see those papers and did not know their claims or proofs, he said that he could not use or quote them.

We were reading and applying Appelrot's papers with pleasure. Petrović's remark on Lagutinski reminded me of one nice encounter in Paris ten years ago, with French mathematician of Polish origin, Jean-Marie Strelcin. On that occasion, Professor Strelcin gave me the collected works of Lagutinski. With great enthusiasm he pointed at the importance of the almost forgotten brilliant papers of the talented Russian mathematician who died too early. Intrigued by Petrović's remark, I browsed Lagutisnki's papers and found references which Petrović was missing: [Lag1911, App1924].

Finally, let us remind that Petrović enjoyed a well-deserved international reputation, which is confirmed by the fact that he had been invited to deliver lectures at the International Congresses of Mathematicians five times: Rome 1908, Cambridge 1912, Toronto 1924, Bologna 1928 and Zurich 1932. Nowadays, probably no living mathematician was given such honor. It happened in the early 20th century, but very seldom.

OUR DAYS

The monograph titled *First Integrals with Restrictions* [Petr1999, vol. 2, 159–202] (*Integrales premieres a restriciones*) from 1929 is very similar by its content to the theory of integrable dynamic systems, which is the main topic of our Seminar. On pages 162–163 Petrović mentioned the canonical equations of dynamics (Hamiltonian systems) and formulated the fundamental Liouville theorem. Petrović's integrals with restrictions are very similar to invariant relations that were, among others, researched by Appelrot, and also by us. Our road to the theory of integrable systems would have been much easier if we had read this Petrović's paper in our young days.

Post festum. Further to my question from the lecture to find out more about Mika's older Parisian colleague Mijalko Ćirić, who is mentioned at *Mathscinet* as Charles Hermite's doctoral student, my colleague, Miloš Milovanović, PhD, responded. He kindly provided a link to Radoje Domanović's story *He Abolished Mechanics* from *Stradija* dated March 20, 1905:

https://domanovic.wordpress.com/tag/mijalko-ciric/

The story is very interesting for its juicy, sharp and as relevant as ever Domanović's language, but also because of its moral that being a student of a great mathematician and being a "Parisian" in Belgrade at the beginning of the 20th century did not automatically guarantee a successful academic career. This does not represent the views of the author of this article or of the Editorial board on historical persons that are mentioned in Domanović's text and on their creative achievements and merits.

Mika Alas Street in Dorćol

REFERENCES

[App1924] Аппелрот, Г., *Мат. сборник*, т. 32, 1924, 9–21.

[Ber1957] Бертолино, М., Неке функционалне неједнакости добијене применом Чаплигинове методе и упоређивање са резултатима М. Петровића, *Весник Друштва МФ*, Београд 1957, 87–94.

[Ber1967] Bertolino, M., Priorite de Michel Petrovitch relative an theoreme de Tchaplyguine sur les integralites differentielles du premier ordere, *Mat. vesnik*, 4, 1967, 165–168.

[BerTrif1971] Bertolino, M., Trifunović D., Sur le theoreme fondamentale de S. A. Čapligin sur l'integralite differentielle du premier ordre, *Mathematica balcanica*, 1971, 11–18.

[Chap1919] Чаплыгин, С. А., *Основания новоīо сӣособа ӣриближенноīо инӣеīрирования дифференциальных уравнений*, 1919.

[Fuch1906] Richard, F., *Sur quelques equations differentielles lineaires du second ordre, Comptes Rendus*, 1906.

[Gamb1910] Gambier, B., Sur les équations différentielles du troisième ordre et d'ordre supérieur dont l'intégrale générale a ses points critiques fixes, *Acta Mathematica*, 1910.

[Gol1950] Голубев, В. В., *Лекции ӣо аналиӣической ӣеории дифференциальных уравнений*, Москва 1950.

[Gol1911] Голубев, В. В., Об одном приложении теоремы Picard'a к теории дифф. уравнений, *Маӣ сборник*, 1911.

[Kow1899] Kowalevski, S., Sur le probleme de la rotation d'un corps solide autour d'un point fixe, *Acta Mathematica*, 1899.

[Lag1911] Лагутинский, Н., *Маӣ. сборник*, т. 27, 1909–1911, 420–423.

[Mal1914] Malmquist, I., Sur les fonctions a un nombre fini de branches definies par les equations differentielles du premier ordre, *Acta Mathematica*, V. 36, 1914.

[Pain1902] Painleve, P., Sur les equations differentielles du second ordre et d'ordre superieur dont l'integrale generale est uniforme, *Acta Mathematica*, 1902.

[Pea1886] Peano, G., *Sull' interabilita delle equazioni differenziali di primo ordine*, Torino, 1886.

[Petr1894] Petrovitch, M., *Sur les zeros et les infinis des integrales des equations differentielles algebriques*, Sorbonne, 1894.

[Petr1899Annalen] Petrovitch M., Sur une maniere d'etendre le theorem de la moyenne aux equations differentielles du premier ordre, *Math. Annalen*, 1899.

[Petr1899] Petrovitch, M., Sur une propriete des equations diff. a l'aide des fonctions meromorphs doublement periodiques, *Acta Mathematica*, 1899.

[Petr2018] Petrovitch, M,. On a property of differential equations integrable using meromorphic double-periodic functions, *Theoretical and Applied Mechanics*, 2018, vol. 45 (2018) Issue 1, 121–127, DOI: https://doi.org/10.2298/TAM1801121P.

[Petr1929] Petrovitch, M., *Integrales premieres a restriciones*, 1929.

[Petr1938] Petrovitch, M., Theoremes generaux sur les equations differentielles algebriques, *Publications de l'Institut Mathématique*, 1938, t. 6–7, 290–325.

[Petr1999] *Сабрана дела Михаила Пеӣровића*: у 15 томова, Завод за издавање уџбеника, Београд, 1999.

[Pic1899] Picard, E., Memoire sur la theorie des fonctions algebriques de deux variables, J. de *Math. pures appl*., 1899.

[Pic1908] E. Picard, *Traite d'analyse*, Paris 1908, III, 378.

[Pil2018] Пилиповић, С., *Академик Михаило Пеӣровић – доӣриноси у науци и насӣави*, 2018.

[Prot2015] Протић, Љ., Живот, дело и научни рад Михаила Петровића Аласа у: *Срӣски маӣемаӣичари*, зборник предавања одржаних на скупу Српски математичари у оквиру манифестације Мај месец математике 2012, САНУ, Универзитет у Београду, Завод за уџбенике, Београд 2015, 17–28.

[Stan1999] Станковић, Б., *Аналиӣичка ӣеорија диференцијалних једначина Михаила Пеӣровића*, Сабрана дела Михаила Петровића, том 1, 367–378.

GROUP FOR NUMERICAL MATHEMATICS IN NOVI SAD

Tradition of applied mathematics started by Mihailo Petrović Alas

Nataša KREJIĆ
Department of Mathematics and Informatics,
Faculty of Sciences, Novi Sad

The founding father of mathematics in Serbia, Mihailo Petrović Alas, devoted a significant portion of his career to applied mathematics. Mathematical modelling requires a broad spectrum of mathematical knowledge alongside a broad education, which academician Petrović had, and the focus of his work in the applied (or, to put it better, motivated) mathematical research was on phenomenology, i.e. defining the certain systems of equations that describe phenomena, regardless of objects or phenomena that are being modelled. Today, mathematical modelling is a well-developed and very important branch of mathematics, so this allows the conclusion that it also belongs to pioneering contributions by Mihailo Petrović Alas.

The Group for Numerical Mathematics at the Department of Mathematics and Informatics of the Faculty of Sciences in Novi Sad gathers mathematicians who deal with numerical mathematics, mathematical modelling and simulation at the University of Novi Sad. Almost all researches are the direct mathematical descendants of Mihailo Petrović Alas. The Group grew into an independent research group out of the Group for Mathematical Analysis during the 1980s, headed by professor Dragoslav Herceg. Professor Herceg earned his PhD on numerical

Faculty of Sciences, University of Novi Sad

solving of singularly perturbed differential equations, under the supervision of academician Bogoljub Stanković, in cooperation with professor Erich Bohl from the University of Konstanz in 1980. A few years later, Prof. Herceg founded the Group for Numerical Mathematics with numerous students, among whom is the author of this paper.

During the 1980s and 1990s, research was primarily focused on two topics – numerical solving of differential equations and numerical methods of linear algebra. In both areas, significant results were achieved and intensive cooperation was established with universities in Germany, Ireland and the USA. In the field of numerical solving of differential equations, research was dealing with singularly perturbed problems on non-equidistant discretization meshes which achieve parameter-uniform convergence, spline collocational methods and differential methods. Later on, the research was expanded to problems of partial differential equations with perturbations and finite element method. In the same period, the research focus in numerical linear algebra was on relaxation iterative methods for solving systems of linear equations. In this period a series of PRIM conferences on applied mathematics was organized (PRIM – Primenjena matematika (applied mathematics)).

At the beginning of this century, numerical optimization became an important part of research, in line with the trends in modern mathematics. Research is mainly focused on the problems of nonlinear optimization, with or without constraints, of continuous type and of large scale, with a strong emphasis on theoretical results on the convergence of iterative methods. The achieved results include a contribution to convergence theories of Quasi-Newton method, definition and the analysis of new Newton-type methods for solving nonlinear complementarity problems, as well as new methods for solving singular systems of nonlinear equations. Close cooperation was established with a research group headed by professor

Petrovaradin fortress in Novi Sad

Martinez from Campinas in Brazil and with many European research groups, which significantly expanded the spectrum of scientific topics.

At the same time, research in the field of numerical linear algebra, under the leadership of professor Ljiljana Cvetković, has been expanded to special classes of matrices (diagonally dominant matrices and their generalizations), which pave the way towards new results in various other areas of numerical linear algebra. Those were primarily results on eigenvalue localization by the Gershgorin-type theorems, which were achieved in cooperation with professor Richard Varga from Kent State University (Ohio), and which were later crowned by the construction of advanced algorithm for drawing a minimal Gershgorin matrix set. In the next few years, the cooperation was significantly expanded, resulting in joint papers published with colleagues from universities in Valencia and Saragosa (Spain), Poznan (Poland), Ioannina (Greece), Rostov (Russia), Beijing (China) and Kansas (USA). Besides results in localization of eigenvalues of matrix pencils, an efficient evaluation of norms of matrix inverses were achieved, which were later used for obtaining first results in the

Monument to Mileva Marić Einstein
(Author: Lazar Lazić, 2018)

Group photograph of mathematicians from Novi Sad from the 1970s. Second row, the tallest figure: Academician Mirko Stojaković (1915–1985), one of the leading mathematicians from Novi Sad at the time.

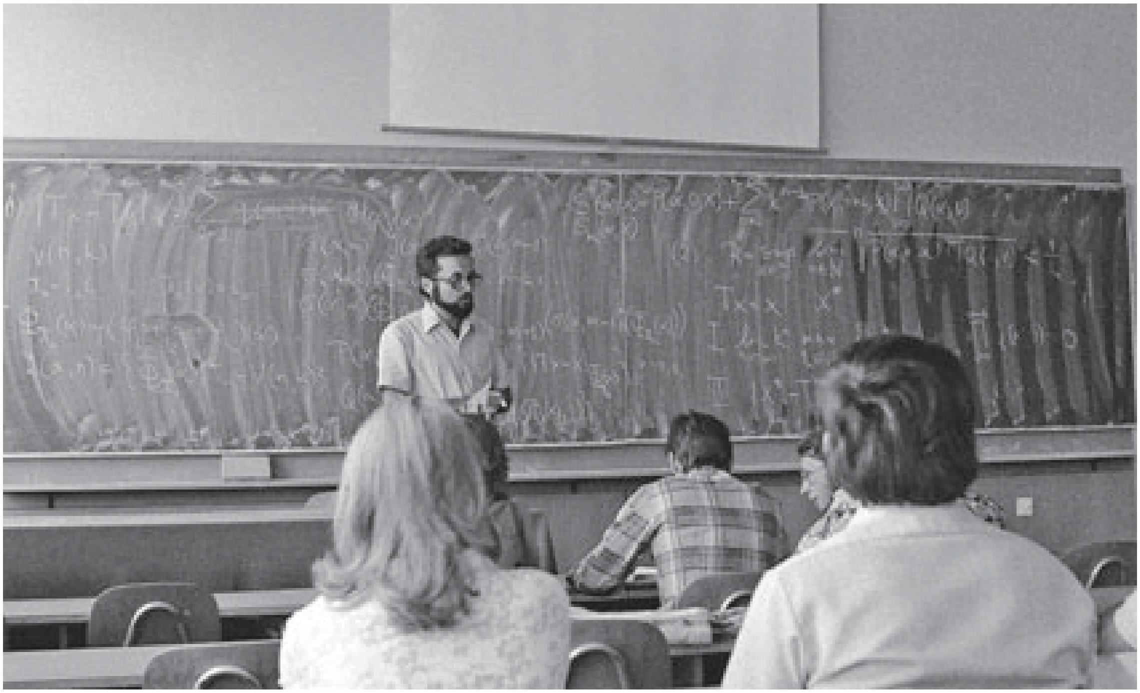
localizations of matrix pseudo-spectrum in various norms. As per numerical methods, the iterative methods for solving problems of linear complementarity stood out based on matrix decomposition, for which convergence areas were improved. In addition, new optimization algorithms for problem of distance to general instability for medium– and large-scale matrices were developed. Finally, the mentioned results in the field of numerical linear algebra were efficiently applied in the area of stability of dynamic systems, especially for optimization of operation of wireless sensor networks, for obtaining robust indicators of ecological stability of empirical food networks, overcoming the Schmidt paradox in climate modelling, as well as the analysis of the impact of nanotubes on intercellular communication. Research activities in the area of numerical methods of linear algebra on an international level resulted in the organization of the GAMM 2013 conference in Novi Sad, as well as in a series of the ALA conferences over the previous twenty years.

Over the last ten or so years, a large number of activities of the Group have been focused on mathematical modelling and the development of applied mathematics, in line with contemporary European and global trends. The Department of Mathematics and Informatics joined ECMI – European Consortium for Mathematics in Industry in 2003 as an educational and scientific center. A master's program in applied mathematics has been established with the focus on mathematical modelling and education of young mathematicians who have additional knowledge in economy and technics, and who are capable of working in multidisciplinary teams and applying fundamental mathematical knowledge to real problems.

Group photo of mathematicians. First from the left: Academician Mirko Stojaković.
Second from the right: Academician Đuro Kurepa

Significant efforts were made to establish cooperation with non-academic institutions aiming at applying mathematical knowledge and methods to real problems. These efforts resulted in raising some new research topics, mainly concerning the application of numerical mathematics methods in economy, medicine, chemistry, biology, meteorology and technics. A series of results achieved in the field of numerical optimization is motivated by problems stemming from algorithmic trading, which is a dominant method for the execution of transactions on stock exchanges, which is, by its definition, based on a mathematical model and statistical characteristics of the market.

Contemporary trends in numerical optimization, as well as the importance of modelling and solving of real problems, stirred up interest in problems of stochastic optimization and in the mathematical simulations of real systems. Given the fact that in the real systems random parameters are often present, or values of parameters are known only with some degree of certainty, the minimization of stochastic functions is of great importance. Basic problem in solving problems of stochastic optimization is the efficiency of numerical methods, because good approximation of stochastic objective function requires working with very large samples, which is very challenging computing-wise, and often inapplicable. By using the apparatus of numerical optimization and stochastic analysis, a series of results with variable sample sizes is obtained, which greatly increased the efficiency of a method. At the same time, international cooperation has been broadened, and scientific cooperation with numerous ECMI research groups in this field has been established.



A lecture at the Department for Mathematics, University of Novi Sad

In the Big Data era in which we live today, machine learning and deep neural networks are very important topics. Both topics are fundamentally mathematical problems and very often a significant part of the method comes down to the problem of function minimization. With the explosion of data quantity, a need for distributed optimization appeared. Namely, the available data is often stored in different geographical locations and its centralization is not technically feasible, or the data quantity exceeds computer's capacity or data centralization is avoided due to data privacy challenges. Besides, concepts of federated and online learning involve partial data processing at the source, and partial cooperation within a group of agents in order to get the final result. Therefore the methods of distributed optimization are being developed, which implies that there is a network of computers that are mutually interconnected by architectures of various types, and each computer in the network has access to a specific set of data and its local objective function, whereas the objective is the minimization of aggregate objective function. A reformulation of the standard methods of continuous optimization in distributed environment is a non-trivial challenge; the lack of central node makes

A chess tournament at the Department of Mathematics, University of Novi Sad

classical methods inapplicable, the mutual communication of each node with all other nodes is impossible because it is either too expensive or is opposed to various privacy limitations, and communication within the network, when possible, is expensive, can be slow and interruption-prone. Therefore, the non-trivial redesigning of classical (efficient) methods of numerical optimization is necessary. Research in this field has been in the focus of the scientific efforts of the Group for Numerical Optimization over the last few years. Its current activities encompass several research projects in the field of Big data realized within the H2020 and IPA programs. Besides, a doctoral program focusing on the mathematical challenges in the field of Big data is formed, and it is being implemented in cooperation with universities in Milan, Lisbon and Eindhoven and seven European companies, within the Marie Sklodowska Curie H2020 program.

Researchers from the Group for Numerical Mathematics are active in the Center of Research Excellence for Mathematical Research of Non-linear Phenomena, headed by academician Stevan Pilipović and SKALA center headed by professor Ljiljana Cvetković.

MIHAILO PETROVIĆ ALAS – SCIENTIFIC LEGACY AND MODERN ACHIEVEMENTS IN PROBABILITY THEORY

Dora SELEŠI
University of Novi Sad,
Faculty of Sciences

"Problems, nowadays intractable by the tools of mathematical analysis, do not have to stay that way for all eternity; difficulties, invincible for today's mathematics, may become no more than a piece of cake for tomorrow's one."

Mihailo Petrović, 1914.

The Danube and the Sava – two rivers, two shores where the prominent Alas grew up and reached adulthood, found his inspiration, purpose and home. Calm, dignified and humble at their source, they turn into mighty giants. Their continuous flows carry great riches. Differential equations and special functions – the two fields that were the main focus of Mihailo Petrović's research where he made substantial contributions. The results, which may seem simple and modest in terms of today's evaluation criteria, are continuously evolving and turning into mighty achievements, embodying a source of great scientific knowledge. Time flows and carries centuries in its stream, the generations of mathematicians are maturing and working all over the world. The scientific legacy of professor Petrović remains as relevant today as ever. In this article I will try to briefly present the current state of the art of mathematical

A photo of the Danube in Zemun, an old picture postcard (source: https://www.kolekcionar.net/articles.php?id=278795)

analysis, with an emphasis on stochastic analysis and some modern scientific achievements resulting from Mihailo Petrović's papers.

Mihailo Petrović (1868–1943), a doyen of mathematics in Serbia, took his first steps in scientific research with his papers elaborating on the theory of ordinary differential equations and he left a vast amount of papers in this field. Soon he discovered his other field of permanent interest: the theory of special functions, mainly elliptic functions. He made a significant contribution to the theory of differential equations and their solving via power series, as well as to the approximation of functions and the qualitative analysis of differential equations. The value of these works exceeds the time in which they were written; the modelling of physical phenomena nowadays relies not only on a deterministic approach via ordinary differential equations, but also on quantifying the uncertain factors by probabilities and incorporating them into the equation as stochastic (random) processes. Hence, many natural, social, economic and other phenomena can be appropriately described via *stochastic differential equations*. One of the most advanced methods for solving stochastic differential equations is the method of polynomial chaos expansions, whose essence lies in expressing a stochastic process as a Fourier series via the base of stochastic orthogonal polynomials (Hermite, Jacobi, Legendre and other polynomials). In this idea one can

*9. ASYMPTOTIC PROPERTIES OF REGULARLY VARYING FUNCTIONS

The purpose of this section is to investigate the relations between the tails and the truncated moments of distributions with regularly varying tails. The main result is that if $1 - F(x)$ and $F(-x)$ vary regularly so do all the truncated moments. This is asserted by theorem 2, which contains more than what we shall need for the theory of stable distributions. It could be proved directly, but it may also be considered a corollary to theorem 1 which embodies Karamata's[21] striking characterization of regular variation. It seems therefore best to give a complete exposition of the theory in particular since the arguments can now be significantly simplified.[22]

We introduce the formal abbreviations

$$(9.1) \qquad Z_p(x) = \int_0^x y^p\, Z(y)\, dy, \qquad Z_p^*(x) = \int_x^\infty y^p\, Z(y)\, dy.$$

* This section is used only for the theory of stable distributions, but the use of theorem 2 would simplify many lengthy calculations in the literature.

[20] Special cases were noticed by S. Port.

[21] J. Karamata, *Sur un mode de croissance regulière*, Mathematica (Cluj), vol. 4 (1930) pp. 38–53. Despite frequent references to this paper, no newer exposition seems to exist. For recent generalizations and applications to Tauberian theorems see W. Feller, *One-sided analogues of Karamata's regular variation*, in the Karamata memorial volume (1968) of L'Enseignement Mathématique.

[22] Although new, our proof of theorem 1 uses Karamata's ideas.

William Feller was the first to apply Karamata's theory of regularly varying functions in probability theory. The image depicts a page from Feller's book.

perceive not only a modern solving technique that originated from the theory of approximations and special functions, but also the essence of Petrović's phenomenology in the very setup of the model. Henri Poincare (1854–1912) claimed that differential equations are inherently involved in all laws of nature and science. Petrović adopted this idea and used it to develop his theory that he termed mathematical phenomenology. A contemporary designation of the same concept is *mathematical modeling*. The modeling of stochastic nonlinear dynamical systems is the main focal point of research related to modern mechanics, astronomy, sociology, biology, economy and many other fields of science. Problems related to stochastic differential equations that describe the models of fluid flow, blood flow in the vessels, optimal aircraft design, spreading of epidemics, price changes of financial derivatives, etc., are just some of the most interesting examples of equations the solving of which occupies today's mathematicians.

Jovan Karamata (1903–1967), who was Mihailo Petrović's student and the creator of the theory of regularly varying functions and slowly varying functions, was the one who made the most significant contribution to probability theory. The importance of this class of functions came to the center of attention in Feller's book titled *An Introduction to Probability*

A coin of the Deutsche Mark 20 DEM, produced in 1977, on the occasion of Gauss's birthday, depicting Gauss's curve of normal distribution.

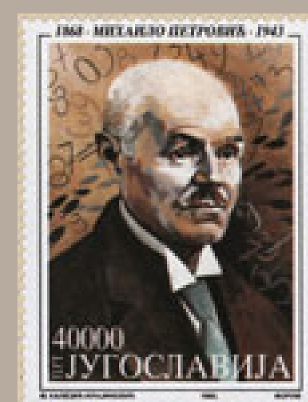
Theory and its Applications (*W. Feller, An Introduction to Probability Theory and its Applications I and II, Wiley, New York, 1969 and 1971*). The famous central limit theorem in probability theory states that sums of independent random variables with finite variances behave as the normal (Gaussian) distribution in limit process. It is only in the last few years that random variables with infinite variances were also proven to be realistic models for various phenomena, e.g. for insurance claims arising from natural disasters (earthquakes, floods, hurricanes, etc.), for magnitude levels of ground shaking during earthquakes, for file sizes sent via e-mail, for the time spent on social networks, etc. The central limit theorem does not necessarily hold for variables with infinite variances, thus in order to determine the distribution of sums and of maxima/minima of such variables, the classes of Karamata's distribution functions with slowly varying tails must be used. A modern theory covering this topic is *the theory of large deviations* and it is being applied in insurance theory, risk management, information theory, queueing theory, renewal theory, thermodynamics and statistical mechanics.

One branch of mathematicians in Serbia who deal with probability theory and stochastic analysis has grown out of Karamata's node in a genealogical tree of Mihailo Petrović. Karamata's student Bogoljub Stanković (1924–2018), followed by his student Stevan Pilipović, has brought the research of functional analysis and generalized functions to Serbia, whereas the study of generalized stochastic processes has been formed as a special sub-field of this research. Within Stanković's Novi Sad school of mathematics at the University of Novi Sad, contributions to probability theory and generalized stochastic processes were made by Olga Hadžić, Stevan Pilipović, Mila Stojaković, Zagorka Lozanov Crvenković, Danijela Rajter Ćirić and the author of this article. The other branch of probabilisticians in Serbia and Petrović's successors has grown out of the node of Tadija Pejović (1892–1982) and his PhD candidate Zoran Ivković (1934–2011). The vast majority of the professors at the Universities in Belgrade and Niš who work in the field of probability theory and statistics are Ivković's successors: Jovan Mališić and Svetlana Janković from the first generation, then Pavle Mladenović, Biljana Popović and Miljana Jovanović as the second generation of mathematicians, and many of their PhD candidates and mathematical descendants of the third and fourth generation: Miroslav Ristić, Marija Milošević as well as many others. A detailed outline of the genealogical tree can be seen on the *Mathematics Genealogy Project* portal. Of course, we must mention the names of those in our school of probability who do not stem from Petrović, such as Milan Merkle who obtained his PhD degree in Michigan or Slobodanka Janković who, via Stevan Stojanović, stems from Đuro Kurepa (1907–1993), who was of great influence in our region. It is possible that many names are missing in this retrospective, but, as this work is rather a personal impression, and not a document which claims to be precise in all details, the author apologizes to all those whose names are unjustifiably omitted.

Mihailo Petrović was a universal creator and thinker, a true polymath, scientist, mathematician, inventor, philosopher, ichthyologist, musician, writer, world traveler. Many authors regard him, along with Norbert Wiener (1894–1964) and Johann von Neumann (1903–1957), as a forefather of cybernetics and computer science. The far-reaching influence of his works reaches even today's most modern fields of science such as neural networks, artificial intelligence, machine learning, deep learning, which are all inspired by the cognitive functioning of the human brain trying to imitate its activity by using mathematical algorithms. Google's algorithm for searching and indexing of web pages, the formalization of natural language learning and the creation of a universal translator are just a few examples of a more or less successful implementation of these techniques. Integral transformations, function approximations and power series are the roots of what is today used in signal processing, digital image processing, with applications in mobile communications, biomedicine, computerized tomography and many other areas of science, engineering and economy. An integral part of all these models are noises that occur in transformation channels, errors in instrument readings, etc., which are quantified as uncertainties i.e. as random processes, and they introduce probability, statistics and stochastic analysis into the contemporary models.

The man who spent his life at the confluence of the Sava and the Danube was a mathematician who was equally engaged with analysis and algorithms and who understood the intrinsic ties among continuity, continuum and discrete structures. Regardless whether he was inspired by the calm, continuous flows of rivers to embark on the analysis of differential equations and continuous structures, or the smooth water streams to notice direction fields and integral curves within them, or his favorite leisure activity – fishing – inspired him to adopt a more algorithmic way of thinking, it will forever remain a mystery. The fact is

Jovan Karamata postmark, published in 2002



Mihailo Petrović Alas postmark, published in 1993 that there is a huge body of scientific papers he left behind, in which continuous objects are approximated by discrete ones (let us remember, for instance, the Riemann integral or the expansion of a function into a power series, the numerical solving of equations, etc.) and discrete objects are approximated by continuous (for instance, in analytical number theory). The large gap between various areas of mathematics needs to be bridged, as much more can be achieved by accepting the natural unity of mathematics and the unity in its duality. Algebraic topology is a brilliant example which underlines the needs and benefits of the synthesis of a broader spectrum of mathematical disciplines. The explosive development of probability theory in the last few years and the fact that it permeates all spheres of mathematics might play the role of this connecting bridge. Probabilistic logic, random graphs and algorithms of random decision trees, Monte Carlo methods in modeling, distributed optimization, quantum computers are just a few examples in which probability theory meets classical discrete mathematics or computer science. On the specific level of stochastic analysis, positive examples that point to a synthesis of various areas are Markov processes which are connected to semigroup theory, singular stochastic processes that are defined via Colombeau's algebras of generalized functions, regularity structures that are used for solving stochastic partial differential equations by regularization methods and quantum field theory methods, and it is exactly these examples that are determining the further direction for development of modern stochastic analysis as an integral part of the scientific legacy of Mihailo Petrović Alas.

Mihailo Petrović's memory from the Danube (SASA Archive, 14188/12)


НАШ НАЈБОЉИ МАТЕМАТИЧАР
г. Мика Петровић
бави се риболовом
У КОЈОЈ ОБЛАСТИ НАУКЕ ЊЕГОВО ИМЕ ЈЕ ДАВНО ПРЕШЛО ГРАНИЦЕ НАШЕ ЗЕМЉЕ
9 маја
AEROPUT

MIHAILO PETROVIĆ
IN THE MEDIA AND ACHIVES

DIGITIZATION OF THE LEGACY OF MIHAILO PETROVIĆ ALAS

Maja NOVAKOVIĆ
Mathematical Institute of SASA

This paper includes the digital media that have recorded the life and work of Mihailo Petrović Alas. Apart from the analysis of individual records, emphasis is placed on their significance as a whole, that is, a database that is archived in the digital treasury keeping the memory of the eminent scientist alive. They are accessed as documents that have immortalized and made the scientist's work and information concerning him more easily accessible. The concept of the digital media includes the following:

- audio-visual media (cultural-educational programmes, documentary and feature films);
- newspaper articles (taken from the digital repository of the "Svetozar Marković" University Library and the Mathematical Institute of SASA);
- the digital legacy of Mihailo Petrović Alas (created by the Faculty of Mathematics of the Belgrade University);
- the virtual library of the Faculty of Mathematics of the Belgrade University.

What is analysed and emphasized is how significant the media that offer online interactive approach to the material are, and how much they facilitate the accessibility of information to the researchers.

A frame from the film *Professor Kosta Vujić's Hat*, 1971, Vladimir Andrić

Graduates of Prof. Kosta Vujić from the First Belgrade Gymnasium

The paper represents the continuation of the research whose results on this topic have been published in the catalogue *Mihailo Petrović Alas: the Founding Father of the Serbian School of Mathematics,* Belgrade, 2018, under the title "Mihailo Petrović Alas in Audio-visual Media". The catalogue traces the historical sequence of video media, comprising the following formats: cultural-educational programmes, documentary and feature films that have recorded and presented the life and work of Mihailo Petrović. From the short documentary film from 1968, produced at the occasion of the centenary of Petrović's birth, entitled *Mika Alas,* to a number of scientific-educational programmes, which give a concise and richly-documented account of the life and work of Mihailo Petrović.

As for the feature film format, there is a film that presents the famous generation of the graduates of the First Male Gymnasium from 1878–1885, to which Petrović belonged, as well. It is the feature film *Professor Kosta Vujić's Hat* (*Šešir profesora Koste Vujića*) from 1971, directed by Vladimir Andrić, based on the script by Milovan Vitezović that was adapted by Bojana Andrić. Though it had been written in the form of a novel, the film was premiered on 24 February 1972, and since then it has been broadcast six more times, making it the most popular TV drama of Belgrade Television. The script for the TV drama was subsequently turned into a novel bearing the same title by Milovan Vitezović in 1983. The film portrays the times of the generation of the First Gymnasium's graduates of the German language teacher Kosta Vujić that would go on to become prominent members of the Serbian science and culture.[172]

On the occasion of the celebration of the centenary of the First Belgrade Gymnasium (1939) Mihailo Petrović wrote the following about his generation:

Graduates of First Belgrade Gymnasium, June 1885 (SASA Archive, 14197/II-15)

> "About the schoolmates, some of which are still alive, we do not think to speak individually. Fate had given us various roles in life and we went our separate ways after graduation, meeting up occasionally, getting to know about one another and recalling common old memories. That generation did not give our land a single statesman, but it provided university professors, writers, lawyers, diplomats, doctors and journalists. One fact noted in other generations of high-schoolers became evident in ours, as well: what some of them showed to be capable of as high-schoolers and what they were to contribute to their area of work after graduating and becoming independent was not always in direct corellation. Some of us, whom the teachers had given up on, and even the schoolmates predicted nothing will become of them, went on to become what could not have been expected: great writers, prominent journalists, etc. Conversely, those with all the highest grades did not always turn out to be what they had been expected to become, and they left a much fainter mark than those that had been little counted upon."[173]

In the last scene of the film *Professor Kosta Vujić's Hat* of 1971, the graduates of the First Belgrade Gymnasium were shown, where it is striking that the inspiration for the scene had been taken from an archived document, that is, from a photograph from the SASA Archives. Similarity is noticeable in the arrangement of the teacher and students (Mihailo Petrović is standing in both cases in the same place at the right-hand side), as well as of the objects laid out before them. For instance, the configuration of geometrical bodies, the globe and a set of tools from the chemistry lab, run by a Chemistry teacher, Marko Leko.

The procedure for taking this photograph is reproduced and repeated in two more photographs in which Petrović was present. This practice was upheld when anniversaries of the

Twenty five years from graduation, 8 June 1910 (SASA Archive, 14197/32)

First Belgrade Gymnasium were celebrated. The photograph was reproduced on the occasion of celebrating the 25th anniversary of their graduation, on 8 June 1910. The same practice, when the photographs are entered and used as a testimony to a previous meeting, is shown in the photograph on the occasion of the 40th anniversary of their graduation, with the Mathematics teacher, Sreten Stojaković, and teachers of the pre-military training, on 8 June 1925. The photographs showing the graduates of the First Belgrade Gymnasium of 1885 is carried over through generations and reproduced in the same medium, until it was re-visualized, this time in the new medium – the feature film *Professor Kosta Vujić's Hat,* in which it served as a source of inspiration. In a word, apart from the preservation of the memory of a photograph, we have here an example of a reinterpretation of its contents in the new medium – film.

Apart from the audio-visual records, another facet of the digital legacy of Mihailo Petrović is represented by newspaper articles, taken from the digital "Svetozar Marković" University Library, as well as by an article from the Mathematical Institute of SASA. From those digitized newspaper articles, which allow for going back to Petrović's time, we can be informed not only about the work of Petrović, but also about the events and celebration in which he himself participated and gave interviews. The press, including *Vreme, Pravda, Politika, Srpski narod, Bodljikavo prase,* wrote about the great scientist and his eclecticism.

As an illustration of Petrović's eclecticism, a newspaper article was published in *Pravda* of 11 January 1939, with the headline "Our best mathematician, Mr Mika Petrović, practices fishing with as much intensity and passion as mathematics".

Forty years of graduation with professor of mathematics Sreten Stojaković and the professors of pre-military training, 8 June 1925 (SASA Archive, 14188/9)

Certain parts of that interview extracted from the article point out that passion Petrović had for science and mathematics was no greater than that for fishing and travel. As Petrović himself asserts with regard to mathematics:

> "I am wholly content with the younger generation of our mathematicians, which for me as a mathematician represents (placing great emphasis on this, because he practised fishing at least as much as mathematics, and maybe even more, which he is later to discuss) the greatest satisfaction. There are only – Petrović goes on to say – fourteen doctoral dissertations in mathematics, defended here or abroad. When we take into account the state of affairs in that respect up until recent times, as well as the state in other countries, then we can optimistically conclude that this number is considerably high and commendable. What is especially commendable and interesting is the fact that there are doctors of mathematical sciences teaching in grammar schools, as well. Once, every doctor of mathematics entered Belgrade Grand School and University, which was understandable, as there was only a handful of them. However, they now deal with mathematics as a science and work systematically and successfully in that field, even though they have no prospect of promoting the fruit of their research and expertise in the university departments."[174]

Then he goes on to speak about fishing:

> "I am involved in fishing unusually intensively. I have worked on it more than I did on mathematics – you may freely write it – and I will finish my working life in that area, and not in mathematics. Only, let's make it clear – Petrović points out further on – I am not engaged in fishing recreationally, but professionally. One should differentiate between fishing and fishery or,


НАШ НАЈБОЉИ МАТЕМАТИЧАР

г. Мика Петровић

бави се риболовом

ИСТО ТАКО ИНТЕНЗИВНО И СТРАСНО КАО И МАТЕМАТИКОМ

У КОЈОЈ ОБЛАСТИ НАУКЕ ЊЕГОВО ИМЕ ЈЕ ДАВНО ПРЕШЛО ГРАНИЦЕ НАШЕ ЗЕМЉЕ

Г. др. Михаило Петровић

A newspaper clipping: Milo Vasović, "Our greatest mathematician, Mr Mika Petrović practices fishery as intensively and passionately as mathematics", *Pravda*, 11 January 1939, 32. ("Svetozar Marković" University Library)

as is often termed, fish breeding. The latter term also includes trading activities related to fish as a product.[175] I have dealt with fishing since 1895, that is, for as long as 43 years, I am fishing now, and I will continue to do so for the rest of my life..."[176]

On travel:

"They regularly invite me to participate in those travels – says Mr Petrović – and I regularly go. Thus, I was away almost every year, and I am certainly going to go this year. Every one of those travels was expectedly filled with learning some new things and having interesting experiences. Regarding my first journey, the public was somewhat familiar with what I could bring and report about from those parts. I have travelled across the North and South Pole, which is for me, as well as for all the other scientists who took part in it, of enormous interest.

Last summer I also travelled. We came to the eels' breeding ground not far from the Bermuda Islands, and we lingered there for a good while. That journey was exceptionally interesting and an account of it in book form is soon to be published by Srpska književna zadruga (Serbian

Literary Cooperative). Still, some of those expeditions I had to cancel or delay – recounts Mr Petrović. It makes no sense, however, to expose oneself to dangerous places when it evidently means putting one's life at risk. Once, I barely survived on an expedition to Madagascar, while one of our crew died instantly from a poisonous insect bite and another hardly managed to escape."[177]

Even this article would suffice for us to become familiar with the versatility, curiosity and complexity of Petrović's character.

There are also other articles describing Petrović and telling stories about him. Apart from the interviews, there are newspaper articles that speak about Mihailo Petrović and his fish specialties in a humorous way. One of these came out on 5 September 1942 in the magazine *Bodljikavo prase (Porcupine),* with a headline "A Tried and True Recipe", describing Mika's recipe for 'drunken carp'.

> "The dinner was prepared by our eminent scientist and a famous expert in fish, Mika Alas. He prepared 'drunken carp'. Somebody asked Ben-Akiba why Alas keeps his secrets so. It's clear! Fish dishes are seasoned with wine. Or at least this is how the story goes. In reality, the wine is drunk by the cook and the fish served to the guests. This is what Mika knows, as well, and that is the reason why he keeps preparing 'drunken carp'. He locks himself in the kitchen and sips his wine while the carp sizzles in the oven. At least that is how I, for one, used to do it – he said, finishing his explanation – which was, naturally, only a joke, since Mika Alas has always been moderate not only in drinking, but also in eating."[178]

Unlike other documents referring to Petrović, newspaper articles tell us about the scientist in a direct way. Apart from the information on the success, inventions and achievements, it is often the case that the newspaper articles contain anecdotes related to Petrović, thereby revealing the humorous side describing the atmosphere and the spirit of the times.

One of the articles describes the tradition of dinners at the rectorate and how they changed in the post-war period. The question was how the custom of the rector's or dean's dinner came about. One of the responses came in the form of an anecdote told by the then president of the Academy of Sciences and a former rector Bogdan Gavrilović. He cited as an example the foundation of the higher education school Lyceum in Kragujevac in 1838. The story opened with how the first teachers of the school wanted to celebrate that important moment and the arrangement involved organizing dinner. Since then, with thousands of students and hundreds of teachers, much has changed, but that little tradition has remained unchanged.[179]

What is printed in the press and preserved is a speech of the then president of the Academy of Sciences and the former rector professor Bogdan Gavrilović and professor Mihailo Petrović Alas, in which we can discern Mika's satire:

> "His voice got lost in the resounding applause. Then the president of the Academy of Sciences rose and began his speech:
>
> 'Fifty years ago to the day I attended a rector's dinner for the first time and was the youngest person among the invitees. All the old professors then looked upon the young affectionately, and sincerely wished us success in science. Now, after all those years, I look upon you in the same

way, I am glad to see you gathered here regardless of the fact that you come from one faculty, and with as much hope and love I wish you to surpass us by far and be worthy of your successors...

The University has always been the hub of science, it has educated and then sent abroad the representatives of knowledge and skills...'

'Yes', interrupted Mika Alas, whose satire never spared anyone, 'I concur – with our renowned Bogdan. I was also told by the elders that in the vicinity of the Rtanj Mountain there were a lot of bears and bear cubs. But there were also two villages, and in the villages well-known bear-baiters versed in the trade. The bear-baiters from the whole region collected bear cubs, taught them various skills and when they grew up and became adult bears, they brought them as well-trained animals into wild regions to disseminate their expertise. (Roaring laughter) No, I do not intend to draw any analogy', the old professor added facetiously. 'I only wanted to express agreement with my predecessor in speaking and to tell you something about the bear-baiting hub that had once been there, at the foot of Rtanj...' Once the elderly and eminent scientist ended his speech, the audience broke into a resounding applause that took a long while to quieten down. After that came the toasts, clinking of glasses, dishes were taken out, and the musical ensemble "Suz" played incessantly. When people started dispersing at dawn, they did not need a watch to tell the time because they could see the morning sun. And maybe even this time there were carriages..."[180]

Декански ћевапчићи
у Ботаничкој башти

К традиционалној вечери професора Универзитета „Декански ћевапчићи" присуствовали

A newspaper clipping: M. Mihailović, "Dean's Kebabs in the Botanical Garden", *Vreme*, 3 July 1937, 10.

Mihailo Petrović (with a hat) “Suz”, atmosphere in the tavern (SASA Archive, 14197/II-1)

To testify that there were festivities, music and song and to bring the atmosphere, there are illustrative pictures adjoining the articles, as well as photographs of the musical ensemble “Suz”, of which Petrović was one of the founders.

An article in *Pravda* of 20 February 1939, reports about the event of marking the 70th anniversary of the scientist and pedagogue Jelenko Mihailović. A number of prominent scientific and educational staff paid homage to Jelenko Mihailović, the rector of the Higher School of Pedagogy. Among the attendees were Mihailo Petrović Alas and his orchestra “Suz”. According to the article: “Professor Petrović was playing the violin, and Mr Mihailović beat the tambourine.”[181] Attendees at the celebration were also entertained by Mr Siniša Stojković, a professor of the Trading Academy, who asked Petrović about the origins of the orchestra “Suz”. “And professor Petrović replied: ʿI am ashamed to say. We founded it in the latter half of our lives. And the principle is: while the wise were getting one up on each other, the silly lived out their lives...’”[182]

The newspaper articles in which speeches have been recorded and published, those given by Mihailo Petrović in this case, are evaluated and viewed as direct testimonies and witnesses of a certain atmosphere, special occasions and celebrations.

Not only did others write about Petrović in the newspapers, but Petrović himself wrote for the press. One of the illustrations represents an article that he wrote anonymously about the

ПОЛИТИКА, недеља 8 мај 1938

ПОВОДОМ ПЕНЗИОНИСАЊА ЈЕДНЕ ГРУПЕ УНИВЕРЗИТЕТСКИХ ПРОФЕСОРА

Математички институт на Београдском универзитету — кошница научног рада

Једна наша велика научна установа за коју треба да сазна и шира јавност

Anonym. “Mathematical Institute at the University of Belgrade – the beehive of scientific work”, *Politika*, 8 May 1938, 9. (Mathematical Institute SASA)

Mathematical Institute of the Belgrade University, which he called a beehive of activity, educational and scientific, as well as public, and had it published in the daily newspaper *Politika*[183] in 1938.

Apart from the mathematical contributions, he writes and publishes an article about fishing just as vehemently, this time under his name. The article was published in 1940 in the paper *Beogradske opštinske novine (Belgrade Municipal Paper),* with the headline "Beograd – negdašnji centar velikog ribarstva" ("Belgrade: A Former Centre of Large-scale Fishing"), in which are listed all the fish species on the Sava and Danube rivers in the region of Belgrade, accompanied by the description and features of certain fish species.[184]

Eleven days after the death of Mihailo Petrović, an obituary was published in his memory in the paper *Srpski narod (Serbian People)* on 19 June 1943, written by Mladen St. Đuričić, a writer and Petrović's friend. The article opens with the following words: "Let them stop, for a moment, science and literature, and music and society to which our great deceased belonged", and ends with the following speech: "That is – Mika Alas! With that name – he neither died, nor will he die easily, our great deceased. In the name of those small people, fishermen and boatsmen, in the name of those you loved the most, oh great Master, I bow to the magnificent memory of you, to which there is no predecessor, nor will there be a successor for long, in the whole wide world."[185]

Such an account of digitized newspapers and other publications from the repository of the "Svetozar Marković" University Library and the National Library of Serbia provides its users with fast searchability and accessibility of the issues of Serbian historical press and periodicals, where it is possible to collect data about a given period, historical event or, as in this case, about a certain person.

With regard to the digital legacy, it is also important to mention the digital legacy[186] of Mihailo Petrović Alas, created by the Faculty of Mathematics in Belgrade. The digital legacy itself can be treated as a digital database or repository, which gives an overview of the existing bibliography related to Petrović. It includes the digitized version of his collected works, manuscripts and photographs. In the library of the Mathematical Institute of SASA there are fourteen volumes by Mihailo Petrović in manuscript form. Notes from the lectures were taken by a student Borivoje B. Pujić from 1910 to 1914. That manuscript has been digitized and is now available in the digital legacy.

A drawing of N. Bešević, in the memory of Mika Alas. Published in: Mladen St. Đuričić, "In the memory of Mika Alas", *Srpski narod*, 19 June 1946, 12. ("Svetozar Marković" University Library)

Digital Legacy of Mihailo Petrović Alas

The digital legacy contains not only biography but also materials in various digital formats about Petrović's life and scientific work. The materials from Petrović's work include professional and scientific papers, books and photographs from the SASA Archives, SASA Library, Association "Adligat", Foundation "Mihailo Petrović Alas" and the Primary School "Mihailo Petrović Alas"

Digital copies of the presented files are placed in the virtual library of the Faculty of Mathematics of the Belgrade University. Next to each file in the digital legacy there is a link to its copy in the virtual library. The virtual library owns several digital legacies of more prominent Serbian mathematicians.[187]

In relation to Petrović and his work, the virtual library grew into a monumental database. The bulk of the presented material has been collected thanks to the professor Žarko Mijajlović, while a large part of the collected material belongs to the library and archives of the Serbian Academy of Sciences and Arts.[188] What is incorporated in the digital legacy is a web-presentation of the catalogue of the exhibition marking the 150th anniversary of the birth of Petrović, under the title: *Mihailo Petrović Alas: the Founding Father of the Serbian School of Mathematics,* showing the sections underlying the exhibition's concept and structure. Creation of the website for the web-presentation

Web-presentation of *Mihailo Petrović Alas – the founder of Serbian school of mathematics* catalogue

of the catalogue of the exhibition is authored by “ARhiMedia group” and the Faculty of Electronics in Niš.

Within the digital legacy there is a 3D model of a bust of Mihailo Petrović, made by the Mathematical Institute of SASA. The original bust is located in front of the family house of Petrović at 22, Kosančićev venac Street, and is the work of the sculptor Aleksandar Zarin from 1969. Technology enables us to treasure media that are not in the print or paper form and to create their digital imprint.

Digital libraries and databases keep and present at one place electronic archiving of all manuscripts, photographs and information related to cultural and scientific legacy and make them more accessible to a wider auditorium. Reflections on Petrović are memorized and continue to live in different media and thanks to the digitization of its legacy we have an easily accessible foundation for information and further research.

The 3D model of Mihailo Petrović Alas's bust
Authors of the 3D model: Vanja Korać and Dragan Aćimović (Mathematical Institute SASA, 2018)

REFERENCES

Д. Трифуновић, *Михаило Петровић Алас – живот и дело*, Дечје новине, Горњи Милановац, 1982.
Д. Трифуновић, *Летопис живота и рада Михаила Петровића*, САНУ, Београд 1969.
Mirjana Maljković, Biljana Stojanović, Žarko Mijajlović "Digital legacy of Mihailo Petrović Alas", *Преглед НЦД* 31 (2017), 10–17.

THE PRESS

Аноним., „Математички институт на Београдском универзитету – кошница научног рада", *Политика,* 8. мај 1938, 9. (Mathematical Institute of SASA)
Михајло Петровић, „Београд – негдашњи центар великога рибарства", *Београдске општинске новине*, 3. јануар 1940, 188–196. ("Svetozar Marković" University Library)
Мило Васовић, „Наш најбољи математичар г. Мика Петровић бави се риболовом исто тако интензивно и страсно као и математиком", *Правда*, 11. јануар 1939, 32. ("Svetozar Marković" University Library)
Аноним., „Вечера у част научника и педагога г. Јеленка Михаиловића", *Правда,* 20. фебруар 1939, 7. ("Svetozar Marković" University Library)
Младен Ст. Ђуричић, „У спомен Мике Аласа", *Српски народ*, 19. јун 1943, 12. ("Svetozar Marković" University Library)
М. Михаиловић, „Декански ћевапчићи у Ботаничкој башти", *Време*, 3. јул 1937, 10. ("Svetozar Marković" University Library)
Аноним., „Опробан рецепт", *Бодљикаво прасе,* 41, 5. септембар 1942, 4. ("Svetozar Marković" University Library)

ELECTRONIC SOURCES

The Digital University Library "Svetozar Marković"

http://www.unilib.rs/sadrzaji/digitalna-biblioteka/

The Digital Legacy of Mihailo Petrović Alas, the Faculty of Mathematics of Belgrade University

http://alas.matf.bg.ac.rs/~websites/digitalnilegatmpalas

Virtual Library of the Faculty of Mathematics in Belgrade

http://elibrary.matf.bg.ac.rs

Digital Legacies of the Faculty of Mathematics of Belgrade University

http://legati.matf.bg.ac.rs/

DOCUMENTS ON MIHAILO PETROVIĆ IN THE ARCHIVES OF THE MATHEMATICAL INSTITUTE SASA (1946–1954)

Marija ŠEGAN-RADONJIĆ
Mathematical Institute of SASA

Mihailo Petrović was one of the first advocates of the idea to set up a specialized institution charged with the task of developing and spreading mathematical knowledge in the territory of Serbia and South East Europe. He was, however, aware that due to a lack of financing and expert staff it was not possible to set up such an institution immediately:

> "There is no genuine mathematical institute, precisely because the Grand School does not have at its disposal the means to maintain such an Institute and to purchase books and journals necessary for it to be sustained." [Petrović, 1898]

Though the conditions for founding an institution of the sort were not created until much later, it should be noted that the name "Mathematical Institute" had already been in use at an earlier time. Namely, after World War I, Petrović and his colleagues used this term to denote mathematical organisations of the Faculty of Philosophy at the University of Belgrade [Milanković, 2012, 67; Milanković, 1957, 1]. What is more, they printed forms with the letterhead of the Mathematical Institute, while Petrović reported about the work of the Institute in the daily newspaper *Politika*, describing it as a "hive of scientific work" [Petrović, 1938, 9].

The first more concrete step towards setting up a separate institute was made towards mid-1938, immediately after Petrović was retired. At that time, his colleagues, as a token of their respect and gratitude, proposed that the mathematical seminar at the University of Belgrade be

Brankova Street, the odd number side. The building in which the Academy and Mathematical Institute were placed for some time is on the left, Jeremija Stanojević, 1929–1932 (Belgrade City Museum, Ур_6396)

divided into two independent institutes – Institute for Theoretical Mathematics: Dr Mihailo Petrović and Institute for Applied Mathematics [Milanković, 1938]. Although the proposal to "set up the Institute Mihailo Petrović for Theoretical Mathematics" was adopted [Jovanović, 1938], this did not take place because World War II broke out.

Petrović did not live to see the end of the war and the founding of the Mathematical Institute under the auspices of the Academy in 1946. This institution, however, rested on the foundations laid earlier by Petrović and his colleagues: the library, continued publication of the journal *Publications* and gathering of mathematicians for the purpose of exchanging views and expertise. In that sense, he is considered to be one of the founders of this institution, as confirmed by a number of documents in the Archives of the Mathematical Institute SASA (MISASA).

MISASA's archives contain archival materials that have resulted from the work of the Institute's members and are in the ownership of the Institute. These materials have not been taken over by the State Archives and have, in all likelihood, not yet been recognised as cultural heritage of general interest[189]. The archives contain manuscript, typewritten, photographed and printed documentary materials spanning a period of one hundred years, from the 1890s until the 1990s. As they officially do not have the status of archival materials, they have therefore not been recorded, classified or described yet, there are no strict guidelines for their preservation and management, and they are kept in the premises of the Mathematical Institute SASA.

The broader scientific community is for a greater part not aware that these archives exist. A good starting point for gaining insight into their content is a group of documents (statutes, decisions and reports) published on the official web page of the Mathematical Institute ("History", n.d.). Here, users can find digital copies of originals which however, except for titles, do not contain other data that would describe them further, and are therefore not easily navigable or visible. In addition to these documents, a part of the archival materials was published in the publications of the Mathematical Institute [*see* Vujičić, 1972; Čavčić, 1990], and in the journal *Pregled NCD (NCD Review)* [*see* Mijajlović, 2014; Pejović, 2015]. Also, the wider public had the chance to see some of these materials as part of the exhibition "Mihailo Petrović Alas – The Founding Father of the Serbian School of Mathematics" [SASA Gallery, 2018]. They are significant as a contribution to studying the history of mathematics and related disciplines in Serbia and Yugoslavia.

Mileva Prvanović and Bogoljub Stanković, who translated Petrović's thesis into the Serbian language, were the first scholarship holders of Mathematical Institute (Personal archive of Professor Zoran Stojaković)

Manuscript notebooks from Mihailo Petrović's lectures are the most well-known archive materials relating to Petrović kept in the archives of the MISASA. These are fourteen hard cover notebooks, discovered by accident in 2012 [Mijajlović, 2014, 30]. The notes were compiled by Petrović's student Borivoj J. Pujić between 1910 and 1914 [*Ibid.*]. It has been assumed that Pujić personally bequeathed these notebooks to the Mathematical Institute, and they were recorded in the library catalogue in the 1960s. Today, thanks to the efforts of Professor Žarko Mijajlović and his associates, they are available to the wider audience as part of the Digital Legacy of Mihailo Petrović Alas.

When it comes to materials that have not been published so far, we should mention a group of documents from the time of establishing of the Mathematical Institute, kept in a file dated 1946. It contains, among other things, a manuscript draft of the opening address of the then director Anton Bilimović, drawn up on the eve of the first gathering of Board members on 22 June 1946. This source shows that Bilimović saw the Board as the "most senior forum of mathematicians–experts in this country" and that in doing so he adhered to the principles advocated by Petrović as the founder of the Belgrade School of Mathematics:

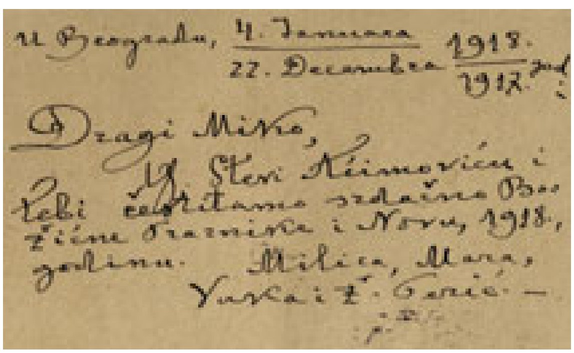
> "After World War I, for more than 20 years, mathematics and its related sciences made a major progress in the circle of Belgrade mathematicians. War circumstances and the death of Mihailo Petrović took a heavy toll on the work of the mathematicians of Belgrade. Today we gather again to continue our work dedicated to mathematics." [MB, p.1, 1946]

Another interesting document from this group are the manuscript minutes from the sixteenth meeting of the Board held on 3 September 1947. Mentioned here is Bilimović's visit to the Committee for Scientific Institutions, University and High Schools of the NRS (the body which, after the war, took over responsibility for high education and science in Serbia) when, in all likelihood, the suggestion was for the first time made to assign Mihailo Petrović Alas' house at No. 22 Kosančićev Venac Street to the Institute [MB, p. 16, 1947]. As the Institute was then located in a single office of the Academy House, in the endowment of Sima Igumanov at No. 15 Brankova Street, it was necessary to procure additional premises for work [MC, 14 October 1947]. Despite the efforts of the Institute's management, which would repeatedly address official authorities, Petrović's house was not turned into a museum and a mathematical library with a reading room [Čavčić, 1990, 185]. However, in 1968, on the occasion of celebrating the 100-year


Délivré par :
Le Chef des ... de la Direction,
N° 23667 CLASSE
BON POUR
BILLET DE DEMI-PLACE
Petrovich
(Ecole Normale)
De Paris
A Rouen
Le Directeur de la Compagnie,
Signature du Titulaire :
Validité pour UN SEUL VOYAGE
Jusqu'au 31 Xbre 1892

N° 23667 CLASSE
BON POUR
BILLET DE DEMI-PLACE
Petrovich
(Ecole Normale)
De Rouen
A Paris
Le Directeur de la Compagnie,
Signature du Titulaire :
Validité pour UN SEUL VOYAGE
Jusqu'au 31 Xbre 1892

Petrović's traveling ticket Paris–Rouen, 1892 (“Adligat” Society)


u Beogradu, 4. Januara 1918.
22. Decembra 1917 god.
Dragi Miko,
Mi Steva Klimović i tebi čestitamo srdačno Božićne praznike i Novu, 1918. godinu. Milica, Mara, Vasa i Ž. Carić —

A New Year's card to Mihailo Petrović from his sister Mara and her family, 1918 (Foundation “Mihailo Petrović”)

anniversary of Petrović's birth, the Institute would be one of the initiators of the proposal for his house to be declared a cultural monument [Trifunović, 1968, 400]. On that occasion, suggestion was also made that the name of Mihailo Petrović be included in the name of the Mathematical Institute [*Ibid.*].

An important testimony on the preservation of memory of Petrović's work is to be found in the Book of Minutes from the meetings of the Council of the Mathematical Institute, which covers 53 meetings of this managing body held between 1948 and 1954. Here we learn that, already in November 1949, plans were made to translate and print the selected works of Mihailo Petrović, as part of the special editions of the Mathematical Institute [MC, p. 21, 1949]. A Committee comprising Radivoj Kašanin, Tadija Pejović and Vojislav Avakumović had the task to make a selection of Petrović's works [*Ibid.*]. Next year, teaching assistant Bogoljub Stanković was entrusted with the task of translating Petrović's doctoral dissertation [MC, p. 35, 1950] and he travelled to Zagreb to collect the data necessary for solving the problems it covered [MC, p. 36, 1950]. In addition to activities relating to the translation of Petrović's works, other activities were also organised to commemorate the founding father of the Serbian School of Mathematics. For instance, members of the Institute took part in organising the commemoration ceremony to mark the ten-year anniversary of Petrović's death, held in the Main Hall of the Academy on 8 June 1953 [MC, p. 55, 1953].

It has already been mentioned that the Mathematical Institute continued to publish the journal *Publications,* with a slight change in the name and a new numeration. MISASA Archives have preserved the original copy of the first issue, published in 1947. In the foreword written in the French language, readers were informed that the renowned and active associate, retired professor Mihailo Petrović, was imprisoned during the war as reserve lieutenant colonel, that despite old age he was taken to a prison camp in Germany and that, after repatriation and a longer illness, he died in 1943 [*Preface*, 1947, VII]. Hence, in his honour, a paper of his was printed as the first out of fifteen papers in this issue [Petrovitch, 1947]. In addition, one of the first issues of the Collection of Papers of the Mathematical Institute, which, by contrast to *Publications,* was printed in the Serbian language, was also dedicated to the memory of Mihailo Petrović. In the foreword to the

Petrović's vineyard on Topčider hill. On the far right: Mihailo Petrović with a dog named Beka.
(SASA Archive, 14188/26)

Collection published to mark the ten-year anniversary of Petrović's death, Bilimović pointed to an important task of members of the Mathematical Institute as Petrović's successors:

> "Petrović's numerous students, who have felt his immense care for the talented mathematical youth, should take care to shed light from all possible angles on the colossally important role played by Petrović in raising the level of mathematical culture in this country." [Bilimović as quoted in the *Memory of Mihailo Petrović*, 1953, XII]

Whereas a number of other documents in the MISASA Archives are also directly or indirectly related to Petrović, we have focussed here on the documents created in the initial years of activity of the Mathematical Institute in order to highlight the intention of its founders to continue Petrović's mission of developing and spreading mathematical knowledge in the territory of Serbia and South East Europe. It should also be noted that the cataloguing and digitisation of these archives is under way, which will, in the future, enable a better insight into their funds and perhaps encourage some new research. In the meantime, the Mathematical Institute, as one of the organisers of the 150-year anniversary of birth of Mihailo Petrović, continues the tradition of commemorating important events from his life and work.

REFERENCES

Вујичић, В., & Чавчић, М. (Eds.) (1972). *Четврт века Математичког института 1946–1971*. Београд: Математички институт.

Галерија САНУ. (15. маја – 22. јуна 2018). *Михаило Петровић Алас – родоначелник српске математичке школе*. Београд.

Minutes of the first meeting of the Board of the Mathematical Institute SAS (22 June 1946). (2 pages). Archives of the Mathematical Institute SASA, F1946.

Minutes of the fifth meeting of the Council of the Mathematical Institute SAS (14 October 1947). (2 pages). Archives of the Mathematical Institute SASA, F1946.

Minutes of the sixteenth meeting of the Board of the Mathematical Institute SAS (3 September 1947). (2 pages). Archives of the Mathematical Institute SASA, F1946.

Јовановић, Д. Б. (1938). *Ректор Универзитета о одлуци Универзитетског сената да се образује Институт „Михаило Петровић"*. Архив Србије, УБ–X–1/1938.

Book of Minutes from the meetings of the Council of the Mathematical Institute SAS (1948–1954). Belgrade: archives of the Mathematical Institute SASA.

Mijajlović, Ž., Pejović, N. (2014). Twenty Four Manuscripts in the Virtual Library of the Faculty of Mathematics in Belgrade. *Review of the National Center for Digitization, 25*, 29–35.

Миланковић, М., Мишковић, В., Пејовић, Т., Билимовић, А., Жардецки, В., Салтиков, Н., & Карамата, Ј. (1938). *Предлог да се универзитетски професор Михаило Петровић изабере за почасног доктора филозофије и да се образује Завод Михаило Петровић за теоријску математику.* Архив Србије, Г-208, ф 10, р4203/1938.

Миланковић, М. (1957). Успомене, доживљаји и сазнања после 1944. године. У В. В. Мишковић (Ed.), *Посебна издања Одељења природно-математичких наука Српске академије наука, 16*. Београд: Научно дело.

Миланковић, М., & Михаиловић, Ј. (2012). *Мика Алас – Белешке о животу великог математичара Михаила Петровића*. Београд: Фонд др Милићевић: Удружење Милутин Миланковић: Књижарско предузеће Жиравац.

Pejović, N., Ninković, S. (2015). A Manuscript on Astronomy and Geodesy of an Unknown Author. *Review of the National Center for Digitization, 26,* 27–36.

Петровић, М. (1898). *Извештај о раду Михаила Петровића за 1897/1898 школску годину*. Архив Србије, ф 1, р 272/1898.

Петровић, М. (1938). Математички институт на Београдском универзитету – кошница научног рада. (8. мај 1938). *Политика*, стр. 9.

Petrovitch, M. (1947). Addition Au Mémorie Sur Les Équations Différentielles Algébriques. *Publications de l'Institut Mathématique, I*, 1–4.

Preface. (1947). *Publications de l'Institut Mathématique*, I, VII–IX.

Трифуновић, Д. (1968). Прослава Михаила Петровића. У: Д. Трифуновић (Ed.) Михаило Петровић 1868–1943. Београд: Републички одбор за прославу стогодишњице рођења Михаила Петровића.

Успомени Михаила Петровића. (1953). Зборник радова. Математички институт, 3, 6–13.

History. (n. d.). Accessed on 16 January 2019, http://www.mi.sanu.ac.rs/novi_sajt/aboutus/history.php.

Чавчић, М. (1990). *Саопштења научних резултата у Математичком институту 1946–1961*. Београд: Српска академија наука и уметности: Математички институт.

Correspondence of Mihailo Petrović on a paper with the logo of Mathematical Institute, 1938 (Archive of Serbia, Г208, Ф8, 1938, 2559).

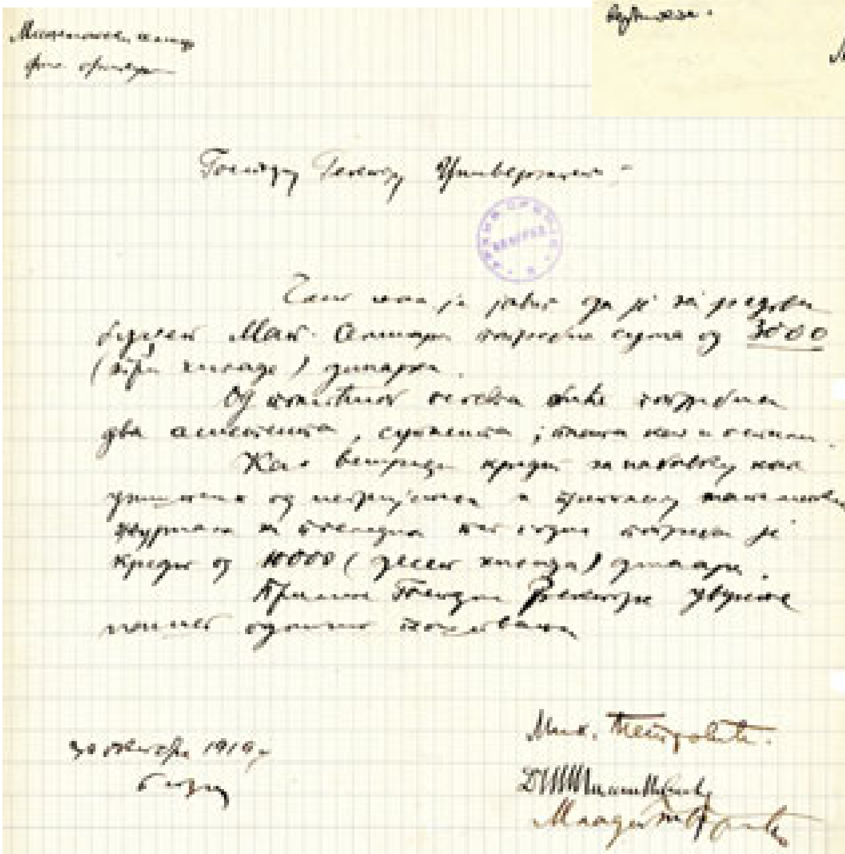
УНИВЕРСИТЕТ

Математички Институт

3. Марта 1938.

Господину Декану Философског факултета.

Мих. Петровић.



Господину Ректору Универзитета.

Мих. Петровић.

30 октобра 1919.

Београд

Mihailo Petrović and Milutin Milanković's appeal to rector of the University of Belgrade ahead of the Mathematical seminar of the Faculty of Philosophy, 30 October 1919
(Archive of Serbia, Faculty of Philosophy, Г-208, III, 1919)

GENEALOGY

MATHEMATICAL GENEALOGY OF MIHAILO PETROVIĆ ALAS

Boško JOVANOVIĆ
University of Belgrade, Faculty of Mathematics

We all came out from Gogol's "Overcoat"

F. M. Dostoyevsky

Mihailo Petrović is deservedly considered the founder of the Belgrade (and Serbian) school of mathematics. Furthermore, the vast majority of today's mathematicians in Serbia have been his successors and followers.

Mihailo Petrović Alas was born on May 6th, 1868 in Belgrade, as the first of five children of his father Nikodim, professor at the Faculty of Theology, and his mother Milica (maiden name Lazarević), the daughter of the parish priest Novica Lazarević. He completed the First Belgrade Gymnasium in 1885, and graduated from the Department of Science and Mathematics of the Faculty of Philosophy at the Belgrade Grand School in 1889. After graduation, he left for Paris where he earned diplomas in the area of mathematics (1891) and physics (1893) at the École Normale Superieure. As the best student in his generation, he attended a reception given by the President of the Republic of France in 1893 and 1894. He defended his doctoral thesis titled "About Zeroes and Poles of Integrals of Algebraic Differential Equations" (Sur les zéros et les infinis des intégrales des equations différentielles algébriques) at Sorbonne (Université Paris IV – Sorbonne), on June 21st, 1894, before a committee consisting of Charles

Hermite, Emile Picard and Paul Painleve, and thus he obtained his PhD degree in mathematical sciences (Docteurés sciences mathématiques) [1]. The ambassador (envoy) of the Kingdom of Serbia to France, Milutin Garašanin, also attended the defense of his doctoral thesis. Let us note that Milutin Garašanin was very important person in the public and political life of Serbia at the time: he was Prime Minister, minister of several ministries, envoy to Vienna and Paris, Speaker of the Parliament, founder and president of the Serbian Progressive Party, academician of the Serbian Royal Academy (SRA) and the son of Ilija Garašanin [2].

MIHAILO PETROVIĆ'S PREDECESSORS

By using the *Mathematics Genealogy Project (MGP)* database of the North Dakota State University [3] that contains data about more than 235,000 defended doctoral dissertations in the field of mathematics, and by following the line doctorand – mentor, it is easy to get the following genealogy of Mihailo Petrović's mathematical predecessors:

Mihailo Petrović (1894)	Johann Bernoulli
Charles Hermite	Jacob Bernoulli
Eugène-Charles Catalan	Nicolas Malebranche
Joseph Liouville	Gottfried Wilhelm Leibniz
Simeon Denis Poisson	Jakob Thomasius
Joseph Louis Lagrange	Friedrich Leibniz (1622)
Leonhard Euler	unknown

This "genealogy" spans over 270 years (from Leibniz the elder to M. Petrović) and contains a sequence of notable French, Swiss and German mathematicians.

The situation changes when, instead of one (first) mentor, we include co-mentors and mentors of several doctorates in the list. Then we get the following:

Mihailo Petrović (1894)
Charles Hermite, *Charles Émile Picard*(1)
Eugène-Charles Catalan
Joseph Liouville
Simeon Denis Poisson[a], *Louis Jacques Thenard*(2)
Joseph Louis Lagrange, *Pierre-Simon Laplace*(3)
Leonhard Euler; *Giovanni Battista (Giambattista) Beccaria**
Johann Bernoulli
Jacob Bernoulli, *Nikolaus Eglinger*(4)
Nicolas Malebranche; *Peter Werenfels*(5)
Gottfried Wilhelm Leibniz
Jakob Thomasius, *Erhard Weigel*[b]; *Bartholomäus Leonhard Schwendendörffer**; *Christiaan Huygens*(6)
Friedrich Leibniz* (1622)

In this list, the co-mentors are separated by a comma, whereas the mentors of different dissertations are separated by a semicolon. For instance, G. W. Leibniz had three doctorates: Dr

Phil. (1666), Dr Jur. (1667) and doctorate published at the Royal Academy of Sciences in Paris (Académie royale des sciences de Paris) (1676).The mentor of his last doctorate was famous physicist Christiaan Huygens.

It is interesting to mention that J. L. Lagrange had never officially defended any doctoral dissertation (there is only a piece of information about B.A. Università di Torino, 1754). Nevertheless, he had provided some fundamental scientific contributions to mathematics and mechanics. On the recommendation of Euler and d'Alembert, Lagrange succeeded Euler as head of the Department of Mathematics of the Prussian Academy of Sciences in Berlin, where he stayed for more than twenty years. Thus, the relation between Euler and Lagrange is tighter in comparison to many official relations between mentors and doctorands [4].

From each co-mentor in the previous list, next to whose name is a number in round brackets, a new "branch" of predecessors starts. Some branches "flow" back into the main tree or to some of the previous branches. Appropriate spots are marked with letters in square brackets. Scientists whose mentor is unknown are marked with a star. As before, the names of co-mentors are italicized, while the name of the originator of each branch is bolded. For some members of the sequence (most often the first and the last member in a respective branch) the year in which they obtained their PhD degree is in the brackets. Thus, we got the following sequence with 30 branches:

Charles Émile Picard(1) (1877)
Gaston Darboux
Michel Chasles
Simeon Denis Poisson[a] (1800)

Louis Jacques Thenard(2)
Nicolas Louis Vauquelin
Antoine Francois de Fourcroy
Jean Baptiste Michel Bucquet (1768)
Pierre Joseph Macquer
Gillaume-Francois Rouelle
Johann Gottlob Spitzley
Nicolas Lemery
Christoph Jacob Glaser
Johann Christoph Sturm
Erhard Weigel[b], *Johannes de Raey*(7)
Philipp Müller
Christoph Meurer
Moritz Valentin Steinmetz
Georg Joachim von Leuchen Rheticus, *Johann Hoffmann**
Johannes Volmar, *Nicolaus Copernicus*(8)
Bonifazius Erasmi* (1509)

Pierre-Simon Laplace(3) (1769)
Jean Le Rond d'Alembert* (1735)

Nikolaus Eglinger(4) (1660)
Emmanuel Stupanus[c], *Johann Caspar Bauhin*(9)
Petrus Ryff
Felix Plater
Guillaume Rondelet
Johannes Winter von Andernach[d]
Rutger Rescius; *Jacobus Sylvius (Jacques Dubois)*(10)

Girolamo Aleandro (Hieronymus Aleander) (1499; 1508)

Moses Perez*, *Scipione Fortiguerra*(11)

Peter Werenfels(5) (1649)

Theodor Zwinger, Jr.

Sebastian Beck

Johann Jacob Grynaeus

Simon Sulzer

Wolfgang Fabricius Capito

Desiderius Erasmus[e]

Jan Standonck* (1474; 1490), *Alexander Hegius*(12)

Christiaan Huygens(6) (1647; 1655)

Frans van Schooten, Jr.; *Jan Jansz Stampioen, Jr.**

Jacobus Golius, Marin Mersenne*

Willebrord Snellius, *Thomas Erpenius*(13)

Ludolph van Ceulen*, *Rudolph Snellius (Snel van Royen)*(14)[f] (1572)

Johannes de Raey(7) (1641)

Henricus Regius (Hendrik de Roy), *Adriaan Heereboord*(15)

Otho Heurnius (Otto van Heurne); *Adriaan van den Spieghel*(16)

Johannes Heurnius, *Petrus Molinaeus (Pierre du Moulin)**

Petrus Ramus (Pierre de La Ramée))[g]; *Hieronymus Fabricius (Girolamo Fabrici d'Acquapendente)*[h]

Johannes Sturmius (Johann Sturm), *Jacques Toussain*[i]

Nicolas Clénard (Nicolaes Cleynaerts), *Johannes Winter von Andernach*[d]

Jacobus Latomus (Jacques Masson), *Jan van Campen (Johannes Campensis)*(17)

Jan Standonck* (1474; 1490); *Unknown*

Nicolaus Copernicus(8) (1499)

Leonhard von Dobschütz*, *Domenico Maria Novara da Ferrara*(18) (1483)

Johann Caspar Bauhin(9) (1649)

Emmanuel Stupanus[c] (1613)

Jacobus Sylvius (Jacques Dubois)(10) (1530)

Jean Tagault*; *François Dubois** (1516)

Scipione Fortiguerra(11) (1493)

Angelo Poliziano

Marsilio Ficino, *Cristoforo Landino**

Johannes Argyropoulos[j]

Basilios Bessarion[k]

Georgios Plethon Gemistos

Demetrios Kydones[l], *Elissaeus Judaeus**

Nilos Kabasilas

Gregory Palamas

Theodore Metochites

Manuel Bryennios

Gregory Chioniadis (1296)

Shams ad-Din Al-Bukhari

Nasir al-Din al-Tusi

Kamal al Din Ibn Yunus

Sharaf al-Din al-Tusi*

Alexander Hegius(12) (1474)

Rudolf Agricola, *Thomas von Kempen à Kempis*(19)

Theodoros Gazes[m]

Vittorino da Feltre[n]

Guarino da Verona (1408)
Manuel Chrysoloras
Demetrios Kydones[l]

Thomas Erpenius(13) (1608)
Joseph Justus Scaliger
Adrien Turnèbe
Jacques Toussain[i]
Guillaume Budé (1486; 1491)
Georgius Hermonymus*, *Janus Lascaris*(20)[o]

Rudolph Snellius (Snel van Royen)(14)[f] (1572)
Valentine Naibod, *Immanuel Tremellius*(21)
Erasmus Reinhold[p]
Jakob Milich
Desiderius Erasmus[e] (1506); *Ulrich Zasius**

Adriaan Heereboord(15) (1631)
Franck Pieterszoon Burgersdijk
Gilbert Jacchaeus
Duncan Liddel; *Jacobus Arminius (Jacob Harmensz)*(22)
John Craig, *Paul Wittich*(23); *Unknown*
Theodor Zwinger
Petrus Ramus (Pierre de La Ramée)[g] (1536); *Bassiano Landi*(24), *Vittore Trincavelli*(25)

Adriaan van den Spieghel(16) (1603)
Hieronymus Fabricius (Girolamo Fabricid' Acquapendente)[h]
Gabriele Falloppio
Antonio Musa Brasavola, *Matteo Realdo Colombo (Renaldus Columbus)*(26)
Niccolò Leoniceno[q]
Ognibene Bonisoli da Lonigo (Omnibonus Leonicenus); *Pelope**, *Pietro Roccabonella*(27)
Vittorino da Feltre[n] (1416)

Jan van Campen (Johannes Campensis)(17) (1519)
Johann Reuchlin (Johannes Kapnion)[r]; *Matthaeus Adrianus**
Johannes Argyropoulos[j] (1444); *Jacob ben Jehiel Loans**

Domenico Maria Novara da Ferrara(18) (1483)
Johannes Müller Regiomontanus, *Luca Pacioli**
Georg von Peuerbach, *Basilios Bessarion*[k]
Johannes von Gmunden
Heinrich von Langenstein (1363; 1375)
Nicole Oresme*; *Unknown*

Thomas von Kempen à Kempis(19)
Geert Gerardus Magnus Groote*, *Florens Florentius Radwyn Radewyns**

Janus Lascaris(20)[o] (1472)
Basilios Bessarion [k] (1436), *Demetrios Chalcocondyles*(28)

Immanuel Tremellius(21) (1549; 1561)
Thomas Cranmer* (1515; 1526); *Unknown*

Jacobus Arminius (Jacob Harmensz)(22) (1582)
Rudolph Snellius (Snel van Royen)(14)[f] (1572)

Paul Wittich(23) (1566)
Valentin Thau

Johannes Hommel

Erasmus Reinhold[p] (1535), *Philipp Melanchthon*(29)

Bassiano Landi(24) (1542)

Giovanni Battista della Monte[s], *Vittore Trincavelli*(25)

Marco Musuro, *Pietro Pomponazzi*[t]; *Niccolò Leoniceno*[q]

Janus Lascaris(20)[o] (1472)

Vittore Trincavelli(25)

Pietro Pomponazzi[t] (1487)

Nicoletto Vernia, *Pietro Roccabonella*(27)

Gaetano da Thiene[u]

Paolo (Nicoletti) da Venezia*

Matteo Realdo Colombo (Renaldus Columbus)(26) (1544)

Andreas Vesalius (Andries van Wesel)

Johannes Winter von Andernach[d] (1527; 1532), *Gemma Frisius (Jemme Reinerszoon)*(30), *Giovanni Battista della Monte*[s]

Pietro Roccabonella(27)

Gaetano da Thiene[u], *Sigismondo Polcastro**

Demetrios Chalcocondyles(28) (1452)

Theodoros Gazes[m] (1433)

Philipp Melanchthon(29) (1511; 1514)

Johannes Stöffler* (1476); *Johann Reuchlin (Johannes Kapnion)*[r]

Gemma Frisius(30) (1529)

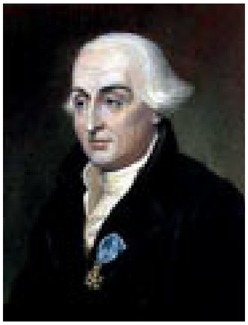
Petrus Curtius (Pieter de Corte)

Maarten van Dorp (Martinus Dorpius)

Leo Outers* (1485)

These "branches" contain around 200 names and by connecting them we get a complex graph showing a "genealogical tree" of the mathematical predecessors of Mihailo Petrović. Let us pay attention to some of them. Among mathematicians we need to mention

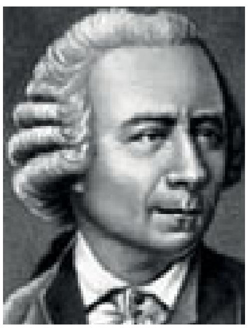
Charles Hermite (1822–1901), French mathematician who made a significant contribution to the numbers theory, square forms, invariant theory, orthogonal polynomials, elliptic functions and algebra. He proved the transcendence of the number *e*. Hermite polynomials, Hermite interpolation, Hermite normal form, Hermitian matrix, Hermitian function, Hermitian operator, etc. were named after him. [5].



Joseph-Louis Lagrange
(1736–1813)

Joseph Liouville (1809–1882), French mathematician; he was dealing with the numbers theory, complex analysis, differential geometry, topology, mathematical physics and astronomy. Sturm–Liouville theory, Liouville's theorem (in complex analysis), Liouville's theorem (in theory of Hamiltonian systems), Riemann–Liouville integrals, Liouville numbers, etc. were named after him. [6].

Simeon Denis Poisson (1781–1840), French mathematician and physicist; he was dealing with mathematical physics and rational mechanics, especially with Fourier integrals, calculus of variations, probability theory, problems in the field of electrostatics and magnetism. Poisson's equation, Poisson distribution, Poisson quotient, etc. were named after him. [7].



Leonhard Euler
(1707–1783)

Joseph Louis Lagrange (1736–1813), Italian-French mathematician and astronomer; he made a significant contribution in all fields of analysis and numbers theory, as well as in classical and celestial mechanics. He is regarded as the greatest mathematician of the 18th century. Lagrange's mean value theorem, Lagrange's theorem (in group theory), Lagrange interpolating polynomial, Lagrangian mechanics (a reformulation of classical mechanics), etc. were named after him. [4].

Leonhard Euler (1707–1783), Swiss mathematician and physicist; he lived and worked in Berlin and Sankt Petersburg. He made great discoveries in many fields of mathematics (mathematical analysis,graph theory, etc.). He introduced a number of terms and representations that are used even today. He also made a significant contribution in the fields of mechanics, optics and astronomy. Euler is regarded as one of the most important mathematicians of the 18th century, and one of the greatest mathematicians of all time. Euler's formula, Euler's theorem, Euler numbers, Euler diagrams, Euler integral, Euler polynomials, etc. were named after him.[8].

Johann Bernoulli (1667–1748), Swiss mathematician, the brother of Jakob Bernoulli and the father of Daniel and Nicolaus II Bernoulli. He is known for his contributions to the development of mathematical analysis; after Newton's death, he was the leader of European mathematicians [9].

Jakob Bernoulli (1655–1705), Swiss mathematician. Made important contributions to the theory of infinite series, solved some of the basic problems of calculus of variations and significantly improved probability theory. He gave analytical terms for several curves (e.g. catenary curve, logarithmic and parabolic spiral). He found solutions for several differential equations. Bernoulli equation, Bernoulli distribution, Bernoulli's formula, Bernoulli polynomials, Bernoulli numbers were named after him[10].

Gottfried Wilhelm Leibniz (1646–1716), German philosopher, mathematician, inventor, lawyer, historian, diplomat and political advisor. He made important contributions to optics and mechanics. He is regarded as the Western civilization's last person possessing encyclopedical knowledge. He introduced infinitesimal calculus independently from Newton, as well as a binary numeral system. Newton-Leibniz formula was named after him, as well as several formulas of differential calculus (about differentiating products of two functions, about differentiating integrals with variable limits, etc.), and several theorems (about medians, about convergence of alternating series) etc.[11].

Gottfried Wilhelm Leibniz
(1646–1716)

Charles Émile Picard (1856–1941), one of the leading French mathematicians of his time. He is best known for his two theorems in the area of functions of complex variables, which were named after him. He contributed significantly to the theory of differential equations. He was one of the first mathematicians to use the ideas of algebraic topology [12]. (Branch 1)

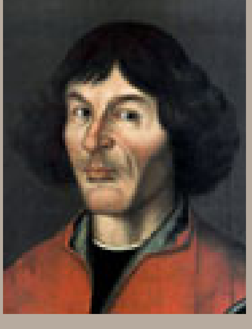
Pierre-Simon Laplace
(1749–1827)

Jean Gaston Darboux (1842–1917), French mathematician. He is best known for his achievements in mathematical analysis (theory of integration, partial differential equations) and differential geometry. Darboux integral, Darboux sums, Darboux function, several theorems (in topology, real analysis), Christofel-Darboux identity (and formula), Darboux's formula, Euler-Darboux equation, Darboux (or Goursat) problem, etc. were named after him. [13]. (Branch 1)

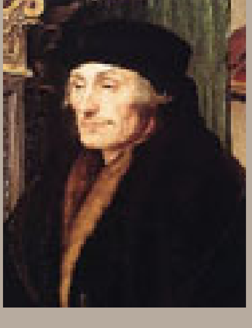
Pierre-Simon Laplace (1749–1827), French mathematician and astronomer; he placed the final stone of mathematical astronomy by summing up and expanding works of his predecessors in his five-volume work *Celestial Mechanics* (Mécanique Céleste). In this masterpiece, instead of geometrical methods of classical mechanic, Laplace used differential and integral calculus, and thus opened much wider range of problems. In solving the applied problems he developed methods of mathematical physics that are widely used even nowadays. His especially important achievements are related to the potential theory and the theory of special functions. Laplace's equation, Laplace transform, Laplace operator, etc. were named after him. [14]. (Branch 3)

Jean Le Rondd'Alembert (1717–1783), French mathematician, physicist and philosopher. Along with Denis Diderot, he is one of the most famous encyclopedists. He is also known for his mathematical research (differential equations, partial differentiations). A well-known criterion for convergence of series was named after him [15]. (Branch 3)

Christiaan Huygens
(1629–1695)



Nicolaus Copernicus
(1473–1543)



Erasmus Roterodamus
(1466–1536)

Alongside mathematicians, there are many scientists who were dealing with other disciplines among Mihailo Petrović's predecessors. Let us mention some of them.

Christiaan Huygens (1629–1695), Dutch mathematician, astronomer and physicist. Historians of science often speak about Huygens as one of the most versatile scientists. He is relatively under-recognized for his role in the development of integral and differential calculus. He claimed that light consists of waves (Huygens principle). He discovered Saturn's moon Titan, was researching Saturn's rings, discovered and described the Orion Nebula, several interstellar nebulas and some binary stars. He wrote the first book on probability theory. His invention of pendulum clock was the turning point in timekeeping [16]. (Branch 6)

Willebrord Snellius (Snel van Royen, 1580–1626), Dutch mathematician, physicist and astronomer. He mathematically formulated the law of light refraction. He set out to measure the length of meridian arc in order to determine Earth's circumference. In this, he used the triangulation method [17]. (Branch 6)

Nicolaus Copernicus (Mikołaj Kopernik, 1473–1543), Polish astronomer, mathematician, lawyer, physician and economist.He formulated a heliocentric model of celestial motion, which was a revolutionary turning point in astronomy, inspiring major discoveries by Kepler and Newton and changing our understanding of the world[18]. (Branch 8)

Domenico Maria Novara da Ferrara (1454–1504), Italian astronomer, mathematician, astrologist, teacher and the friend of Nicolaus Copernicus. Professor of astronomy at the University of Bologna. Among the astronomers of the 15th century, he was regarded as a first-class observer. Copernicus used his observations of the Moon for the refutal of Ptolemy's model [19]. (Branch 8)

Fra Bartolomeo Luca de Pacioli (1445–1517), Italian mathematician, Franciscan, the associate of Leonardo da Vinci and one of the founders of modern accounting. He is often called "father of accounting", because he was the first to publish detailed description of double-entry bookkeeping system. The most important European algebraist of the 15th century [20]. (Branch 18)

Nicolas Lémery (1645–1715), French chemist, pharmacist and physician. He was one of the first to develop the chemical theory of acids and alkalis [21]. (Branch 2)

Desiderius Erasmus (1466–1536), notable Dutch Augustinian theologist, philosopher, philologist and prolific writer. He was a great thinker at the time of the Renaissance, and was considered the leader of European humanists. His most famous work is *In Praise of Folly* [22]. (Branch 5)

Joseph Justus Scaliger (1540–1609), French humanist, philologist, historian and warrior, one of the founders of modern historical chronology, publisher and commentator of ancient manuscripts [23]. (Branch 13)

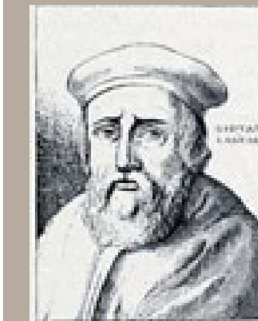
Gabriele Falloppio (1523–1562), Italian surgeon, anatomist and botanist. He greatly contributed to what was known about ear and genitals (cochlea, Fallopian tube) [24]. (Branch 16)

Janus Lascaris (Ἰανὸς Λάσκαρις, 1445–1535), well-known Greek scientist in the Renaissance period. Came from a noble family. After the fall of Constantinople, he came to Italy via Peloponnese and Crete. He was teaching at universities in Italy, mainly dealing with Greek studies, and was collecting Greek manuscripts. He started working for France and was its ambassador to Venice for a while. He participated in the establishment of libraries in Blois and Fontainebleau. One of his students was Dimitrije Ljubavić(1519–1564), Serbian Orthodox priest, humanist, writer and publisher who established the first formal contact between the Eastern Orthodox church and Lutherans in 1559 [25]. (Branch 13)

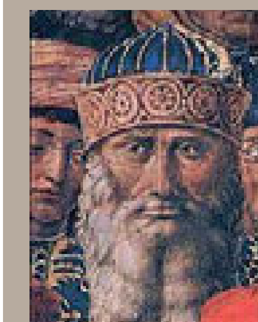
Basilios Bessarion (Βασίλειος Βησσαρίων, 1403–1472), one of the most known Byzantine humanists in Italy, strongly contributed to the revival of the research of ancient literature, especially Greek one, in the 15th century's Western Europe. He was a strong advocate of the union of the Catholic Church and the Orthodox churches, he was Metropolitan bishop of Nicaea, cardinal, Latin patriarch of Constantinople, the candidate for Pope on two occasions. He was looking after the members of the Byzantine imperial family of Palaeologus, participated in negotiations about the marriage of Russian Grand Prince Ivan III Vasilyevich and Sophia Palaeologus [26]. (Branch 11)

Georgios Plethon Gemistos (Γεώργιος Πλήθων Γεμιστός, 1355–1452), Byzantine neoplatonic philosopher, professor in Mistras and the advocate of the renewal of Hellenism as a basis for resistance against the Ottoman Empire. He renewed platonic philosophy and gathered students, thus creating a school of ancient philosophy [27]. (Branch 11)

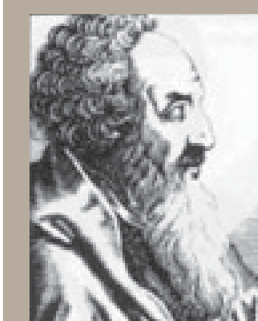
Demetrios Kydones (Δημήτριος Κυδώνης, 1324–1398), Byzantine humanistic writer, statesman and theologist, who significantly contributed to the beginning of research of the Greek language, literature and culture within the Italian Renaissance. He translated the most important works of West European writers into Greek, including the works of Aurelius Augustinus and Thomas Aquinas. He advocated the union of the Western and Eastern churches [28]. (Branch 11)



Janus Lascaris
(1445–1535)



Georgius Gemistus Plethon
(1355–1452)



Basilios Bessarion
(1403–1472)

Gregory Palamas
(1296–1359)

Gregory Palamas (Γρηγόριος Παλαμάς, 1296–1359), Byzantine theologist, monk at Athos and Archbishop of Thessaloniki. In Orthodox theology he developed a theory on hesychasm and Divine uncreated energies. A few years after his death, he was canonized by the Orthodox Church [29]. (Branch 11)

Shams ad-Din Al-Bukhari, Persian astronomer from the end of the 13th century [30]. (Branch 11)

Nasir al-Din al-Tusi (نصیرالدینطوسی; 1201–1274), Persian polymath, architect, philosopher, physician, scientist and theologist, often regarded as the creator of trigonometry as a mathematical discipline. One of the greatest Persian scientists of the later period [31]. (Branch11)

Kamal al Din IbnYunus (1156–1241), prominent Persian mathematician and jurist, head of the well-known observatory in Mosul [32]. (Branch 11)

Sharaf al-Din al-Tusi (شرفالدینمظفربنمحمدبنمظفرتوسی; 1135–1213), Persian mathematician and astronomer, worked during the so-called Islamic Golden Age. He was born in the town of Tus, after which he was named. He taught ephemerides and astrology in Aleppo and Mosul [33], (Branch 11). According to the MGP data, Sharaf al-Din al-Tusi has more than 154,000 successors.

It is from this short overview that one can note that in the mathematical genealogy of Mihailo Petrović there are representatives from many countries and nations, who were dealing with various disciplines – there are no state or national borders, nor borders between various scientific disciplines. *Gens una sumus*!

STUDENTS AND SUCCESSORS OF MIHAILO PETROVIĆ

According to MGP data and data from the Faculty of Mathematics of the University of Belgrade (BU), 11 doctoral dissertations were defended under the mentorship of Mihailo Petrović, all of them at the University of Belgrade, and there are 903 "successors" of Mihailo Petrović in total at the moment. We are providing the list of Mihailo Petrović's doctorands. Numbers next to their names mark the year in which the dissertation was defended and the number of successors.

Mladen Berić	1912.	Dragoslav Mitrinović	1933, 121
Sima Marković	1913.	Danilo Mihnjević	1934.
Tadija Pejović	1923, 195	Konstantin Orlov	1934, 84
Radivoj Kašanin	1924, 16	Petar Muzen	1937.
Jovan Karamata	1926, 481	Dragoljub Marković	1938, 1
Miloš Radojčić	1928.		

Let us take a closer look at Mihailo Petrović's students and successors.

Mladen Berić (1885–1935). Associate professor ("extraordinary professor") at the BU in 1919. He was head of the General State Statistics. In 1921 he left the University, and was not dealing with mathematics anymore[34].

Sima Marković
(1888–1935)

Sima Marković (1888–1939). He graduated and got his PhD degree under Mihailo Petrović's mentorship. Professor at the Second Belgrade Gymnasium, Assistant professor ("Docent") at the BU. Secretary of the Communist Party of Yugoslavia (CPY), a member of parliament and head of the Parliamentary group of CPY, a member of the Executive Committee of Comintern. He was killed in Stalin's purges, and was rehabilitated in 1958[35].

Tadija Pejović
(1892–1982)

Tadija Pejović (1892–1982). Full professor ("Ordinary professor") at the BU, Dean of the Faculty of Science and Mathematics, the first president of the Society of Mathematicians and Physicists of Serbia, an author of several university textbooks. He was mainly dealing with the theory of differential equations. He was one of the 1300 Corporals in World War I. When World War II broke out, he was mobilized as a reserve Lt. Colonel, was captured and remained in captivity for the rest of the war. He was the president of the Association of 1300 Corporals [36,37]. He mentored 17 doctorands:

Vojin Dajović	1956, 25	Zagorka Sakl-Šnajder	1960, 1
Dobrivoje Mihajlović	1956.	Milorad Bertolino	1961, 3
Rastko Stojanović	1956, 1	Blažo Okiljević	1962.
Ernest Stipanić	1957.	Nedeljko Parezanović	1962, 10

Slaviša Prešić	1963, 94	Rade Dacić	1965.
Zoran Ivković	1964, 36	Milica Dajović	1965.
Milosav Marjanović	1964, 12	Borivoje Mihajlović	1965.
Zoran Popstojanović	1964.	Časlav Đaja	1967.
Petar Todorović	1964.		

Total number of his successors at this point is 194. Among them are academicians Milosav Marjanović (SASA), Vojin Dajović (MASA), Miodrag Mateljević (SASA), professors Milorad Bertolino, Nedeljko Parezanović, Slaviša Prešić, Zoran Ivković, Svetozar Milić, Jovan Mališić, Žarko Mijajlović, Miroljub Jevtić, Milutin Obradović, Pavle Mladenović, Rade Živaljević, Miodrag Živković and others.

Radivoj Kašanin
(1892–1989)

Radivoj Kašanin (1892–1989). Full professor at the BU, SASA academician, rector of the Great Technical School, one of the founders of the Mathematical Institute of the SASA and its head. He was dealing with mathematical analysis (function theory, differential equations), mechanics, astronomy and geo-physics. In World War I, as a volunteer in the Serbian army, he took part in fights in Dobruja, Bessarabia and on the Macedonian front [38].

He mentored two doctorands (Tatomir Anđelić, 1946,14 and Radmilo Đorđević, 1963). The list of his successors includes academician Tatomir Anđelić, professors Đorđe Mušicki, Marko Leko, Ilija Lukačević, Milan Plavšić, etc.

Jovan Karamata (1903–1967). One of the greatest Serbian mathematicians of the 20th century. Full professor at the BU, a corresponding member of the Serbian Royal Academy, an academician of the SASA. He was invited to move to the University of Geneva in 1950, where he stayed till his death. He is the founder of the school of theory of real functions. He is the author of the theory of regularly varying functions [39,40]. He mentored 12 doctorands:

Vojislav Avakumović	1939, 294	Šefkija Raljević	1955.
Miodrag Tomić	1950.	Bogdan Bajšanski	1956, 12
Slobodan Aljančić	1953, 32	Milenko Steković	1956.
Ranko Bojanić	1953, 17	Monique Vuilleumier	1965.
Vladeta Vučković	1953, 1	H. Baumann	1965.
Bogoljub Stanković	1954, 95	Ronald Coifman	1965, 115

Total number of his successors at this point is 481. Among them are academicians Vojislav Avakumović, Miodrag Tomić, Slobodan Aljančić, Bogoljub Stanković, Olga Hadžić, Stevan Pilipović (SASA), Manojlo Maravić, Mirjana Vuković (ASA of

Bosnia and Herzegovina), Endre Pap (ASA of Vojvodina), professors Ronald Coifman, Ranko Bojanić, Bogdan Bajšanski, Dušan Adamović, Dragoslav Herceg, Mila Mršević, Arpad Takacsi, Ljiljana Cvetković, Nataša Krejić and others.

Miloš Radojčić (1903–1975). Full professor at the BU until 1959, after that he worked at the University of Khartoum (Sudan) and in the National Centre for Scientific Research in Paris. A corresponding member of the SASA. He was one of the most important Serbian mathematicians and one of the greatest intellectuals of the first half of the 20th century. He introduced synthetic and descriptive geometry into university curriculum. He wrote two high-ranking textbooks. In his scientific work, he was dealing with the theory of complex analytic functions and the theory of relativity [41].

Dragoslav Mitrinović (1908–1995). Full professor at the University of Skopje and at the BU, academician of the Macedonian Academy of Sciences and Arts. In his scientific work, he was dealing with inequalities, functional inequalities, number theory, special functions, differential equations and complex analysis. He published a huge number of university textbooks, books and scientific papers (430 bibliographical units) [42,43]. He mentored 33 doctorands:

Blagoj Popov	1952.	Živko Madevski	1973.
Ivan Bandić	1958.	Ivan Lacković	1975, 2
Lazar Karadžić	1958.	Dušan Slavić	1975.
Dragomir Đoković	1963, 10	Ljubomir Stanković	1975.
Kovina Milošević-Rakočević	1963.	Budimir Zarić	1975.
Danica Perčinkova	1963.	Gradimir Milovanović	1976, 32
Petar Vasić	1963, 29	Žarko Mitrović	1976.
Ilija Šapkarev	1964.	Ismet Dehiri	1977.
Velimir Penavin	1965.	Petar Lazov	1977.
Radosav Đorđević	1966.	Lazar Đorđević	1978.
Dragan Dimitrovski	1968.	Miomir Stanković	1979.
Radovan Janić	1968.	Nikola Azanjac	1980.
Savo Jovanović	1968.	Igor Milovanović	1980, 1
Jovan Kečkić	1970.	Miodrag Petković	1980, 3
Dragoš Cvetković	1971, 11	Vlajko Kocić	1981.
Ionel Stamate	1971.	Behdžet Mesihović	1987.
Živko Tošić	1971.		

Miloš Radojčić
(1903–1975)

Total number of his successors at this point is 121. Among them are academicians Dragoš Cvetković, Gradimir Milovanović, Ivan Gutman (SASA), Blagoj Popov (MASA), and Josip Pečarić (CASA), professors Dragomir Đoković, Petar Vasić, Dragan Dimitrovski, Jovan Kečkić, Slobodan Simić, Miodrag Petković, Dragan Stevanović and others.

Danilo Mihnjević. There is no much information about Danilo Mihnjević. He moved to Yugoslavia (i.e. the Kingdom of Serbs, Croats and Slovenians) as a refugee from Russia after 1917. He pursued his career as a professor in gymnasiums in Kragujevac and Zrenjanin (until 1953) [44,45].

Konstantin Orlov (1907–1985). Full professor at the BU. A refugee from Russia, just like D. Mihnjević. After obtaining his PhD degree, he worked for a long time in the Fifth and the Third Boys Gymnasiums in Belgrade, and in 1947 he was elected a teaching assistant at the Faculty of Philosophy of the BU. That same year he moved to the newly-founded Faculty of Science and Mathematics, where he stayed until his retirement, advancing through all the academic ranks. He was teaching several courses, mainly in numerical and applied mathematics. As an UNESCO expert and invited lecturer he visited several foreign universities [46]. He mentored nine doctorands:

Petar Madić	1965.	Boško Jovanović	1976, 67
Momčilo Ušćumlić	1965.	Arif Zolić	1977.
Mihail Arsenović	1972.	Ljubomir Protić	1978, 8
Miroslava Stojanović	1973.	Branko Savić	1978.
Max Wotulo	1973.		

Total number of his successors at this point is 84. Among them are EndreSüli, professor at the University of Oxford, a foreign member of the SASA and a member of the European Academy of Sciences (EurASc), professor Boško Jovanović and others.

Petar Muzen. There is no much information about Petar Muzen. He was an associate of the Astronomical Observatory in Belgrade [47].

Dragoljub Marković (1903–1965). Full professor at the BU. The main field of his scientific work was algebra, especially problems related to the limits of roots in algebraic equations, calculating roots, i.e. factorization [48]. He mentored one doctorand (Jovan Petrić, 1960).

INDIRECT RELATIONS

Đuro Kurepa
(1907–1993)

Đuro Kurepa (1907–1993). Full professor at the University of Zagreb (1946–65) and at the BU (1965–77), corresponding member of the YASA, academician of the ASA of Bosnia and Herzegovina and of the SASA. He went for specialization in Paris, Warsaw and Princeton, and spent some time as a guest professor at the University of Colorado Boulder (the USA). He was the president of the Society of Mathematicians and Physicists of Croatia, president of the Association of Societies of Mathematicians and Physicists of Yugoslavia, vice president of the International Commission on Mathematical Instruction, president of the Yugoslav National Committee for Mathematics, president of the Balkan Mathematical Union, president of the Commission for Scientific Work of the Association of Mathematicians, Physicists and Astronomers of Yugoslavia. He was dealing with the set theory, algebra and number theory, general topology, etc. In the set theory, he contributed to the theory of trees, so terms such as Kurepa tree, Kurepa hypothesis, Kurepa continuum, etc. are used even today. He introduced the operation of the left factorial in mathematics [49,50].

Đuro Kurepa defended his doctoral dissertation "Ordered and Ramified Sets" (Ensembles ordonné set ramifiés) in 1935 at Sorbonne. His mentor was René Maurice Fréchet, and Fréchet's mentor was Jacques Salomon Hadamard, while Hadamard's (co)-mentors were Charles Émile Picard and Jules Tannery. And since Charles Émile Picard was the second mentor of Mihailo Petrović, we can conclude that Mihailo Petrović is a "mathematical great-uncle" of Đuro Kurepa.

Đuro Kurepa mentored 27 doctorands, and total number of his successors at this point is 162. Among them are academicians Aleksandar Ivić, Stevo Todorčević (SASA), Vladimir Rakočević (a corresponding member of SASA), Veselin Perić (ASA of Bosnia and Herzegovina, ASA of the Republic of Srpska, a honorary member of MASA), professors Stevan Stojanović, Ljubomir Ćirić, Pavle Miličić, Branislav Mirković, Ratko Tošić, Ljubiša Kočinac, Đorđe Dugošija, Aleksandar Lipkovski, Stojan Radenović, Zoran Kadelburg, Miroslav Pavlović, Milutin Dostanić and others.

REFERENCES

[1] https://sr.wikipedia.org/sr-el/Михаило_Петровић_Алас[190]
[2] https://sr.wikipedia.org/sr-el/Милутин_Гарашанин_(политичар)
[3] https://www.genealogy.math.ndsu.nodak.edu/id.php?id=53473
[4] https://en.wikipedia.org/wiki/Joseph-Louis_Lagrange
[5] https://en.wikipedia.org/wiki/Charles_Hermite
[6] https://en.wikipedia.org/wiki/Joseph_Liouville
[7] https://en.wikipedia.org/wiki/Siméon_Denis_Poisson
[8] https://en.wikipedia.org/wiki/Leonhard_Euler
[9] https://en.wikipedia.org/wiki/Johann_Bernoulli
[10] https://en.wikipedia.org/wiki/Jacob_Bernoulli
[11] https://en.wikipedia.org/wiki/Gottfried_Wilhelm_Leibniz
[12] https://en.wikipedia.org/wiki/Émile_Picard
[13] https://en.wikipedia.org/wiki/Jean_Gaston_Darboux
[14] https://en.wikipedia.org/wiki/Pierre-Simon_Laplace
[15] https://en.wikipedia.org/wiki/Jean_le_Rond_d'Alembert
[16] https://en.wikipedia.org/wiki/Christiaan_Huygens
[17] https://en.wikipedia.org/wiki/Willebrord_Snellius
[18] https://en.wikipedia.org/wiki/Nicolaus_Copernicus
[19] https://en.wikipedia.org/wiki/Domenico_Maria_Novara_da_Ferrara
[20] https://en.wikipedia.org/wiki/Luca_Pacioli
[21] https://en.wikipedia.org/wiki/Nicolas_Lemery
[22] https://en.wikipedia.org/wiki/Erasmus
[23] https://en.wikipedia.org/wiki/Joseph_Justus_Scaliger
[24] https://en.wikipedia.org/wiki/Gabriele_Falloppio
[25] https://en.wikipedia.org/wiki/Janus_Lascaris
[26] https://en.wikipedia.org/wiki/Basilios_Bessarion
[27] https://en.wikipedia.org/wiki/Gemistus_Pletho
[28] https://en.wikipedia.org/wiki/Demetrios_Kydones
[29] https://en.wikipedia.org/wiki/Gregory_Palamas
[30] R. Mercier: Shams al-Dīn al-Bukhārī, In: Thomas Hockey et al. (eds.). The Biographical Encyclopedia of Astronomers, Springer Reference. New York: Springer, 2007, pp. 1047–1048.
[31] http://www-history.mcs.st-andrews.ac.uk/Biographies/Al-Tusi_Nasir.html
[32] https://adonis49.wordpress.com/tag/kamal-al-din-ibn-yunus/
[33] http://www-history.mcs.st-andrews.ac.uk/Biographies/Al-Tusi_Sharaf.html
[34] E. Stipanić: Mladen Berić (1885–1935). U: Spomenica 130 godina Matematičkog fakulteta (http://www.math.rs/files/Mladen_Beric.pdf).
[35] https://sr.wikipedia.org/wiki/Сима_Марковић
[36] Lj. Protić: Tadija Pejović. U: Spomenica 125 godina Matematičkog fakulteta, Beograd 1998 (http://www.matf.bg.ac.rs/files/PejovicT.pdf).
[37] https://sr.wikipedia.org/sr-el/Тадија_Пејовић
[38] https://sr.wikipedia.org/sr-el/Радивој_Кашанин

[39] A. Nikolić: Jovan Karamata. U: Spomenica 130 godina Matematičkog fakulteta, Beograd 2003 (http://www.math.rs/files/Jovan_Karamata.pdf).
[40] https://en.wikipedia.org/wiki/Jovan_Karamata
[41] R. Dacić, M. Mateljević: Miloš Radojčić (http://www.mi.sanu.ac.rs/History/radojcic.htm)
[42] G. V. Milovanović: Dragoslav S. Mitrinović (1908–1995), U: Život i delo srpskih naučnika, Biografije i bibliografije, 6 (M. Sarić, ed.), SANU, Beograd, 2000, 519–581.
[43] https://sr.wikipedia.org/wiki/Драгослав_Митриновић
[44] Радојка Марковић, Лично саопштење, око 1980.
[45] http://www.zrenjaninskagimnazija.edu.rs/?page_id=23
[46] A. Zolić: Konstantin Pavlovič Orlov. U: Spomenica 125 godina Matematičkog fakulteta, Beograd 1998 (http://www.matf.bg.ac.rs/files/KonstantinOrlov.pdf).
[47] J. Milogradov-Turin: Department of Astronomy of the University of Belgrade, Publ. Astron. Obs. Belgrade, No. 75 (2003), 289–292.
[48] S. Prešić: Dragoljub Marković. U: Spomenica 130 godina Matematičkog fakulteta, Beograd 2003 (http://www.math.rs/files/Markovic_Dragoljub.pdf).
[49] Ž. Mijajlović: Đuro Kurepa. U: Spomenica 130 godina Matematičkog fakulteta, Beograd 2003 (http://www.math.rs/files/Djuro_Kurepa.pdf).
[50] https://en.wikipedia.org/wiki/Đuro_Kurepa

MATHEMATICAL GENEALOGICAL TREE OF MIHAILO PETROVIĆ

Compiled by Žarko MIJAJLOVIĆ
University of Belgrade, Faculty of Mathematics

This list contains the names of doctors of mathematical sciences, linked through a supervisorial relationship to Mihailo Petrović. This lineage can be graphically represented as a genealogical tree structure. The nodes in the tree represent mathematicians, while the links between them indicate the mentoring relationship between a doctoral candidate and his or her mentor. The name of a node includes the names of mathematicians, the university and the year of defending their doctorate theses. The name of other mentor, if existent, is indicated beside the candidate's name and marked with a star *. When it comes to some foreign mathematical successors of Mihailo Petrović, due to their large number, the names of their successors have not been included in this list. In such cases, next to a mathematician's name is indicated the number of his mathematical successors. Around 450 doctoral disertations of the authors from this list, including the ones defended before World War II, can be found in the Virtual Library of the Faculty of Mathematics in Belgrade (VL).

Sources:
internet databases of the Faculty of Mathematics in Belgrade
http://poincare.matf.bg.ac.rs/informacije/dok.htm
VL: http://elibrary.matf.bg.ac.rs
Mathematics Genealogy Project,
https://www.genealogy.math.ndsu.nodak.edu
Software: S. Simić, R. Popović

Mihailo Petrović, U. Pariz 1894.

- Mladen Berić, U. Belgrade, 1912.
- Sima Marković, U. Belgrade, 1913.
- Tadija Pejović, U. Belgrade, 1923.
    - Vojin Dajović, U. Belgrade, 1956.
        - Mioljub Nikić, U. Belgrade, 1972.
            - Žarko Pavićević, U. Belgrade, 1983.
        - Vladimir Mićić, U. Belgrade, 1973.
            - Milutin Obradović, U. Belgrade, 1984.
                - Nikola Tuneski, U. Belgrade, 1999.
        - Miodrag Perović, U. Belgrade, 1978.
        - Dušan Georgijević, U. Belgrade, 1979.
        - Miroljub Jeftić, U. Belgrade, 1979.
            - Jovo Šarović, U. Belgrade, 1988.
            - Ivan Jovanović, U. Belgrade, 1992.
        - Miodrag Mateljević, U. Belgrade, 1979.
            - Vladimir Marković, U. Belgrade, 1998.
                - Alastair Fletcher, U. Warwick, 2006.
                - Anestis Fletcher, U. Warwick, 2006.
            - David Kaljaj, U. Belgrade, 2002.
                - Marijan Marković, U. Belgrade, 2013.
                - Đorđije Vujadinović, U. Belgrade, 2014.
            - Vesna Manojlović, U. Belgrade, 2008.
            - Marijan Marković, U. Belgrade, 2013.
            - Miljan Knežević, U. Belgrade, 2014.
        - Enes Udovičić, U. Belgrade, 1980.
        - Stojan Duborija, U. Belgrade, 1981.
        - Miloje Rajović, U. Belgrade, 1985.
    - Dobrivoje Mihajlović, U. Belgrade, 1956.
    - Rastko Stojanović, U. Belgrade, 1956.
        - Dragovan Blagojević, U. Belgrade, 1969.
    - Ernest Stipanić, U. Belgrade, 1957.
    - Zagorka Sakl-Šnajder, U. Belgrade, 1960.
        - Dragoljub Cvetković, U. Belgrade, 1985.
    - Milorad Bertolino, U. Belgrade, 1961.
        - Pavle Pejović, U. Belgrade, 1971.
        - Božo Vrdoljak, U. Belgrade, 1975.
        - Julka Knežević-Miljanović, U. Belgrade, 1979.
            - Jelena Manojlović, U. Belgrade, 2000.
    - Blažo Okiljević, U. Belgrade, 1962.
    - Nedeljko Parezanović, U. Belgrade, 1962.
        - Vojislav Stojković, U. Belgrade, 1981.
        - Ivan Obradović, U. Belgrade, 1991.
        - Milan Tuba, U. Belgrade, 1993.
            - Raka Jovanović, U. Belgrade, 2012.

        - ◆ Nebojša Baćanin, U. Belgrade, 2015.
        - ◆ Adis Alihodžić, U. Belgrade, 2016.
    - ▹ Duško Vitas, U. Belgrade, 1993.
        - ◆ Ranka Stanković, U. Belgrade, 2009.
        - ◆ Miljana Mladenović, U. Belgrade, 2017.
        - ◆ Staša Vujičić-Stanković, U. Belgrade, 2017.
    - ▹ Sanja Petrović, U. Belgrade, 1997.
    - ▹ Davor Radenović, U. Belgrade, 1998.
- ➢ Slaviša Prešić, U. Belgrade, 1963.
    - ▹ Koriolan Gilezan, U. Belgrade, 1971.
    - ▹ Janez Ušan, U. Belgrade, 1971.
    - ▹ Svetozar Milić, U. Belgrade, 1972.
        - ◆ Zoran Stojaković, U. Belgrade, 1974.
            - ➤ Đura Paunić, U. Novi Sad, 1986.
                - ❒ Mirjana Ivanović, U. Novi Sad (1992)
                    - Δ Borislav Jošanov, U. Novi Sad, 2001.
                    - Δ Miha Badjonski, U. Novi Sad, 2004.
                    - Δ Vladimir Kurbalija, U. Novi Sad, 2009.
                    - Δ Miloš Radovanović, U. Novi Sad, 2011.
                    - Δ Aleksandra Klašnja-Milićević, U. Novi Sad, 2013.
                    - Δ Boban Vesin, U. South-Eastern Norway, 2014.
                    - Δ Zoltan Geler, U. Novi Sad, 2015.
                    - Δ Miloš Savić, U. Novi Sad, 2015.
                    - Δ Dejan Mitrović, U. Novi Sad, 2015.
                    - Δ Milinko Mandić, U. Novi Sad, 2016.
                    - Δ Veljko Aleksić, U. Kragujevac, 2017.
                - ❒ Zoran Budimac, U. Novi Sad, 1994.
                    - Δ Dragan Simić, U. Novi Sad, 2005.
                    - Δ Dragoslav Pešović, U. Novi Sad, 2007.
                    - Δ Zoran Putnik, U. Novi Sad, 2014.
                    - Δ Gordana Rakić, U. Novi Sad, 2015.
        - ◆ Stojan Bogdanović, U. Novi Sad, 1980.
            - ➤ Veljko Vuković, U. Priština, 1985.
            - ➤ Todor Malinović, U. Novi Sad, 1986.
            - ➤ Petar Protić, U. Novi Sad, 1986.
                - ❒ Milan Božinović, U. Niš, 1997.
                - ❒ Nebojša Stevanović, U. Niš, 2006.
            - ➤ Miroslav Ćirić, U. Belgrade, 1991.
                - ❒ Tatjana Petkovič, U. Niš, 1998.
                - ❒ Žarko Popović, U. Niš, 2001.
                - ❒ Jelena Ignjatović, U. Niš, 2007.
                    - Δ Ivana Micić, U. Niš, 2014.
                    - Δ Zorana Jančić, U. Niš, 2014.
                    - Δ Ivona Brajović, U. Niš, 2015.
                - ❒ Aleksandar Stamenković, U. Niš, 2010.
                - ❒ Milan Bašić, U. Niš, 2011.

- ❐ Nada Damljanović, U. Niš, 2012.
- ❐ Velimir Ilić, U. Niš, 2012.
- ❐ Vesna Veličković, U. Niš, 2012.
- ❐ Ivan Stanković, U. Niš, 2017.
- ➤ Melanija Mitrović, U. Niš, 2000.
- ➤ Nela Malinović-Jovanović, U. Niš, 2008.
- ◆ Siniša Crvenković, U. Novi Sad, 1981.
- ➤ Rozália Madarász, U. Novi Sad, 1989.
- ❐ Ivica Bošnjak, U. Novi Sad, 2002.
- ➤ Igor Dolinka, U. Novi Sad, 2000.
- ➤ Milovan Vinčić, U. Novi Sad, 2001.
- ➤ Ivan Pavkov, U. Novi Sad, 2001 (*Nebojša Ralević)
- ◆ Branimir Šešelja, U. Novi Sad, 1981.
- ➤ Vjekoslav Budimirović, U. Novi Sad, 2001.
- ➤ Mirna Udovičić, U. Novi Sad, 2014.
- ➤ Elijah Eghosa Edeghabha, U. Novi Sad, 2017.
- ➤ Ana Slivkova, U. Novi Sad, 2018.
- ◆ Velimir Pavlović, U. Belgrade, 1982.
- ◆ Dragan Blagojević, U. Belgrade, 1987.
- ◆ Andreja Tepavčević, U. Novi Sad, 1993.
- ➤ Vera Lazarević, U. Novi Sad, 2001.
- ➤ Branka Budimirović, U. Novi Sad, 2011.
- ➤ Vanja Stepanović, U. Novi Sad, 2012.
- ➤ Marijana Gorjanac-Ranitović, U. Novi Sad, 2015.
- ➤ Omalkhear Salem Almabruk Bleblou, U. Novi Sad, 2017.
- ➤ Marija Đukić, U. Novi Sad, 2018.
- ▹ Branka Alimpić, U. Belgrade, 1973.
- ◆ Aleksandar Krapež, U. Belgrade, 1980.
- ➤ Sava Krstić, U. Belgrade, 1986.
- ◆ Blagoje Stamenković, U. Belgrade, 1989.
- ▹ Nataša Božović, U. Belgrade, 1975.
- ◆ Ranko Šćepanović, U. Belgrade, 1985.
- ▹ Žarko Mijajlović, U. Belgrade, 1977.
- ◆ Aleksandar Jovanović, U. Belgrade, 1982.
- ➤ Aleksandar Perović, U. Belgrade, 2008.
- ➤ Slađana Spasić, U. Belgrade.
- ➤ Zoran Nikolić, U. Belgrade.
- ➤ Srđan Stamenković, U. Belgrade.
- ◆ Slobodan Vujošević, U. Belgrade, 1982.
- ◆ Miodrag Rašković, U. Belgrade, 1983.
- ➤ Radosav Đorđević, U. Kragujevac, 1991.
- ❐ Nebojša Ikodinović, U. Kragujevac, 2006.
- Δ Vladimir Ristić, U. Kragujevac, 2013.

- Δ Marija Boričić, U. Belgrade, 2016.
- ➤ Zoran Ognjanović, U. Kragujevac, 1999.
  - ☐ Dragan Doder, U. Belgrade, 2011.
  - ☐ Angelina Ilić Stepić, U. Belgrade, 2012.
  - ☐ Bojan Marinković, U. Novi Sad, 2014.
  - ☐ Dejan Masliković, U. Belgrade, 2016.
  - ☐ Tatjana Stojanović, U. Kragujevac, 2016.
- ◆ Rade Živaljević, U. Belgrade, 1983. (*Dragoljub Aranđelović)
  - ➤ Žana Kovijanić, U. Belgrade, 2000.
  - ➤ Vladimir Grujić, U. Belgrade, 2002.
  - ➤ Đorđe Baralić, U. Belgrade, 2013.
- ◆ Milan Grulović, U. Belgrade, 1984.
  - ➤ Miloš Kurilić, U. Novi Sad, 1994.
    - ☐ Aleksandar Pavlović, U. Novi Sad, 2009.
    - ☐ Boris Šobot, U. Novi Sad, 2009.
    - ☐ Boriša Kuzeljević, U. Novi Sad, 2014.
    - ☐ Nenad Morača, U. Novi Sad, 2018.
- ◆ Miodrag Živković, U. Belgrade, 1990.
  - ➤ Bojan Vučković, U. Belgrade, 2018.
- ◆ Milenko Mosurović, U. Belgrade, 2000.
- ◆ Predrag Janičić, U. Belgrade, 2001. (*Alan Richard Bundy)
  - ➤ Filip Marić, U. Belgrade, 2009.
    - ☐ Milan Banković, U. Belgrade, 2016.
    - ☐ Mirko Stojadinović, U. Belgrade, 2016.
    - ☐ Danijela Simić, U. Belgrade, 2017.
  - ➤ Mladen Nikolić, U. Belgrade, 2013.
  - ➤ Vesna Marinković, U. Belgrade, 2015.
  - ➤ Sana Stojanović Đorđević, U. Belgrade, 2016.
- ◆ Radoš Bakić, U. Belgrade, 2002.
- ◆ Branko Malešević, U. Belgrade, 2007.
  - ➤ Ivana Jovović, U. Belgrade, 2013.
  - ➤ Milica Savatović, U. Belgrade, 2018.
- ◆ Dragan Stankov, U. Belgrade, 2008. (*Aleksandar Ivić)
- ◆ Dušan Jokanović, U. Podgorica, 2009.
- ◆ Vesna Vučković, U. Belgrade, 2010.
- ◆ Tanja Stojadinović, U. Belgrade, 2014.
- ◆ Vanja Korać, U. Belgrade, 2014.
- ◆ Stevan Kordić, U. Belgrade, 2016. (*Tatjana Davidović)

▷ Gradimir Vojvodić, 1979.
- ◆ Jovanka Pantović, 2000. (*Ratko Tošić)
  - ➤ Svetlana Jakšić, U. Novi Sad, 2016.

▷ Dragić Banković, 1980.
- ◆ Ljubica Diković, U. Kragujevac, 2009.

        - ◆ Silvana Marinković, U. Kragujevac, 2011.
        - ◆ Verica Milutinović, U. Kragujevac, 2015.
    - ▹ Miloš Miličić, 1982.
    - ▹ Milan Božić, 1983.
        - ◆ Daniel Romano, 1986.
        - ◆ Zoran Stokić, 1996.
    - ▹ Branislav Boričić, 1984.
    - ▹ Mile Tasić, 1992.
    - ▹ Miodrag Kapetanović, 1996.
    - ▹ Zoran Đorđević, 1998.
- ➢ Zoran Ivković, U. Belgrade, 1964.
    - ▹ Jovan Mališić, U. Belgrade, 1973.
        - ◆ Pavle Mladenović, U. Belgrade, 1985.
            - ➤ Siniša Stamatović, U. Belgrade, 1993.
            - ➤ Bojana Milošević, U. Belgrade, 2016.
            - ➤ Ivana Ilić, U. Belgrade, 2013.
        - ◆ Tibor Poganj, U. Belgrade, 1986.
            - ➤ Joško Dvornik, U. Rijeka, 2006.
            - ➤ Biserka Draščić, U. Zagreb, 2009.
            - ➤ Dragana Jankov, U. Zagreb, 2011.
            - ➤ Josip Orović, U. Rijeka, 2012.
            - ➤ Miljenko Petrović, U. Rijeka, 2012.
        - ◆ Biljana Popović, U. Belgrade, 1990.
            - ➤ Miroslav Ristić, U. Niš, 2002.
                - ❐ Božidar Popović, U. Niš, 2011.
                - ❐ Aleksandar Nastić, U. Niš, 2012.
            - ➤ Vladica Stojanović, U. Priština, 2007.
        - ◆ Vesna Jevremović, U. Belgrade, 1991.
        - ◆ Dragan Đorić, U. Belgrade, 2002.
    - ▹ Jelena Bulatović, U. Belgrade, 1975.
        - ◆ Ljiljana Petrović, U. Kragujevac, 1988.
            - ➤ Slađana Dimitrijević, U. Kragujevac, 2013.
            - ➤ Dragana Valjarević, U. Kragujevac, 2013.
    - ▹ Slobodan Janković, U. Belgrade, 1979.
    - ▹ Ratomir Pažanin, U. Belgrade, 1980.
    - ▹ Fuat Rizvanoli, U. Belgrade, 1982.
    - ▹ Blagota Lučić, U. Belgrade, 1985.
    - ▹ Ljiljana Petruševski, U. Belgrade, 1986.
    - ▹ Svetlana Janković, U. Belgrade, 1987.
    - ▹ Zoran Glišić, U. Belgrade, 1987.
    - ▹ Slobodanka Mitrović, U. Belgrade, 1987.
    - ▹ Dražen Pantić, U. Belgrade, 1988.

      - Viktor Obuljen, U. Belgrade, 1997.
    - Milosav Marjanović, U. Belgrade, 1964.
      - Miodrag Mišić, U. Belgrade, 1971.
      - Milan Drešević, U. Belgrade, 1973.
      - Momir Stanojević, U. Belgrade, 1976.
      - Ante Vučemilović, U. Belgrade, 1979.
      - Dušan Milovančević, U. Belgrade, 1982.
      - Rade Živaljević, U. Belgrade, 1983.
        - Žana Kovijanić, U. Belgrade, 2000.
        - Vladimir Grujić, U. Belgrade, 2002.
        - Đorđe Baralić, U. Belgrade, 2013.
      - Siniša Vrećica, U. Belgrade, 1984.
        - Duško Jojić, U. Belgrade, 2007.
      - Momčilo Bjelica, U. Belgrade, 1991.
    - Zoran Popstojanović, 1964.
    - Petar Todorović, 1964.
    - Rade Dacić, 1965.
    - Milica Dajović, 1965.
    - Borivoje Mihajlović, 1965.
    - Časlav Đaja, U. Belgrade, 1967.
- Radivoj Kašanin, U. Belgrade, 1924.
    - Tatomir Anđelić, U. Belgrade, 1946.
      - Đorđe Mušicki, U. Belgrade, 1956.
      - Marko Leko, U. Belgrade, 1963.
      - Stevo Komljenović, U. Belgrade, 1964.
      - Lazar Rusov, U. Belgrade, 1964.
      - Luka Vujošević, U. Belgrade, 1964.
      - Milan Plavšić, U. Belgrade, 1967.
        - Milan Gligorić, U. Belgrade, 1973.
        - Zoran Golubović, U. Belgrade, 1984.
        - Dragan Milosavljević, U. Belgrade, 1986.
        - Ivan Šestak, U. Belgrade, 1987.
      - Ilija Lukačević, U. Belgrade, 1968.
        - Dragi Radojević, U. Belgrade, 1992.
      - Božidar Jovanović, U. Belgrade, 1973.
      - Mirjana Lukačević, U. Belgrade, 1974.
    - Radmilo Đorđević, U. Belgrade, 1963.
- Jovan Karamata, U. Belgrade, 1926.
    - Vojislav Avakumović, U. Belgrade, 1939.
      - Stanislav Fempl, U. Sarajevo, 1956.
      - Manojlo Maravić, U. Sarajevo, 1956.
        - Mihailo Galić, U. Sarajevo, 1974.

- ◆ Kalmi A. Finci, U. Sarajevo, 1977.
- ◆ Semiha Šlaković, U. Sarajevo, 1978.
- ◆ Mirjana Vuković, U. Sarajevo, 1979.
  - ➤ Olivera Marković, U. East Sarajevo, 2008.
  - ➤ Emil Ilić-Georgijević, U. East Sarajevo, 2011.
- ▷ Branislav Martić, U. Sarajevo, 1961.
- ▷ Walter Eberhard, Rhein, Westfälische Techn. Hochsch. Aachen, 1964. (16)
  - ◆ Gerhard Freiling, Philipps U. Marburg, 1975. (12)
- ▷ Helmut Neunzert, Rhein, Westfälische Techn. Hochsch. Aachen, 1965. (130)
  - ◆ Axel Klar, U. Kaiserslautern, 1994. (56)
    - ➤ Michael Herty, Techn. U. Darmstadt, 2004. (6)
      - ☐ Veronika Schleper, Techn. U. Keiserslautern, 2009 (2)
        - Δ Jochen Neusser, U. Stutgart, 2016.
- ▷ Wolfgang Gromes, Philipps U. Marburg, 1969. (5)
- ▷ Peter Bautsch, Philipps U. Marburg, 1970.
- ▷ Jochen Brüning, Philipps U. Marburg, 1972. (6)
- ▷ Rickleff Brübach, Philipps U. Marburg, 1973
- ▷ Bogoljub Stanković, SASA, 1954 (*J. Karamata)
- ▷ Vojislav Marić, U. Sarajevo
  - ◆ Bwee Tjong, U. Kentucky, 1968. (*Wimberly Calvin Royster)
  - ◆ Detki Jožef, U. Novi Sad, 1977.
  - ◆ Mirko Budinčević, U. Novi Sad, 1985.
  - ◆ Aleksandar Nikolić, U. Novi Sad, 1997.
  - ◆ T. A. Ramayan, U. Madras, India
- ➢ Miodrag Tomić, U. Belgrade, 1950.
- ➢ Slobodan Aljančić, U. Belgrade, 1953.
  - ▷ Dušan Adamović, U. Belgrade, 1965.
    - ◆ Jovan Malešević, U. Belgrade, 1975.
    - ◆ Slavko Simić, U. Belgrade, 1997.
    - ◆ Milorad Stevanović, U. Belgrade, 1994.
    - ◆ Stevo Stević, U. Belgrade, 2001.
      - ➤ Bratislav Iričanin, U. Novi Sad, 2009.
  - ▷ Milivoje Lazić, U. Belgrade, 1973.
  - ▷ Dimitrije Hajduković, U. Belgrade, 1974.
  - ▷ Dragoljub Aranđelović, U. Belgrade, 1975.
  - ▷ Ilija Lazarević, U. Belgrade, 1975.
  - ▷ Miodrag Ivović, U. Belgrade, 1976.
  - ▷ Vladimir Savić, U. Belgrade, 1978.
    - ◆ Dragan Đurčić, U. Belgrade, 1999.
  - ▷ Milan Tasković, U. Belgrade, 1978.
    - ◆ Ivan Aranđelović, U. Belgrade, 1999.
    - ◆ Siniša Ješić, U. Belgrade, 2006.
      - ➤ Rale Nikolić, U. Kragujevac, 2012.

- Beriša Muharem, U. Belgrade, 1979.
- Bosiljka Laković, U. Belgrade, 1980.
- Mirko Janc, U. Belgrade, 1981.
- Slobodan Miloradović, U. Belgrade, 1982.
- Tatjana Ostrogorski, U. Belgrade, 1987.

- Vladeta Vučković, U. Belgrade, 1953.
  - Leon Harkleroad, U. Notre Dame, 1976.
- Ranko Bojanić, U. Belgrade, 1953.
  - Ronald DeVore, Ohio State U., 1967.
    - Yingkang Hu, U. South Carolina, 1989.
    - George Kyriazis, U. South Carolina, 1992.
    - Florin Saban, U. South Carolina, 1995.
    - Zesheng Yang, U. South Carolina, 1995.
    - Wei Shao, U. South Carolina, 1996.
    - Guergana Petrova, U. South Carolina, 1999.
    - Bojan Popov, U. South Carolina, 1999.
      - Orhan Mehmetoglu, Texas A&M U., 2012.
  - Victor Ekong, Ohio State U., 1972.
  - You-Hwa Lee, Ohio State U., 1972.
  - John KLippert, Ohio State U., 1973.
  - Rada Higgins, Ohio State U., 1974.
  - Radwan Al-Jarrah, Ohio State U., 1980.
  - Fuhua Cheng, Ohio State U., 1982.
  - Chin-Chi Shan, Ohio State U., 1982.
  - Xiaoming Huang, Ohio State U., 1993.
- Bogoljub Stanković, SASA, 1954.
  - Marija Skendžić, U. Novi Sad, 1971.
  - Olga Hadžić, U. Novi Sad, 1972.
    - Mila Stojaković, U. Novi Sad, 1980.
      - Biljana Carić, U. Novi Sad, 2018.
    - Ljiljana Gajić, U. Novi Sad, 1982.
    - Tatjana Došenović, U. Novi Sad, 2002.
    - Zoran Mitrović, U. Novi Sad, 2002.
  - Danica Nikolić-Despotović, U. Novi Sad, 1972.
  - Endre Pap, U. Novi Sad, 1975.
    - Nebojša Ralević, U, Novi Sad, 1997.
      - Goran Bojanić, U. Novi Sad, 2013.
      - Vladimir Bojanić, U. Novi Sad, 2013.
      - Lidija Čomić, U. Novi Sad, 2014. (*)
      - Ljubo Nedović, U. Novi Sad, 2017.
      - Lidija Krstanović, U. Novi Sad, 2017.

        - Dejan Ćebić, U. Novi Sad, 2018.
        - Dejan Mirčetić, U. Novi Sad, 2018. (*)
    - Aleksandar Kerenji, U. Novi Sad, 1991.
    - Ivana Štajner-Papuga, U. Novi Sad, 2001.
        - Dragan Jočić, U. Novi Sad, 2015.
    - Marta Takač, U. Novi Sad, 2004.
        - Tóthné Laufer Edit, Obuda U. Budapest, 2014.
        - Šram Norbert, Obuda U. Budapest, 2017.
    - Aleksandar Takači, U. Novi Sad, 2006.
        - Darko Drakulić, U. East Sarajevo, 2016. (*Tatjana Grbić)
    - Tatjana Grbić, U. Novi Sad, 2008.
        - Slavica Medić, U. Novi Sad, 2014.
        - Bojan Jovanović, U. Novi Sad, 2018.
    - Biljana Mihajlović, U. Novi Sad, 2009.
        - Vera Miler Jerković, U. Novi Sad, 2018 (*Branko Malešević)
    - Branka Janković, U. Novi Sad, 2011.
    - Mirjana Štrboja, U. Novi Sad, 2011.
- Ibrahim Eid Sharkavi, U. Novi Sad, 1979.
- Stevan Pilipović, U. Novi Sad, 1979.
    - Zagorka Lozanov Crvenković, U. Novi Sad, 1989.
    - Mirjana Stojanović, U. Novi Sad, 1991.
    - Dušanka Perišić, U. Novi Sad, 1992.
    - Marko Nedeljkov, U. Novi Sad, 1995.
        - Danijela Rajter-Ćirić, U. Novi Sad, 2002.
            - Miloš Japundžić, U. Novi Sad, 2016.
        - Nebojša Dedović, U. Novi Sad, 2014.
        - Tanja Krunić, U. Novi Sad, 2016.
        - Daw Dalal, U. Novi Sad, 2017.
    - Milorad Mijatović, U. Novi Sad, 1996.
    - Ratko Kravarušić, U. Novi Sad, 1998.
    - Aneta Buchkovska, U. Novi Sad, 2000.
    - Nenad Teofanov, U. Novi Sad, 2000.
        - Dušan Rakić, U. Novi Sad, 2010.
        - Filip Tomić, U. Novi Sad, 2016.
    - Mirjana Vidanović, U. Novi Sad, 2003.
    - Marko Kostić, U. Novi Sad, 2004.
        - Milica Žigić, U. Novi Sad, 2014.
    - Ljupčo Nastovski, U. Novi Sad, 2004.
    - Dora Seleši, U. Novi Sad, 2007.
        - Tijana Levajković, U. Novi Sad, 2012.
    - Katerina Saneva, U. Novi Sad, 2008.
    - Jelena Aleksić, U. Novi Sad, 2009.
        - Ivana Vojnović, U. Novi Sad, 2017.

- Diana Dolićanin, U. Novi Sad, 2009.
    - Dženis Pučoć, U. Novi Pazar, 2014.
- Vasko Reckovski, U. Novi Sad, 2009.
- Branko Sarić, U. Novi Sad, 2009.
- Dušan Zorica, U. Novi Sad, 2009.
- Bratislav Iričanin, U. Novi Sad, 2009.
- Marko Janev, U. Novi Sad, 2011.
- Suzana Simić, U. Novi Sad, 2011.
- Bojan Prangoski, U. Novi Sad, 2013.
- Sanja Kostadinova, U. Novi Sad, 2014.
- Daniel Velinov, U. Novi Sad, 2014.
- Milica Žigić, U. Novi Sad, 2014.
- Pavel Dimovski, U. Novi Sad, 2015.
- Smiljana Jakšić, U. Novi Sad, 2016.
- Snježana Maksimović, U. Novi Pazar, 2016.
- Petar Sokolski, U. Novi Pazar, 2016.
- Đorđe Vučković, U. Ghent, 2018.
- Snežana Gordić, U. Novi Sad, 2018.

▹ Dragoslav Herceg, U. Novi Sad, 1980.

- Relja Vulanović, U. Novi Sad, 1986.
- Ljiljana Cvetković, U. Novi Sad, 1987.
    - Vladimir Kostić, U. Novi Sad, 2010.
    - Ksenija Doroslovački, U. Novi Sad, 2014.
    - Maja Nedović, U. Novi Sad, 2016.
- Nenad Petrović, U. Novi Sad, 1988.
- Adžić Nevena, U. Novi Sad, 1990.
- Nataša Krejić, U. Novi Sad, 1994.
    - Sanja Rapajić, U. Novi Sad, 2005.
    - Miles Kumaresan, U. Novi Sad, 2010.
    - Sandra Buhmiler, U. Novi Sad, 2013.
    - Nataša Krklec Jerinkić, U. Novi Sad, 2015.
    - Snežana Đorđević, U. Novi Sad, 2015.
    - Zoran Ovčin, U. Novi Sad, 2016.
    - Sanja Lončar, U. Novi Sad, 2017.
- Zorana Lužanin, U. Novi Sad, 1997.
    - Ivan Anić, U. Novi Sad, 2011.
    - Irena Stojkovska, U. Skopje, 1994.
    - Nataša Dragnič, U. Novi Sad, 2016.
    - Milena Kresoja, U. Novi Sad, 2017.
- Helena Zarin, U. Novi Sad, 2003.
    - Ljiljana Teofanov, U. Novi Sad, 2008.
    - Mirjana Brdar, U. Novi Sad, 2016. (*Helena Zarin)

- Miroljub Miloradović, U. Novi Sad, 2004.
- Predrag Kovačević, U. Novi Sad, 2006.
- Radivoje Stojković, U. Novi Sad, 2006.
- Veselin Mićanović, U. Novi Sad, 2008.
- Goran Radojev, U. Novi Sad, 2015.
- Branko Prentović, U. Novi Sad, 2015.
- Eugen Ljajko, U. Novi Sad, 2016.

- Katarina Surla, U. Novi Sad, 1980.
    - Zorica Uzelac, U. Novi Sad, 1989.
    - Vanja Vukoslavčević, U. Novi Sad, 1999.
- Arpad Takači, U. Novi Sad, 1982.
    - Biljana Jolevska-Tuneska, U. Novi Sad, 2003.
- Radivoje Despotović, U. Novi Sad, 1984.
- Đurđica Takači, U. Novi Sad, 1987.
    - Duška Pešić, U. Novi Sad, 2006.
    - Jelena Tatar, U. Novi Sad, 2007.
    - Jasminka Radovanović, U. Novi Sad, 2011.
    - Igor Dimovski, U. Novi Sad, 2012.
    - Ivana Milanović, U. Novi Sad, 2015.

- Sefkija Raljević, U. Belgrade, 1955.
- Milenko Šteković, U. Sarajevo, 1956.
- Milenko Steković, U. Belgrade, 1956.
- Bogdan Bajšanski, U. Belgrade, 1956.
    - Dennis Girard, Ohio State U., 1968.
    - Charles Heiberg, Ohio State U., 1971.
    - Richard Biberich, Ohio State U., 1973.
    - Bernard Ploeger, Ohio State U., 1975.
    - Sayel Ali, Ohio State U., 1987.
    - Laying Tam, Ohio State U., 1990.
    - William Mariasoosai, Ohio State U., 1991.
    - Noli Reyes, Ohio State U., 1992.
        - Edwin Balila, U. Philipinnes, 2000.
    - Michael Snell, Ohio State U., 1995.
    - Bryan Johnson, Ohio State U., 1997.
    - Natalia Humphreys, Ohio State U., 1999.
- H. Baumann, U. Geneve, 1965.
- Ronald Coifman, U. Geneve, 1965. (106)
    - Mladen Wickerhauser, U. Yale, 1985. (22)
        - Wojciech Czaja, U. Washington in St. Louis, 2000. (14)
            - David Widemann, U. Maryland, Coll. Park, 2008.
- Monique Vuilleumier, U. Geneve, 1965.

- Miloš Radojčić, U. Belgrade, 1928.

- Dragoslav Mitrinović, U. Belgrade, 1933.
    - Blagoj Popov, U. Skopje, 1952.
    - Ivan Bandić, U. Belgrade, 1958.
    - Lazar Karadžić, U. Belgrade, 1958.
    - Dragomir Đoković, U. Belgrade, 1963.
        - Lovell Sweet, U. Waterloo, 1973.
        - Taw-Pin Lim, U. Waterloo, 1976.
        - John McFall, U. Waterloo, 1976.
        - Chiu-Tsin Ralston (Chen), U. Waterloo, 1980.
        - David Casperson, U. Waterloo, 1985.
        - Thang Quoc Nguyen, U. Waterloo, 1994.
            - Tan Nguyen, U. Hanoi, 2008.
            - Phuong-Bac Dao, U. Hanoi, 2010.
            - Ngoan Ngo, Inst. Math. Vietnam Academy Sci., 2017.
            - Ming-Peng Gong, U. Waterloo, 1998.
    - Kovina Milošević-Rakočević, U. Belgrade, 1963.
    - Danica Percinkova, U. Skopje, 1963.
    - Petar Vasić, U. Belgrade, 1963.
        - Josip Pečarić, U. Belgrade, 1982.
            - Sanja Varošanec, U. Zagreb, 1994.
            - Ivan Perić, U. Zagreb, 1997.
                - Anita Matković, U. Zagreb, 2006.
            - Marko Matić, U. Zagreb, 1998.
                - Milica Klaričić Bakula, U. Zagreb, 2005.
                    - Mihaela Ribičić-Penava, U. Zagreb, 2009.
                - Josipa Barić, U. Zagreb, 2011.
                - Sanja Tipurić-Spužević, U. Mostar, 2014.
            - Jadranka Sunde, U. Zagreb, 1998.
                - Brett McLindin, U. South Australia, 2005.
                - Katherine Hanton, U. South Australia, 2012.
            - Vera Čuljak, U. Zagreb, 1999.
            - Jadranka Mićić Hot, U. Zagreb, 2001.
                - Zlatko Pavić, U. Zagreb, 2011.
            - Ana Vukelić, U. Zagreb, 2003.
                - Sanja Kovač, U Zagreb, 2008.
            - Andrea Aglić-Aljinović, U. Zagreb, 2004.
                - Josipa Barić, U. Zagreb, 2011.
                - Sanja Tipurić-Spužević, U. Mostar, 2014.
            - Iva Franjić, U. Zagreb, 2006.
            - Anwar Matloob, U. GC Lahore, 2009.
                - Rabia Bibi, National U. Sci. Techno. Pakistan, 2014.
            - Julije Jakšetić, U. Zagreb, 2010.
                - Anamarija Perušić, U. Zagreb, 2013.

            - Atiq Rehman, U. GC Lahore, 2011.
            - Zlatko Pavić, U. Zagreb, 2011.
            - Ghulam Farid, U. GC Lahore, 2012.
            - Jurica Perić, U. Zagreb, 2012.
            - Khuram Khan, U. GC Lahore, 2013.
            - Asif Khan, U. GC Lahore, 2014.
- Ilija Šapkarev, U. Skopje, 1964.
- Velimir Penivan, U. Belgrade, 1965.
- Radosav Đorđević, U. Belgrade, 1966.
- Dragan Dimitrovski, U. Skopje, 1968.
- Radovan Janić, U. Belgrade, 1968.
- Savo Jovanović, U. Belgrade, 1968.
- Jovan Kečkić, U. Belgrade, 1970.
- Dragoš Cvetković, U. Belgrade, 1971.
    - Slobodan Simić, U. Belgrade, 1979.
        - Francesko Belardo, U. di Messina, 2007.
        - Zoran Stanić, U. Belgrade, 2007.
        - Milica Anđelić, U. Alveiro, 2011.
    - Ivan Gutman, U. Belgrade, 1981.
        - Boris Furtula, U. Kragujevac, 2007.
    - Zoran Radosavljević, U. Belgrade, 1987.
        - Marija Rašajski, U. Belgrade, 2006.
        - Bojan Mihailović, U. Belgrade, 2016.
    - Dragan Stevanović, U. Belgrade, 2000.
        - Marko Milošević, U. Niš, 2008.
        - Milan Bašić, U. Niš, 2011.
        - Aleksandar Ilić, U. Niš, 2011.
        - Vladimir Baltić, U. Niš, 2014.
        - Alexander Vasilyev, U. Primorska, 2014.
        - Paweł Petecki, U. Primorska, 2016.
- Ionel Stamate, U. Belgrade, 1971.
- Živko Tošić, U. Belgrade, 1971.
- Živko Madevski, U. Skoplje, 1973.
- Ivan Lacković, U. Niš, 1975.
    - Ljubiša Kocić, U. Niš, 1985.
        - Elena Babace, U. Skopje, 2009.
- Dušan Slavić, U. Belgrade, 1975.
- Ljubomir Stanković, U. Niš, 1975.
- Budimir Zarić, U. Belgrade, 1975.
- Gradimir Milovanović, U. Niš, 1976.
    - Ljiljana Petković, U. Kragujevac, 1985.
        - Dragan Živković, U. Niš, 2008.
    - Milan Kovačević, U. Niš, 1986.

        - ▹ Gospava Đorđević, U. Niš, 1990.
        - ▹ Nenad Cakić, U. Niš, 1996.
        - ▹ Predrag Stanimirović, U. Niš, 1997.
            - ◆ Nebojša Stojković, U. Niš, 2002.
            - ◆ Milan Tasić, U. Niš, 2003.
                - ➤ Selver Pepić, U. Niš, 2012.
                - ➤ Ivan Stanimirović, U. Niš, 2013.
            - ◆ Predrag Krtolica, U. Niš, 2004.
            - ◆ Marko Petković, U. Niš, 2008.
                - ➤ Radica Bojičić, U. Niš, 2014.
            - ◆ Marko Miladinović, U. Niš, 2010.
            - ◆ Slađana Miljković, U. Niš, 2012.
            - ◆ Muzafer Saračević, U. Niš, 2013.
            - ◆ Igor Stojanović, U. Niš, 2014.
            - ◆ Milena Petrović, U. Niš, 2015.
        - ▹ Đorđe Đorđević, U. Niš, 1997.
        - ▹ Miodrag Spalević, U. Kragujevac, 1997.
            - ◆ Miroslav Pranić, U. Kragujevac, 2007.
            - ◆ Aleksandar Pejčev, U. Kragujevac, 2013.
                - ➤ Ljubica Mihić, U. Belgrade, 2017.
            - ◆ Dušan Đukić, U. Kragujevac, 2018.
        - ▹ Dojčin Petković, U. Priština, 1998.
        - ▹ Predrag Rajković, U. Niš, 1998.
            - ◆ Slađana Marinković, U. Niš, 2005.
        - ▹ Aleksandar Cvetković, U. Niš, 2004.
            - ◆ Zvezdan Marjanović, U. Kragujevac, 2012.
        - ▹ Marija Stanić, U. Kragujevac, 2007.
            - ◆ Marina Milovanović, U. Kragujevac, 2014.
            - ◆ Tatjana Tomović, U. Kragujevac, 2014.
        - ▹ Zlatko Udovičić, U. Sarajevo, 2010.
        - ▹ Marjan Matejić, U. Kragujevac, 2016.
    - ➢ Žarko Mitrović, U. Niš, 1976.
    - ➢ Ismet Dehiri, U. Priština, 1977.
    - ➢ Petar Lazov, U. Belgrade, 1977.
    - ➢ Lazar Đorđević, U. Niš, 1978.
    - ➢ Miomir Stanković, U. Niš, 1979.
    - ➢ Nikola Azanjac, U. Kragujevac, 1980.
    - ➢ Igor Milovanović, U. Niš, 1980.
        - ▹ Branislav Ranđelović, U. Niš, 2015.
    - ➢ Miodrag Petković, U. Niš, 1980.
        - ▹ Lidija Stefanović, U. Niš, 1986.
        - ▹ Slobodan Lakić, U. Novi Sad, 1996. (*D. Herceg)
        - ▹ Slobodan Tričković, U. Novi Sad, 1997.

        - ▹ Đorđe Herceg, U. Novi Sad, 1999.
        - ▹ Dušan Milošević, U. Niš, 2005.
        - ▹ Lidija Rančić, U. Niš, 2005.
        - ▹ Mimica Milošević, U. Niš, 2011.
        - ▹ Jovana Džunić, U. Niš, 2012.
    - ➢ Vlajko Kocić, U. Belgrade, 1981.
    - ➢ Behdžet Mesihović, U. Sarajevo, 1987.
- Danilo Mihnjević, U. Belgrade, 1934.
- Konstantin Orlov, U. Belgrade, 1934.
    - ➢ Petar Madić, U. Belgrade, 1965.
    - ➢ Momčilo Ušćumlić, U. Belgrade, 1965.
    - ➢ Mihail Arsenović, U. Belgrade, 1972.
    - ➢ Miroslava Stojanović, U. Belgrade, 1973.
    - ➢ Max Wotulo, U. Belgrade, 1973.
    - ➢ Boško Jovanović, U. Belgrade, 1976.
        - ▹ Desanka Radunović, U. Belgrade, 1984.
        - ▹ Endre Šili, U. Belgrade, 1984.
            - ◆ Wu Wei, U. Oxford, 1987.
            - ◆ Antony Ware, U. Oxford, 1991.
                - ➤ Gordana Dmitrašinović-Vidović, U. Calgary, 2004.
                - ➤ Hua Li, U. Calgary, 2006.
            - ◆ Hill Adrian, U. Oxford, 1992.
                - ➤ Michael Falcon, U. Bat, 1998.
            - ◆ Mark Baker, U. Oxford, 1994.
            - ◆ Jeffrey Wood, U. Oxford, 1994.
            - ◆ Paul Houston, U. Oxford, 1996.
                - ➤ Edward Hall, U. Leicester, 2007.
                - ➤ Sabine Shamberg, U. Nottingham, 2009.
            - ◆ Catherine Wilkins, U. Oxford, 1998.
            - ◆ Nicolas Jackson, U. Oxford, 1999.
            - ◆ Kan Cheng, U. Oxford, 2000.
            - ◆ Kathryn Harriman, U. Oxford, 2000.
            - ◆ Minh Pham, U. Oxford, 2001.
            - ◆ Emmanuil Georgoulis, U. Oxford, 2003.
                - ➤ Edward Hall, U. Leicester, 2007.
                - ➤ Terhemen Aboiyar, U. Leicester, 2008.
                    - ❒ Patrick Okolo, U. Agriculture, Makurdi, 2015.
                    - ❒ Abu Onoja, U. Agriculture, Makurdi, 2015.
                    - ❒ Eka Ogbaji, U. Agriculture, Makurdi, 2016.
                - ➤ Yuha Virtanen, U. Leicester, 2010.
                - ➤ Fazil Subhan, U. Leicester, 2011.
                - ➤ Stephen Metcalfe, U. Leicester, 2015.

            - Samuel Cox, U. Leicester, 2017.
            - Dong Zhaonan, U. Leicester, 2017.
            - Younis Sabawi, U. Leicester, 2017.
            - Oliver Sutton, U. Leicester, 2017.
        - Janice Robson, U. Oxford, 2003.
        - Andrea Cangiani, U. Oxford, 2004.
            - Stefen Metkaf, U. Leicester, 2015.
            - Semjuel Koks, U. Leicester, 2017.
            - Dong Zaonan, U. Leicester, 2017.
            - Junis Sabavi, U. Leicester, 2017.
            - Oliver Saton, U. Leicester, 2017.
        - Max Jensen, U. Oxford, 2005.
            - John Chapman, U. Daram, 2012.
        - Andris Lasis, U. Oxford, 2006.
        - Cristoph Ortner,U. Oxford, 2006.
            - Siobhan Burke, U. Oxford, 2011.
            - Bernhard Lagwallner, U. Oxford, 2011.
            - Hao Wang, U. Oxford, 2013.
            - Thomas Hadson, U. Oxford, 2014.
            - Simon Bignold, U. Warwick, 2016.
            - Faizan Nazar, U. Warwick, 2016.
        - David Knežević, U. Oxford, 2008.
        - Arranz Carreno Aurelio, U. Oxford, 2011.
        - Leonardo Figueroa, U. Oxford, 2011.
        - Sioban Burke, U. Oxford, 2011.
        - Bernhard Lagvolner, U. Oxford, 2011.
        - Hao Vang, U. Oxford, 2013.
        - Iain Smers, U. Oxford, 2015.
        - Graham Baird, U. Oxford, 2017.
    - Momir Ćelić, U. Belgrade, 1986.
    - Milan Dražić, U. Belgrade, 1996.
        - Ljubica Mihić, U. Belgrade, 2017.
    - Branislav Popović, U. Kragujevac, 1999.
    - Dejan Bojović, U. Kragujevac, 2000.
        - Bratislav Sredojević, U. Kragujevac, 2016.
    - Sonja Gegovska-Zajkova, U. Skoplje, 2004.
    - Slobodanka Boljanović, U. Belgrade, 2012.
    - Zorica Milovanović, U. Belgrade, 2015.
    - Milan Dotlić, U. Belgrade, 2016.
    - Aleksandra Delić, U. Belgrade, 2016.
    - Sandra Hodžić, U. Belgrade, 2016.
- Arif Zolić, U. Belgrade, 1977.

- Ljubomir Protić, U. Belgrade, 1978.
  - Dušan Tošić, U. Belgrade, 1984.
    - Jozef Kratica, U. Belgrade, 2000.
      - Zorica Stanimirović, U. Belgrade, 2007.
    - Vladimir Filipović, U. Belgrade, 2006.
    - Tatjana Davidović, U. Belgrade, 2006.
      - Tatjana Jakšić Kruger, U. Novi Sad, 2017.
    - Miroslav Marić, U. Belgrade, 2008. (*Jozef Kratica)
    - Milena Vujošević Janičić, U. Belgrade, 2013.
  - Liljana Stefanovska, U. Skoplje, 1994
- Branko Savić, U. Belgrade, 1978.

- Petar Muzen, U. Belgrade, 1937.
- Dragoljub Marković, U. Belgrade, 1938.
  - Jovan Petrić, U. Belgrade, 1960.

REMARKS

1 For more detail about Mihailo Petrović's early life, see: Љубомир Протић, *Живот, дело и научни рад Михаила Петровића Аласа*, Зборник Српски математичари, САНУ, Београд, 2015.

2 The "Adligat" Book Museum in Belgrade contains around thirty letters from this correspondence.

3 This Petrović's term paper was described and commented on by Mirko Stojaković and Dragan Trifunović in their article *Петровићева модификација Грефеове методе за решавање алгебарских једначина*, Математички весник 5(20), 1968, 439–446, http://elibrary.matf.bg.ac.rs.

4 *Sur les zéros et les infinis des intégrales des équations différentielles algébriques (On Zeros and Infinities of the Integral of Algebraic Differential Equations),* http://alas.matf.bg.ac.rs/~websites/digitalnilegatmpalas. The Serbian translation of the thesis is contained in the book *Диференцијалне једначине I*, књ. 1 *Сабраних дела Михаила Петровића*. An appendix – the phototype edition of Petrović's dissertation was printed in *The Collected Works*.

5 https://www.genealogy.math.ndsu.nodak.edu, service of *North Dakota State University*, supported by the American Mathematical Society (AMS).

6 J. D. Kečkić, *Serbian doctors of mathematics in the 19th century*, Pub. Inst. Math. N. s. tome 38 (52), 1985, pp. 3–6, http://elib.mi.sanu.ac.rs. In this interesting article, the author presents all doctoral dissertations of Serbian mathematicians in the 19th century and their short biographies.

7 Basic, but precise information about the membership of Klerić, Nešić, Gavrilović and Živković in the Academy, the dates of their births and deaths, are contained, for instance, in *Годишњак* САНУ за 2012, CXIX, Београд 2013, https://www.sanu.ac.rs/Novosti/2013Godisnjak.pdf. The well-known Academy's edition *Живот и дело српских научника (Lives and Works of Serbian Scientists)* contains detailed biographical information about these mathematicians and scientists. This Academy's edition was digitised and can be accessed at: https://www.sanu.ac.rs.

8 Digital copies of Bošković's translation of three Zinger's book on astronomy, geodesy and mathematical cartography can be found in the virtual library http://elibrary.matf.bg.ac.rs. This library also contains the mentioned works of Kosta Stojanović and Milan Andonović. A brief overview of these works was published in *Early astronomical heritage in Virtual library of Faculty of mathematics in Belgrade*, N. Pejović, Ž. Mijajlović, NCD Review, 19 (2011), 11–25, http://elib.mi.sanu.ac.rs.

9 Editors Eleanor Robson and Jacqueline Stedall, Oxford University Press, 2009, 918 p.

10 At the end of the 19th century several Serbian mathematicians studied for doctorates at western universities: Dimitrije Danić in Jena (1885), Bogdan Gavrilović in Budapest (1887), Đorđe Petković in Vienna (1893), Petar Vukičević in Berlin (1894), and finally, the most famous Serbian mathematician, Mihailo Petrović, who completed his thesis in Paris in 1894. It is not known why Petrović chose Paris when all his contemporaries studied in Germany or Austria, but he established important links with the French government during his studies and maintained them later. Thus, although most educational influences in the middle of the century were Austro-Hungarian or German, the most prominent of Serbian mathematicians, who set the future direction of the national mathematical school, introduced French mathematics and French mathematicians to his country. Also see *Mathematics Education in the Balkan Societies Up To the WWI*, Teaching Innovations, 2014, Volume 27, Issue 3, pp. 46–57.

11 СКА Год. 36 (1928) 53.

12 Source: БМС, http://digital.bms.rs/ebiblioteka/publications/view/3835.

13 Source: Faculty of Mathematics in Belgrade, http://poincare.matf.bg.ac.rs/informacije/dipmat.htm#1874–1914. In the 1990s, these data were collected by Jelena Milogradov, an astronomy professor at the Faculty of Mathematics.

14 Two articles deal with this collection of manuscripts: 1. Ž. Mijajlović, N. Pejović, *Twenty four manuscripts in the Virtual library of the Faculty of Mathematics in Belgrade,* Преглед НЦД 25 (2014), 29–35, 2. N. Pejović, S. Ninković, *A manuscript on astronomy and geodesy of an unknown author*, Преглед НЦД 26 (2015), 27–36. The former article contains a general overview of the collection and the latter concerns the interesting destiny of a manuscript, also from this collection, originating from the mid-19th century. The digital version of the journal *Преīлед НЦД* can be accessed at: http://elib.mi.sanu.ac.rs.

15 The function theory is the old name for the functions of a complex variable.

16 Tadija Pejović, Konstantin Orlov.

17 Vojin Dajović, Mihailo Arsenović, Milorad Bertolino, Milosav Marjanović, Nedeljko Parezanović, Slaviša Prešić, Ljubomir Protić, Gradimir Milovanović and others.

18 See the Digital Legacy of Bogdan Gavrilović, http://poincare.matf.bg.ac.rs/~ncd/Bogdan_Gavrilovic_VirtBibl.

19 He was a Serbian representative at the International Mathematical Union (ICMI); see the Portrait gallery on the ICMI website: http://www.icmihistory.unito.it. In the early 20th century, he was also a member of mixed Romanian-Austro-Hungarian-Serbian commissions for fishing on the Danube.

20 Although there are opposite examples, reputable gentlemen could not resist either the charm of "fishery balls" that were often organised by Petrović. Jelenko Mihailović noted an interesting anecdote from 1903 about well-known finance minister Lazar Paču and Mika's catch of a beluga, weighing "200 kilos". Although the minister received ten kilograms of "ajvar" (caviar) from the catch, he did not find it sufficient, and in the evening of the same day he brought the entire government, together with the president, to the "Jasenica" tavern, for a dinner prepared by Mika Alas.

21 In the introductory article in the catalogue of the exhibition of the Archive of Serbia *Belgrade Mathematical School* of 16 May 1968, Božidar Manić wrote that Petrović "did not accept the position of the rector".

22 According to professor Tadija Pejović, the Mathematical Club was founded in 1930. It particularly stepped up its activity in 1932 with the launch of the first Serbian specialised mathematical journal *Publications de l'Institut Mathématique Université de Belgrade.*

23 It is interesting that several renowned professors of the Great School were not included in the election, such as, for instance, mechanics professors Mijalko Ćirić and Kosta Stojanović.

24 In different places and according to different authors, the number of Petrović's doctoral students varied from ten to 15. For instance, the webpage *Mathematical Geneaology Project* does not contain the name of Petar Muzen. The exact number is most probably 11, as enumerated here based on data from the archives of the Faculty of Mathematics. Vojislav Avakumović is often considered Petrović's twelfth doctoral student, although Avakumović's tutor was Karamata, while Petrović was only a member of the defence committee.

25 According to data from the *Mathematical Geneaology Project* and the archives of the Faculty of Mathematics until 2017.

26 *Publications*: http://elib.mi.sanu.ac.rs.

27 *Елеменши машемашичке феноменолоīије*, Београд, 1911, 774 p.; *Mécanismes communs aux phénomènes disparates*, Париз, 1921, 279 p.; *Феноменолошко ūресликавање*, Београд, 1933, 236 p. Also see the article in French: *Le noyau d'analogie*, Revue de Mois, No. 119, 1919, 475–486.

28 *Радови Михаила Пешровића у Алīебри*, Ж. Мијајловић, book 4, *Алīебра*, 262–273, Сабрана дела Михаила Петровића, 1998, Завод за уџбенике и наст. средства, Београд, http://elibrary.matf.bg.ac.rs.

29 Mainly fishery.

30 The book *Велико ūушовање, Михаило Пешровић Алас* edited by Dragan Trifunović (published by Vuk Karadžić, Belgrade 1982) contains a nice selection of Petrović's memories and traveloques.

31 *Мика Алас – Белешке о живошу великоī машемашичара Михаила Пешровића*, приредио Владо Милићевић, 3VM Geo Ltd. Фонд др Милићевић, Калгари и Удружење Милутин Миланковић, Београд, 1912.

32 Hereinafter: *The Collected Works.*

33 It is noteworthy that Pupin was one of the first members-founders of the *American Mathematical Society* in 1889 (source: *A Semicentennial History of the American Mathematical Society*, 1888–1938, R.C. Archibald, AMS, New York, 1938).

34 Mihailo Petrović was born in 1868 in Belgrade and died in 1943 in Belgrade. This year, 2018, is dedicated to the anniversary of 150 years from the birth of Mihailo Petrović.

35 zbMATH, https://zbmath.org/.

36 Mathematical Reviews, https://mathscinet.ams.org/mathscinet/.

37 Milica's mother Marija came from the reputable Nešić family. Her brother, Sima Nešić, who studied at the Trade Academy in Vienna, was intelligent and educated (he spoke German, French, Turkish, Greek, Jewish, Aromanian and Albanian languages), and he worked in the police service. At the time of the conflict near the Čukur Fountain in 1862 due to the wounding of a Serbian boy, he lost his life during the intervention and he was the first victim of this event. The street below the building of the Faculty of Mathematics in Belgrade bears his name – Simina Street.

38 Jules Henri Poincaré, 1854–1912, taught at the Sorbonne. His fields of work included topology, elliptic functions, differential equations, power series, thermodynamics and mechanics. He was member of the Paris Academy of Sciences and a number of other academies. Henri Poincaré was not only one of the most talented mathematicians of all times, but he also had an exceptional gift for writing. He is said to have been unsurpassed in the beauty of writing about mathematics. He had a particular capacity to simply present mathematics to others [Albijanić, 171]. There were also other great professors: Émile Picard (1856–1941), Paul Painlevé (1863–1933), Dénes Kőnig (1884–1944), Charles Hermite (1822–1901), Paul Émile Appell (1855–1930), Édouard Jean-Baptiste Goursat (1858–1936), Jean-Gaston Darboux (1842–1917), Jacques Salomon Hadamard (1865–1963), Paul Tannery (1843–1904).

39 The original title of Mihailo Petrović's doctoral dissertation: *Sur les zéros et les infinis des intégrales des équations différentielles algébriques*. With this examination, Petrović obtained the title of: *Docteur és sciences mathematiques*.

40 Addition: on 3 March, full professors were appointed: to the Faculty of Philosophy – Sava Urošević (mineralogy), Bogdan Popović (general history of literature); to the Technical Faculty – Nikola Stamenković (hydraulic engineering), Vladimir Todorović (mechanics), Bogdan Gavrilović (mathematics); to the Faculty of Law – Slobodan Jovanović (State and International Public Law), Živojin M. Perić (Civil Law). Already on 11 March, heads of departments were appointed.

41 It is interesting to note that Miloš Radojičić studied geometric function theory and had no disciples, but this area of study would be revived in Belgrade somewhere during the 1980s.

42 He was one of the 1300 corporals in World War I. He was conscripted as reserve lieutenant colonel at the start of World War II, was imprisoned and remained in imprisonment during the war until the end of 1945.

43 He wrote a book *My Memories and Events 1892–1945* in two volumes.

44 See papers starting from 1962, "Journal article, sur une équation fonctionnelle cyclique d'ordre supérieur" Dragoslav S. Mitrinović, Slaviša B. Prešić, Publikacije Elektrotehničkog fakulteta. Serija Matematika i fizika, No. 70/76 (1962), pp. 1–2.

45 Karamata's proof found its place in renowned monographs by K. Knopp, *Theorie und Anwendung der unendlichen Reihen*, 1931; G. Doetsch, *Theorie und Anwendung der Laplace Transformation*, 1937; D. V. Widder, *The Laplace Transformation*, 1946; G. H. Hardy, *Divergent Series*, 1949; J. Favard, *Course d'Analyse. Compléments et Exercices d'Analysis*, 1962–1963. It is interesting that, on the occasion of its 60-year anniversary of publication, the editorial board of the journal *Mathematische Zeitschrift*, included this paper by Karamata in its selection of 50 most significant papers, having chosen among several thousand papers published. ("We shall give an extremely elegant proof which has recently been obtained by Karamata." (E. C. Titchmarsh, *The Theory of Functions*, 1939, s.226.))

46 Thereby Karamata's theory grew into a huge mathematical building whose significance is still rising and to which, among other things, three famous world mathematical monographs have been dedicated.

47 Details on published books, monographs and scientific papers of Gradimir Milovanović are available at: http://www.mi.sanu.ac.rs/~gvm/

48 Shanghai Ranking's Global Ranking of Academic Subjects 2018 – Mathematics, http://www.shanghairanking.com/Shanghairanking-Subject-Rankings/mathematics.html.
49 Драган Трифуновић, *Летопис живота и рада Михаила Петровића*, Српска академија наука, Београд, 1969, p. 128, 129, 131, 222, 132.
50 Војин Дајовић, „Предговор", у: Михаило Петровић, *Чланци*, Друштво математичара и физичара Народне републике Србије, Научна књига, Београд 1949, p. IV.
51 Драган Трифуновић, *op. cit.*, p. 131.
52 Михаило Петровић, *Елиптичке функције* (друго издање), Научна књига, Београд, 1969a, p. VII and Драган Трифуновић, *op. cit.*, p. 208.
53 Драган Трифуновић, „Поговор", у: Михаило Петровић, *Интервална математика – диференцијални алгоритам,* [Сабрана дела – књига 8], Завод за уџбенике, Београд, 1997a, p. 410.
54 Зоран Каделбург, „Поговор", у: Михаило Петровић, *Елиптичке функције – интеграција помоћу редова*, [Сабрана дела – књига 9], Завод за уџбенике Београд, 1997б, and Михаило Петровић (1997a), *op. cit.*, p. 319.
55 This information should be taken with reserve as the databases used (https://genealogy.math.ndsu.nodak.edu, http://poincare.matf.bg.ac.rs/informacije/dok.htm) are supplemented with new data and names almost every day.
56 Драган Трифуновић, *op. cit.*, pp. 241, 291–292.
57 *Ibid*., p. 286.
58 Михаило Петровић (1969a), *op. cit.*, p. VI.
59 See: https://sr.wikipedia.org/sr-el/Publications_de_l%27Institut_Math%C3%A9matique.
60 Михаило Петровић (1969a), *op. cit.*, p. VI.
61 Драган Трифуновић, *op. cit.*, pp. 285, 498–499, 280, 360–361.
62 Михаило Петровић (1997a), *op. cit.*, pp. 414–415, 418.
63 Михаило Петровић (1997б), *op. cit.*, p. 316. and Драган Трифуновић, *op. cit.*, p. 515.
64 Михаило Петровић (1997б), *op. cit.*, p. 317. and Драган Трифуновић, *op. cit.*, p. 517.
65 Михаило Петровић, *Рачунање са бројним размацима* (друго издање), Издавачко предузеће „Научна књига", Београд, 1969б, p. V.
66 Михаило Петровић (1997б), *op. cit.*, pp. 404–431 and Михаило Петровић (1997a), *op. cit.*, pp. 316–323.
67 Михаило Петровић (1969a), *op. cit.*, p. VI.
68 Драган Трифуновић, *op. cit.*, p. 215.
69 Михаило Петровић, *Чланци, студије : популарни списи и примењена математика*, [Сабрана дела – књига 10], Завод за уџбенике Београд, 1999, pp. 84, 84–92, 85.
70 *Ibid.*, pp. 90, 90–91.
71 Драган Трифуновић, *op. cit.*, p. 252 (16) and p. 476 (139, 181).
72 After World War II, the *ICMI* continued with its prolific activity. For more information see: Драган Трифуновић, *op. cit.*, p. 287, https://scindeks-clanci.ceon.rs/data/pdf/1452–9343/2014/1452–93431408167M.pdf and https://www.mathunion.org/icmi.
73 Владимир Мићић, Зоран Каделбург, Војислав Андрић и други, *Седамдесет година Друштва математичара Србије*, Друштво математичара Србије, Београд, 2018, p. 3.
74 Драган Трифуновић, *op. cit.*, pp. 172, 177–179, 182–183.
75 Михаило Петровић (1999), *op. cit.*, pp. 9–12, 18–22, 23–26, 27–31.
76 Михаило Петровић (1969a), *op. cit.*, pp. III, V, 12–15, 23–26, 20–24, 16–19.
77 The author of this article presented his paper at the 14th Serbian Mathematical Congress held in Kragujevac in May 2018, elaborating on such highly interesting geometric and non-geometric examples of dependencies in magnitudes in tasks.
78 Михаило Петровић (1999), *op. cit.*, pp. 112–120.
79 *Ibid.*, the paper enumerates only some of Petrović's articles.
80 http://www.novosti.rs/vesti/naslovna/reportaze/aktuelno.293.html:485728-Uzicki-profesor-sestarom-i-lenjirom.
81 Драган Трифуновић, *op. cit.*, pp. 129, 175 and 214, 248–249, 141 and 156, 170, 203, 179, 217, 181, 140, 157, 172, 221.
82 *Ibid.*, pp. 216 and 222, 223, 225, 239 and 244, 321, 325, 275, 342–343, 330–331.

83 *Ibid.*, pp. 330–331.
84 Михаило Петровић (1999), *op. cit.*, pp. 65–69.
85 Драган Трифуновић, *op. cit.*, pp. 431–584.
86 *Ibid.*, pp. 568–584.
87 *Ibid.*, pp. 325, 276, 321, 268 and 430, 381.
88 In 1935, Milutin Milanković wrote a humorous poem "From All Our Scientists", devoted to Mihailo Petrović. See: https://vidovdan.org/istorija/to-su-bili-ljudi-za-ponosrodoljubi-milutin-milankovic-je-posvetio-pesmu-miki-alasu-1935-g/?script=lat.
89 *Сабрана дела Михаила Петровића (Collected Works of Mihailo Petrović),* Завод за уџбенике и наставна средства у Београду, Београд, 1998. His *Collected Works* provide a classification of Petrović's bibliography by subject, specifying a total of twenty papers in the area of mathematical phenomenology.
90 All quotes in this study are from *Collected Works*.
91 Here Petrović is referring to the viewpoint of John Stuart Mill that there is a minimal set of assumptions from which it is possible to derive all laws of nature.
92 In late 19th century, physicians Ludwig Boltzmann, Gustav Kirchhoff and Heinrich Hertz used the term mathematical phenomenology to denote representation of phenomena by mathematical means. We have no confirmation whether and to what extent Petrović relied on their views. For more detail on this, see Nikola Petrović's blog *Mathematical Phenomenology between Myth and Reality,* available at http://nikolamorena.blogspot.com/2012/10/matematicka-fenomenologija-izmeu-mita-i.html.
93 This conceptual apparatus is translatable to the mathematical one, but is nowhere close to it in precision. Petrović belonged to a strong French school of mathematics, but at a time when it accepted with some reservation Cantor's ideas in the theory of sets which gave to the mathematical language the precision to which we are accustomed today. The manner in which Petrović introduces his phenomenological concepts is much closer to the physics of his time.
94 The already referred to, very substantial blog by Nikola Petrović, discusses in detail the position of mathematical phenomenology among sciences that are close to it and in philosophy.
95 In the wording of the second assumption, we speak of a minimum *set* and not the minimum *number* of these assumptions, as stands in Petrović's formulation.
96 Torkel Frantzen, *Godel's Theorem: An Incomplete Guide to its Use and Abuse*, Wellesley, Massachusetts, 2005.
97 Roger Penrose, *Godel, the Mind, and the Laws of Physics*, u *Horizon of Truth*, Kurt Godel and the Foundation of Mathematics, Ed. M. Baaz, C. H. Papadimitriou, H. W. Putnam, D. S. Scott, C. L. Harper, Cambridge University Press, 2011.
98 Kneale, William and Martha, *The development of logic,* Oxford University Press, 1962.
99 Since the relationship of contemporary and Aristotelian logic resembles that of chemistry and alchemy, it is deemed that contemporary logic is unjustifiably termed logic and that it would have been more appropriate if Leibniz's term for the science of the language of science had been accepted: *characteristica universalis*.
100 It is often alleged that Petrović made one of the first computers, which is true. It was an analogue computer, in which one physical process was simulated by another, which is consistent with the spirit of mathematical phenomenology, but this was not a computer in the modern sense of the word.
101 On various meanings of the word *lógos* see more in: Слађана Ристић Горгиев, *Логос и бесмртност,* Центар за црквене студије, 2007. (Slađana Ristić Gorgiev, *Logos and Immortality*).
102 Before Kant, a similar view had been advocated by Plato (ideas as pure mental forms that are inborn in the human mind), and after Kant – by C. G. Jung (personal and collective archetypes as patterns shaping the experience).
103 On the contemporary version of that opinion, the thesis of *non-overlapping magisteria*, see more in Dejan Nikolić's paper published in the first issue of the magazine *Odgovor (The Answer)*.
104 A similar attitude towards metaphysics is adopted in analytical philosophy, one of the dominant trends in contemporary philosophy.
105 A renowned philosopher of science Karl Popper has shown that it is a very hard thing to do [Poper 2002: 79–80].

106 Historians cannot establish with certainty which parts of the Pythagorean legacy are to be ascribed to Pythagoras himself and which to his disciples.

107 There are two alternative Serbian translations of the Latin term *Philosophiæ Naturalis,* which relates to the area of philosophy from which natural sciences are derived: "the philosophy of nature" and "natural philosophy". I prefer the first equivalent for the following two reasons: because it clearly implies that it concerns an area of philosophy and not a philosophical trend and because the other variant implies that there also exists "unnatural philosophy".

108 See more about it in the research of Nikola Petrović entitled *Univerzalni stvaraoci u modernoj Srbiji (Universal Creators in Modern Serbia).*

109 This essay is published in Serbia in the book of collected essays of B. Hamvaš under the title *Estetika (Esthetics).* The publisher is Draslar partner.

110 More details on B. Petronijević's metaphysics are to be found in the book *Branislav Petronijević – univerzalni stvaralac* (*Branislav Petronijević – a Universal Creator)* by Vojislav Gledić, published by Admiral Books.

111 For more details, see: K. G. Jung, *Arhetipovi i kolektivno nesvesno,* Atos, 2001. (C. G. Jung, *Archetypes and the Collective Unconscious).*

112 See more in: Mark Bjukenen, *Neksus – društvene mreže i teorija malog sveta* (Mark Buchanan, *Nexus: Small Worlds and the Groundbreaking Theory of Networks).*

113 Предраг Ђуричић, „Личност Михаила Петровића Аласа у успоменама и анегдотама", *Легенде Београдског универзитета*, Београд: Универзитетска библиотека „Светозар Марковић", 2005.

114 Михаило Петровић Алас, *Сабрана дела I–XV*, (ур.) Драган Трифуновић, Београд: Завод за уџбенике и наставна средства, 1998. (Mihailo Petrović Alas, *Collected Works*, Volumes I–XV, (ed.) Dragan Trifunović).

115 See text on the back cover, in: Мијајловић, Жарко (ур.), *Михаило Петровић Алас: родоначелник српске математичке школе,* Београд: Српска академија наука и уметности, 2018. (For an English edition, see: Mijajlović, Žarko (ed.), *Mihailo Petrović Alas: The Founding Father of the Serbian School of Mathematics,* Belgrade: Serbian Academy of Sciences and Arts, 2018).

116 Михајло Пантић, *Читање воде – српске приче о риболову* (антологија), Београд: Стубови културе, 1998. (Mihajlo Pantić, *Reading Water: Serbian Stories about Fishing* (an anthology)).

117 Лаза К. Лазаревић, *Целокупна дела* (свеска 1; приредили Владан Недић и Бранимир Живојиновић), Београд: Српска академија наука и уметности, 1986. (Laza. K. Lazarević, *Complete Works,* Vol. 1, (eds.) Vladan Nedić and Branimir Živojinović).

118 Михаило Петровић Алас, *Путописи – други део* (*Сабрана дела* књ. XII), Београд: Завод за уџбенике и наставна средства, 1998. (Mihailo Petrović Alas, *Travelogues – Part Two* (*Collected works,* Volume XII)).

119 Ajzak Volton, *Savršeni ribolovac (ili kako da se zabavi čovek sklon razmišljanju)*, translated from English by Radmila B. Šević, Novi Sad: Mediterran publishing, 2015. (Citations taken from: Izaak Walton and Charles Cotton, *The Compleat Angler,* London and New York: John Lane: The Bodley Head, 1897).

120 Михаило Петровић Алас, *Метафоре и алегорије* (*Сабрана дела* књ. XIII), Београд: Завод за уџбенике и наставна средства, 1998. (Mihailo Petrović Alas, *Metaphors and Allegories* (*Collected Works,* Volume XIII).

121 Dragan Trifunović (1930–2009) is the most famous historian of Serbian mathematics. His baptism name was Miodrag, but the hypocorism "Dragan" was so widespread that he used it to sign all written and printed texts after he defended his doctoral dissertation. He published numerous books and papers on Serbian mathematicians, mostly about Mihailo Petrović Alas. He came from a family of distinguished intellectuals. His brother Lazar was the director of the National Museum and one of the most important art theoreticians, and his other brother Đorđe was a world-renowned philologist. All three Trifunović brothers were "historians in their souls" and most their papers were of historiographic nature. Trifunović obtained the majority of biographical data on Petrović that we have today. He was known for his radical attitudes – for instance, he claimed Milutin Milanković was a German agent, which caused vehement reactions. In retrospect, his radicalism, or at least it seems so to the author of this paper, dynamised the provincial, listless atmosphere of the intellectual life of Serbian mathematicians.

122 Mihailo Petrović left an enormous number of photographs, some of which we are still discovering. In all of them, he looks serious, without smiling. Professor Žarko Mijajlović, who initiated the current project of exhibitions and monographs about Mihailo Petrović, says the only photo where Petrović smiles was taken in a tavern. This humorous remark should be complemented with another thought – unfortunately, none of Petrović's photos is of the quality we are used to today and they were taken "as if for a purpose". We have used the quotation marks because it seems to us he was fond of self-promotion. His numerous activities, which he pursued in parallel with mathematics, were a sort of an alternative life. He certainly was not as gifted as Einstein or Tesla, who were probably the champions of self-promotion, but, still, this observation has left a strong impression on me.

123 "Immovable" living species, such as conifers from the northwestern forests of North America are even 6000 years old given that in favourable weather conditions they have a half-year vegetation period and reach the height of 300 metres.

124 According to the estimates of the now already exact biology, Jonathan is 178 years old and is in good health, although his senses are deteriorating and needs assistance while eating. He was brought from the Seychelle islands in 1882, in an already mature age of 50. As other giant tortoises, he belongs to the Testudinidae family, suborder Cryptodira. The oldest tortoise, as it has been confirmed, died aged 189 in Calcutta.

125 The "lady" – the quotation marks are not used by chance – was called Frederica and died around ten years ago. During a veterinary intervention, around thirty years ago, it was determined that Frederica was in fact a male, which is why it is no wonder that she and Jonathan had no offspring.

126 The Madagascar tectonic plate split from Pangea – a supercontinent that incorporated almost all the landmasses on earth – several hundred million years ago, which is very early in term of geology, and the life and evolution had "sufficient time" to develop species not existing elsewhere. They were sufficiently separated by seas so that the majority could not reach other new continents.

127 At least the majority of biographical sources claim so – the main being, of course, Dragutin Trifunović.

128 Pavle Popović (1868–1939) was a Serbian philologist, professor at Belgrade University, rector (1924–1928), founder of a number of Serbian literary gazettes and president of Srpska književna zadruga. Speaking in present-day jargon, he was a sort of Dobrica Ćosić of the time.

129 *Кроз ӣоларну обласӣ,* Српска књижевна задруга, Коло 35, Београд, 1932, p. 248.

130 *У царсӣву ӣусара,* Српска књижевна задруга, Поучник, кв. 7, Београд, 1933, p. 269.

131 *Са океанским рибарима,* Српска књижевна задруга, Савременик, Коло 5, књ. 19, p. 245.

132 *По забаченим осӣрвима,* Српска књижевна задруга, Поучник књ. 9, Београд 1936, p. 294.

133 *Роман јеӣуље,* Српска књижевна задруга, Поучник књ. 11, Београд 1940, p. 187.

134 A considerable part of the *Eel Novel* is a sort of a travelogue.

135 http://alas.matf.bg.ac.rs/~websites/digitalnilegatmpalas/. The Virtual Library, also called "Digital Legacy", contains the works of many other Serbian mathematicians as well. Professor Žarko Mijajlović created the concept and has worked the most on this all-encompassing project. We owe a debt of gratitude to Mijajlović who invested enormous effort in the current project of marking the 150th anniversary of the birth of Mihailo Petrović, and most importantly in the Virtual Library – his life work, whereby he enabled each researcher and interested visitor to easily access online our almost entire mathematical legacy.

136 Belle Epoque (French), "the beautiful epoch", name of the period between the end of Franco-Prussian war in 1871 and the beginning of the World War I in 1914.

137 F. Verhulst, *Henri Poincaré, Impatient Genius*, Springer, 2012. The author of this paper does not know the reason of Poincaré's absence from Defense committee for Petrović's doctoral dissertation.

138 Due to illness, Poincaré did not come to Zurich, and his lecture was read by professor Franel. The other plenary lecturers were A. Hurwitz, G. Peano and F. Klein.

139 D. J. Albers, G. L. Alexanderson, C. Reid, *International mathematical congresses: an illustrated history, 1893–1986*, Springer, 1987.

140 At the lecture, Hilbert stated ten problems, whereas printed versions that were published in several journals included 23 problems. At the moment of writing this article, 20 problems have been either fully or partially solved.
141 Alongside the Congress of Mathematicians, Mihailo Petrović participated at the World Exhibition in Paris, where he received a bronze medal for his hydrointegrator. See: *Михаило Петровић Алас: родоначелних српске математичке школе*, САНУ, Београд, 2018.
142 The solution of the Poincaré Conjecture was published by Russian mathematician Grigori Perelman on the internet in 2003. See: М. Гесен, *Савршена строгост*, Службени гласник, Београд, 2017.
143 "Phenomenology – science on phenomena, description and analysis of phenomena within a certain scientific area considering their development and mutual relation." (И. Клајн, М. Шипка, *Велики речних страних речи и изр*аза, Нови Сад, 2006.)
144 М. Петровић, *Математичка феноменологија*, Сабрана дела 6, Завод за уџбенике и наставна средства, Београд, 1998.
145 J. N. Shive, R. L. Weber, *Similarities in physics*, Adam Hilger Ltd, Bristol 1982.
146 М. Петровић, *Математичка феноменологија*, Сабрана дела 6, Завод за уџбенике и наставна средства, Београд, 1998.
147 B. Greene, *The Hidden Reality: Parallel Universes and the Deep Laws of the Cosmos*, Vintage, 2011.
148 Green wrote that the superstring theory narrowed the gap between the Theory on General Relativity and quantum mechanics and gave hope that gravity could also be put under the umbrella of unified quantum mechanics.
149 М. Петровић, *Рибарство*, Сабрана дела 14, Завод за уџбенике и наставна средства, Београд, 1998.
150 Vito Volterra was one of the four plenary lecturers at the congress in Rome in 1900.
151 М. Пантић, у: *Михаило Петровић Алас: родоначелних српске математичке школе*, САНУ, Београд, 2018.
152 Д. Трифуновић, *Бард српске математике Михаило Петровић Алас*, Завод за уџбенике и наставна средства, Београд, 1991.
153 М. Миланковић, Ј. Михаиловић, *Мика Алас, белешке о животу великог математичара Михаила Петровића*, Фонд др Милићевић, Удружење М. Миланковић, Београд, Калгари, 2012.
154 М. Миланковић, Ј. Михаиловић, *нав. дело.*
155 Stevan Sremac passed away in 1906.
156 Ђ. Карађорђевић, *Истина о моме животу,* Иванка Марковић-Сонтић, Београд, 1988.
157 Ђ. Карађорђевић, *нав. дело.*
158 Ђ. Карађорђевић, *нав. дело.*
159 That same year, 1985, Živojin Pavlović published his novel *Wall of death* for which he received the NIN prize.
160 Карл Густав Јунг, *Aion*, Miba Books, 2018.
161 V. Milićević wrote about this in the introduction of the book by M. Milanković and J. Mihailović.
162 М. Миланковић, Ј. Михаиловић, *Мика Алас, белешке о животу великог математичара Михаила Петровића*, Фонд др Милићевић, Удружење М. Миланковић, Београд, Калгари, 2012.
163 J. Chevalier, A. Gheerbrant, *Rječnik simbola*, Matica Hrvatska, Zagreb, 1983.
164 Ж. Мијајловић у: *Михаило Петровић Алас: родоначелних српске математичке школе*, САНУ, Београд, 2018.
165 В. Јанковић, *Митови и легенде*, Лагуна, Београд, 2017., видети и Матеј, 4: 18–22.
166 Т. Пејовић, *Моје успомене и доживљаји 1919–1945.*
167 Патријарх Павле, *Човек пред лицем Божијим*, Светигора, Цетиње, 2015.
168 М. Петровић, *Чланци – Студије*, Сабрана дела 10, Завод за уџбенике и наставна средства, Београд, 1998.
169 In English *Antikythera mechanis.*
170 Jacopo Francesco Riccati (1676–1754), a mathematician and jurist from Venice.
171 T. Pejović made a mistake here. Jovan Karamata was Vojislav Avakumović's supervisor.
172 Маја Новаковић, „Михаило Петровић Алас у аудио-визуелним медијима", у: *Михаило Петровић Алас, родоначелник српске математичке школе*, Ж. Мијајловић (ур.), Београд: Српска академија наука и уметности, 2018, 119–124.
173 Д. Трифуновић, *Михаило Петровић Алас – живот и дело*, Дечје новине, Горњи Милановац, 1982, 33.

174 Мило Васовић, „Наш најбољи математичар г. Мика Петровић бави се риболовом исто тако интензивно и страсно као и математиком", *Правда*, 11. јануар 1939, 32. (University library "Svetozar Marković").

175 It is often noted that Mihailo Petrović was engaged in professional fishing (*ribarstvo*), and not in the recreational one (*ribolov*). Even the published volume of his collected works is entitled *Ribarstvo.*

176 Мило Васовић, „Наш најбољи математичар г. Мика Петровић бави се риболовом исто тако интензивно и страсно као и математиком", *Правда*, 11. јануар 1939, 32. (University library "Svetozar Marković").

177 *Ibid*.

178 Anon., „Опробан рецепт", *Бодљикаво прасе,* 41, 5. септембар 1942, 4. (University library "Svetozar Marković")

179 М. Михаиловић, „Декански ћевапчићи у Ботаничкој башти", *Време*, 3. јул 1937, 10. (University library "Svetozar Marković")

180 *Ibid*.

181 Anon., „Вечера у част научника и педагога г. Јеленка Михаиловића", *Правда*, 20. фебруар 1939, 7. (University library "Svetozar Marković")

182 *Ibid*.

183 Anon., „Математички институт на Београдском универзитету – кошница научног рада", *Политика*, 8. мај 1938, 9. (Mathematical Institute of SASA)

184 Михајло Петровић, „Београд – негдашњи центар великога рибарства", *Београдске општинске новине*, 3. јануар 1940, 188–196. (University library "Svetozar Marković")

185 Младен Ст. Ђуричић, „У спомен Мике Аласа", *Српски народ*, 19. јун 1943, 12. (University library "Svetozar Marković")

186 The digital legacy of Mihailo Petrović Alas: http://alas.matf.bg.ac.rs/~websites/digitalnilegatmpalas/

187 Apart from Petrović's legacy, there are legacies of other mathematicians: Milutin Milanković, Bogdan Gavrilović, Anton Bilimović, Slaviša Prešić. More details on the digital legacies of the Faculty of Mathematics of Belgrade University can be found at: http://legati.matf.bg.ac.rs/

188 On implementation and architecture of a digital legacy, see more in: Mirjana Maljković, Biljana Stojanović, Žarko Mijajlović, "Digital legacy of Mihailo Petrović Alas", *Преглед НЦД,* 31 (2017), 10–17.

189 In a conversation with Professor Zoran Marković, director of the Mathematical Institute in the period from 1985 until 2014, the author was informed that the representatives of the Archives of Serbia took over a part of the materials, while leaving the rest to the Mathematical Institute. According to Marković, a part of the materials, including, for instance, the minutes from the meetings of the Institute's managing bodies, were planned to be thrown away, and were only preserved thanks to an intervention of the then management of the Institute. Hence the assumption that the above materials are currently not perceived as cultural heritage.

190 All internet sources from the reference list were accessed on October 23, 2018.


LEÇONS
SPECTRES MATHÉMATIQUES
ОБИЧНЕ
ДИФЕРЕНЦИЈАЛНЕ ЈЕДНАЧИНЕ
МАТЕМАТИЧКА
ФЕНОМЕНОЛОГИЈА
THÈSES
A LA FACULTÉ DES SCIENCES DE PARIS
МИХАИЛО ПЕТРОВИЋ
МЕТАФОРЕ
И
АЛЕГОРИЈЕ
У ЦАРСТВУ ГУСАРА

MIHAILO PETROVIĆ: SELECTED BIBLIOGRAPHY

APPENDICES TO BIBLIOGRAPHY AND SOURCES OF DATA

Prepared by Žarko MIJAJLOVIĆ and Stevan PILIPOVIĆ

Mihailo Petrović left behind more than three hundred science works in the field of mathematics and about one hundred works in other areas. He also published around twenty university textbooks and monographies. Petrović has very large opus in numerous fields, from mathematics and its applications to travelogue novels and fishing. He was publishing his works in the most famous world's magazines, books for the leading publishers, but also in domestic publications, and even in local magazines and newspapers. Some of his works were re-printed, while others were collected into thematic wholes and re-published that way. Manuscript *Stereometric inequalities* was printed in Collection of works of SAS XXXV only in 1953, ten years after Petrović's death. After the World War II, summarized compilations of Petrović's travelogues were published as separate editions and under different title. Regardless of editors' efforts in preparation of such editions, those books have to be credited to Mihailo Petrović. Having all this in mind, it is very complicated to gather the complete and accurate bibliography of Mihailo Petrović. Fortunately, this important and hard work was done very well and very thoroughly by Dragan Trifunović, definitely the best expert when it comes to professor Petrović's career. Trifunović's compilation of Petrović's bibliography can be found in several places, but the most complete version, together with numerous comments, registers, indices and other auxiliary texts is in the 15th book of Collected Works of Mihailo Petrović. In the same book there is a big list of sources and contributions of other authors on Mihailo Petrović. This book, thanks to Institute for Textbook Publishing in Belgrade and engagement of academician Gradimir Milovanović, is publicly available via internet (see paragraph *electronic sources* in this appendix) in Digital legacy and Virtual library of Faculty of Mathematics in Belgrade. Let us also mention that almost entire Petrović's legacy is available there in digital form, as well as numerous authors' works on Petrović. The works which have been digitalized and uploaded into Virtual library or Petrović's digital legacy are market in this bibliography with **VB**. Considering the easy availability of the material, before all of the mentioned Trifunović's compilation, we will mention here only excerpts from Petrović's bibliography and authors' contributions about him which are relatively new. Actually, the main goal of this selection is to point to versatility and broadness of Petrović's work. At the end, with the same goal, we are giving two interesting appendices on Petrović's membership in learned societies and his participation in work of various committees and boards.

Petrović published his scientific works and articles in 30 first-class foreign science magazines, and in the same number of domestic ones. The following list is brief excerption (according to Trifunović) from that list, together with number of published works in those magazines:

- *Глас Српске краљевске академије* (60),
- *Годишњак Српске академије наука и уметности* (20),
- *Rad Jugoslavenske akademije znanosti i umjetnosti* (12),
- *Bulletin de l'Académie royale de Serbie* (16),
- *Publications mathématiques de l'Université de Belgrade* (14),
- *Comptes rendus* (30),
- *Bulletin de la Société mathématique de France* (14),
- *Acta Mathematica* (1),
- *Mathematische Annalen* (3),
- *American Journal of Mathematics* (3),
- *L'Intermédiaire des mathématiciens* (9),
- *Věstnik Král. česke společnosti nauk* (6),
- *L'Enseignement mathematique* (8),
- *Rendiconti del Cicolo Matematico di Palermo* (4),
- *Српски књижевни гласник* (13),
- *Гласник Југословенског професорског друштва* (6),
- *Тежак* (6),
- *Ловац* (6),
- *Политика* (27).

Selected scientific works of Mhailo Petrović by 1900

1. О *једној модификацији Грефеовоī меῑода за решавање виших бројних једначина*, first scientific paper, unpublished, 1886.
2. *Intégration graphique de certains types d'équations differentielles du premier ordre*, Bulletin de la Sci. Math. de France, 27, 200–2005, 1890.
3. *Sur les integrales uniformes des équations du premier ordre et du genre zéro*, Comptes rendus, Paris, t. CXVIII, 22, 1190–1193, 1894.
4. *Sur les zéros eῑ les infinis des intégrales des équations différentielles algébriques,* Théses présenтées à la Faculté des Scieпces de Paris, N° 823, Gauthier-Villars, Paris, p. 109, 1894. (Doctoral thesis)
5. *Sommation des séries à l'aide des intégrales définies,* Compt. rendus, Paris, CXX, 15, 819–821, 1895.
6. *Sur l' équation différentielle binôme du premier ordre,* Compt. rendus, Paris, CXXI, 19, 632–635, 1895.
7. *Sur l' équation différentielle de Riccati et applications chimiques.* Vestnik kralovske, Sitzungsberichte der Kőnig – Bőhmischen Gesellschaft der Wissenschaften, Praha, 39, 1–25, 1896.
8. *O diferencijalnim jednačinama prvog reda koje se mogu grafički integraliti pomoću g. Klerićevog šestara*, Glas LI, 18, 313–316, 1896.
9. *Sur les fonctions symetriques et periodiques des diverses determinations d'une fonction algebrique*, Bulletin des Sciences mathematiques, Paris, 2e serie, XX, 108–114, 1896.
10. *Remarques sur les équations de dynamique et sur le mouvement tautochrone*, American Journal of Mathematics, Baltimore, XVIII, 2, 135–144, 1896.
11. *Sur les résidus des fonctions définies par les équations differentielles*, Mathematische Annalen, Leipzig, t. 48, 75–80, 1896.
12. *Contribution* a *la theorie des solutions singulieres des équations différentielles du premier ordre,* Mathematische Annalen, Leipzig, 50, 1–3, 103–112, 1896.
13. *Sur un procede d'integration graphique des equations difterentielles*, Comptes rendus, Paris, 1897, CXXIV, 20, 1081–1084, presented in French Academy of Sciences by professeur P. Appell, 17. 5. 1897, reprinted in *Journal de Physique*, Paris, 476–479, 1897.
14. *Sur l'equation differentielle lineare du second ordre*, Bulletin de la Societe mathematique de France, Paris, XXV, 8–9, 221–235, 1897.
15. *Quelques formules generales relatives au calcul des integrales definies*, Rendiconti del Circolo Matematico di Palermo, Palermo, XI, 6, 247–259, 1897.
16. *Хидраулична инῑеīрација*, Технички лист, Београд, 1898.
17. *Sur l' integration hydraulique des équation différentielle*, American Journal Math, Baltimore, 20, 4, 293–300, 1898.
18. *Sur une propriété des équations différentielles intégrables à l'aide des fonctions méromorphes doublement périodiques*, Acta mathematica, Stockholm, 1898, t. 22, pp. 379–386.

19. *О електричним осцилацијама при испражњавању кондензатора*, Глас СКА, LVI, 27–111, 1898.
20. *Appareil á liquid pour l'intégrration graphique de certains types d'équations différentielles*, American Journal of Mathematics, Baltimore, Vol. XXII, No. 1, 1–12, 1899.
21. *Прилози хемиској кинетици*, Глас СКА, LVII, 207–277, 1899.

Selected scientific works of Mihailo Petrović after 1900

1. *О математичкој теорији активности узрока*, (приступна академска беседа, саопштено 1. децембра 1899), СКА, Глас, књ. LIX, Први разред, књ. 22, Београд, 1900, стр. 183–247.
2. *Jedno pitanje iz teorije funkcija sa dvjema nezavisno promjenljivim količinama*, JAZU, Rad, knj. 143, Razred matematičko-prirodoslovni, knj. 29, Zagreb, 1900, str. 96–106.
3. Sur une *classe d'équations differentielles du premier ordre*, Vestnik Král, české společnosti náuk, Praha, 1901, Trida math. prirodovedecká, t. XXXI, pp. 1–20.
4. *Remarque sur les zeros des series de Taylor,* Bull. Soc. math de France, Paris, 1901, t. XXIX, pp. 303–312.
5. *Generalisation de certaines formules de Stieltjes*, Rendiconti Circ. Mat. Palermo, 1903, t. XVII, 327–334.
6. *Sur une classe de séries entières*, Comptes rendus, Paris, 1906, t, CXLIII, 4, pp. 208–210.
7. *Једна симетрична функција корена и њене особине*, СКА, Глас, књ. LXXV, Први разред, књ. 30. Београд, 1908, стр. 75–100.
8. *Procédé étémentaire d' application des intégrales définies réelles aux equations algébriques et transcendantes,* Nouvelles annales des mathématiques, Paris, 1908, 4e série, t. VIII, pp. 1–15.
9. *Expressions diverses des fonctions associées*, Bulletin de la Societe des Sciences de Bucarest-Roumanie, 1908, t. XVII, 1–2, pp. 11–19.
10. *Télèmétre á sextant,* Brevet d'invention, Paris, 1910, Coll. XII (Instruments de précision, électricité), N° 413730 (avec M. Terzitch).
11. *Allure d'une transcendente entière*, Comptes rendus, Paris, 1912, t. CLIV, 8, pp. 499–501.
12. *Principe de minimum dans les phénomèns éléctrodynamiques et éléléctromagnétiques*, Joumal de Physique théorique et appliquée, Paris, 1912, 5e serie, t. II.
13. *Une transcendente entière et son rôle d'élément de comparaision*, Annales scientifiques de l'École normale supérieure, Paris, 1914, 3e serie, vol. XXXI, octobre 1914, pp. 441–454.
14. *Sur quelques fonctions des* côtes *et des angles d'un triangle*, L'Enseignement mathématique, Geneve, 1916, t. XVIII, 3–4, pp. 153–163.
15. *Remarques sur l'intégrales ∫ uv dx,* L'Enseignement mathematique, Geneve, 1919, t. XX, 4, pp. 268–270.
16. *Problèmes arithmétiques sur les équations différentielles*, Bulletin Soc. mathematique de France, Paris, 1924, t. LII, pp. 514–519.
17. *Correspondance entre la fonction et la fraction décimale*, Proceedings of the V International Congress of Mathematicians, Toronto (Canada), 1924, pp. 449–455.

18. *Problèmes d'intégration qualitative en astronomie*, Annuaire pour l'an 1930, Publications de l'Observatorie astronomique de l'Universite de Belgrade, 1929, t. II, pp. 121–124.
19. *Une application de la résultante de deux fonctions,* Mathematica, Cluj, 1930, t. IV, p. 33–37.
20. *A propos d'une récente application de l' astronomie à la climatologie,* Memoires, Publications de l'Observatoire astronomique de l'Universite de Belgrade, Belgrade, 1932, t. I, pp. 7–12.
21. *Theoreme sur les integrales curvilignes*, Publications mathemat. de l'Univers. de Belgrade, 1933, t. II, 45–59.

Philosophy works

1. *Елементи математичке феноменологије*, СКА, Београд, стр. 788, 1911. **VB**
2. *Méchanismes commums aux Phénomènes disparates*, Nouvelle Collection scientifique, Directeur Émile Borel, Librairie Félix Alcan, Paris, p. 279, 1921. **VB**
3. *Феноменолошко пресликавање*, СКА, Београд, стр. 33. 1933. **VB**
4. *Метафоре и алегорије*, СКЗ, 1967, стр. 196.
   Appendices: **a.** complete bibliography of Mihailo Perović and works of other authors which were a reaction to Petrović's mathematical phenomenology; **b.** Mihailo Petrović (biography). Both appendices were put together by D. Trifunović. **VB**

Scientific monographies and university textbooks:

1. *Les spectres numeriques*, Gauthier-Villars, Paris, 98, 1919. **VB**
2. *Leçons sur les spectres matheatiques,* Paris 1928. **VB**
3. *Integrales premieres a restrictions*, Gauthier-Villars, Paris, 58, 1929. **VB**
4. *Криптографија*, Београд, 1928.
5. *Рачунање са бројним размацима,* предавања на Београдском универзитету, издање Задужбина Луке Ћеловића-Требињца, Београд, стр. 193, 1932. **VB**
6. *Елиптичке функције*, предавања на Београдском универзитету, издање Задужбина Луке Ћеловића-Требињца, Београд, стр. 236, 1937. **VB**
7. *Интеграција диференцијалних једначина помоћу редова,* Београд, 1938; друго издање Универзитет у Београду, стр. 196, 1969. **VB**
8. *Један диференцијални алгоритам и његове примене*, СКА, Београд 1936.

University lectures

In the library of Mathematical Institute of the SASA there are 14 scripts by Mihailo Petrović, all of them manuscripts. Notes from the lectures were written by student Borivoje J. Pujić, in the period from 1910–1914. Titles of the scripts: *Теорија алгебарских једначина*, *Теорија извода са применама*, *Диференцијални рачун*, *Геометријске примене диференцијалног рачуна*, *Интегрални рачун*, *Геометријске примене интегралног рачуна*, *Обичне диференцијалне* *једначине*, *Парцијалне диференцијалне једначине*, *Линеарне диференцијалне једначине*, *Геометријске примене диференцијалних једначина*, *Аналитичка геометрија у равни*, *Аналитичка геометрија у простору*, *Основи теорије детерминаната*, *Теорија функција*. **VB**

Travelogues and novels

1. *Кроз поларну област*, СКЗ, Београд, стр. 248, 1932. **VB**
2. *У царству гусара*, СКЗ, Београд, стр. 270, 1933. **VB**
3. *С океанским рибарима*, СКЗ, Београд, стр. 254, 1935. **VB**
4. *По забаченим острвима,* СКЗ, Београд, стр. 296, 1936. **VB**
5. *Роман јегуље,* СКЗ, стр. 270, Београд 1940. **VB**
6. *Далека копна и мора*, компилација претходно објављених рукописа, приредио Гвидо Тартаља, Просвета, стр. 267, Београд, 1948. **VB**
7. *Велико путовање*, избор из Петровићевих путописа и сећања, приредио Драган Трифуновић, стр. 165, „Вук Караџић", Београд, 1982. **VB**

Works on fishing

1. *Да ли рибе спавају*, Ловац, Београд, 1897, t. II, 9, стр. 61.
2. *La pêche en Serbie*, Exposition Internationale de Turin, Београд, 13, 1911. **VB**
3. *Риболови у тимочкој крајини*, „Млада Србија", Београд, стр. 15, 1933. **VB**
4. *Београд, негдашњи центар великог рибарства*, чланци одштампани у листу „Београдске општинске новине", стр. 94, Београд, 1940. **VB**
5. *Ђердапски риболови у прошлости и у садашњости*, СКА, Београд, стр. 132, 1941. **VB**

Literary and historical works

1. *Једна енглеска књига у нашој преводној књижевности прошлог века*, поводом годишњице Николе Чупића, 1934.
2. *Једна недовршена или изгубљена приповетка Стевана Сремца*, Београд, 1938.
3. *Један велики муслимански гусар*, књ. XLIII, Задужб. Николе Чупића, Београд, 80–127, 1941.

Sources and authors' contributions on Mihailo Petrović

SASA Archives, Libraries: SASA, Mathematical Institute of SASA, MF (Faculty of Mathematics in Belgrade), University Library "Svetozar Marković", Belgrade City Museum, Funds and collections of Archives of Serbia: Ministry of Education, Belgrade University, Faculty of Philosophy, Ministry of People's Economy, Personal Fund of Živojin Perić, Collection *Varia*, *Српске новине.*

Books and catalogues

1. М. Божић, *Преглед историје и филозофије математике*, Завод за уџбенике, 2. издање, Београд, стр. 305, 2010.
2. Г. Малетић, *Катарке Београда*, Лагуна, Београд, стр. 200, 2017.
3. Милутин Миланковић, Јеленко Михаиловић, *Мика Алас – Белешке о животу*, Космос, Београд, 1946; II издање *Личност Михаила Петровића*, приредио В. Милићевић, Београд – Калгари, стр. 124, 2012. **VB**
4. *Михаило Петровић, човек, филозоф, математичар*, зборник радова, у редакцији Д. С. Митриновић, Математичка библиотека 38, Београд, стр. 196, 1968. **VB**
5. М. Петровић, *Notice sur les travaux scientifiques de M. Michel Petrovitch* (1894–1921), Gauthier--Villars, Paris, 1922. **VB**
6. Тадија Пејовић, *Моје успомене и доживљаји 1892–1945*, Породица Пејовић, Београд, 1992. **VB**
7. E. Robson and J. Stedall, *The Oxford handbook of the history of mathematics*, Oxford University Press, 2009.
8. Д. Трифуновић, *Летопис живота и рада Михаила Петровића*, САНУ Београд 1969, стр. VIII+631. **VB**
9. Д. Трифуновић, *Михаило Петровић Алас – живот и дело*, Дечје новине, Горњи Милановац, 1982. **VB**
10. Д. Трифуновић, *Бард српске математике – Михаило Петровић Алас*, Завод за уџбенике и наставна средства, Београд.
11. *Сабрана дела Михаила Петровића*, 15 томова, гл. уредник Д. Трифуновић, Завод за уџбенике и наставна средства, Београд 1998; **Приредили**: Б. Станковић, Д. Адамовић (**1.** *Диференцијалне једначине* 1), В. Марић, Љ. Протић (**2.** *Диференцијалне једначине* 2), Д. Аранђеловић (**3.** *Анализа*), Ж. Мијајловић (**4.** *Алгебра*), Д. Адамовић, Д, Трифуновић (**5.** *Математички спектри*), Д. Трифуновић (**6.** *Математичка феноменологија,* **8.** *Интервална математика, Диференцијални алгоритми*, **14.** *Рибарство*, **15**. *Михаило Петровић – писма, библиографија и летопис*), В. Вујичић (**7.** *Елементи математичке феноменологије*), З. Каделбург (**9.** *Елиптичке функције*, *Интеграција помоћу редова*), М. Марјановић, Д. Трифуновић (**10.** *Чланци – Студије*), С. Пековић (**11,** *Путописи* I, **12**. *Путописи* II, **13**. *Метафоре и алегорије*). **VB**
12. *Српски математичари*, зборник предавања, гл. уредник Д. Којчић, САНУ 2015, Београд, 173 стр.; **Authors' contributions on Mihailo Petrović:** Љ. Протић, К. Хедрих, А. Николић, В. Милићевић. **VB**
13. *Изложба – Београдска математичка школа*, каталог, уводни текст Б. Манић, Архив Србије, Београд, 1968**.** **VB**
14. *Изложба – Легенде Београдског универзитета, Михаило Петровић Алас*, М. Вранић-Игњачевић, Д. Бесара, Д. Трифуновић, Универзитетска библиотека „Светозар Марковић", Београд, 67 стр. 2003.

15. *Изложба – Михаило Петровић Алас, родоначелник српске математичке школе*, каталог, уредник Ж. Мијајловић, САНУ, 166 стр. Београд 2018. **Authors' annexes**: Ж. Мијајловић, С. Пилиповић, С. Вујошевић, М. Пантић, Д. Вучић, И. Живаљевић, М. Божић, Р. Станковић, К. Р. (Стевановић) Хедрих, М. Михаљевић, М. Думнић, М. Новаковић, М. Шеган-Радоњић **VB**

Articles

1. М. Бертолино, *Наш дуг Михаилу Петровићу*, Дијалектика, 1–2, год. XV, Београд, 1980, 87–92.
2. J. Kečkić, *Serbian doctors of mathematics in the 19th century*, Publications de l'Institut Mathematique, N.S. t. 38 (52), 1985, 3–6.
3. Ј. Кечкић, *Михаило Петровић*, Живот и дело Српских научника, књ. 2, САНУ, Београд, 1997.
4. S. Lawrence, *Mathematics Education in the Balkan Societies Up To the WWI*, Teaching Innovations, 2014, Volume 27, Issue 3, pp. 46–57.
5. M. Maljković, B. Stojanović, Ž. Mijajlović, *Digital legacy of Mihailo Petrović Alas,* NCD Review, 31, 2017, 10–17.
6. Ž. Mijajlović, N. Pejović, *Twenty Four Manuscripts in the Virtual Library of the Faculty of Mathematics in Belgrade*, NCD Review, 25, 2014, 29–35.
7. Ž. Mijajlović, Z. Ognjanović, A. Pejović, *Digitization of Mathematical Editions in Serbia,* Mathematics in Computer Science 3(3): 251–263, 2010.
8. A. Nikolić, *The beginnings of mathematical institutions in Serbia*, Publ. Inst. Math. 102 (116), 1–16, 2017.
9. T. Pejović, *Michel Petrovitch (1869–1943*), Publications de l'Institut Mathematique, N.S. 8 (22), 1–4, 1968.
10. A. Petrović, *Development of the first hydraulic analog computer*, Archives internationals d'histoire des sciences, Vol. 54, No. 153, Brepols Publishers, Belgium, 1–14, 2004.
11. N. Pejović, *Digitization of mathematical textbooks used in Serbia in the past*, NCD Review, 12, 2008, 55–64.
12. S. Pilipović, *Naš profesor Mihailo Petrović,* Planeta, 2013.
    http://www.planeta.rs/56/08 Tema4.htm
13. M. Sipos, *The fascinating and forgotten world of hydraulic computing*, June 22, 2016.
    https://www.are.na/block/823545,
14. Миодраг Томић, Математичке науке, Српска академија наука и уметности и развој науке и уметности у Срба, вол. 1, САНУ Београд, 1989.
15. Д. Трифуновић, (1989), *Математички инструменти Љубомира Клерића*, Историјски списи математике и механике, Математички институт САНУ, Историја математичких и механичких наука, књ. 2, 65–84.

16. Снежана Шарбох, Патенти – *Даљинар, направа за шифровање и вечити календар*, Планета, 1. 2. 2018.
17. *Др Михаило Петровић 1868–1943*, У фондовима и збиркама Архива Србије, Приредила: Славица Соломун, Архив Србије, Београд 2018.

Electronic sources – Internet links:

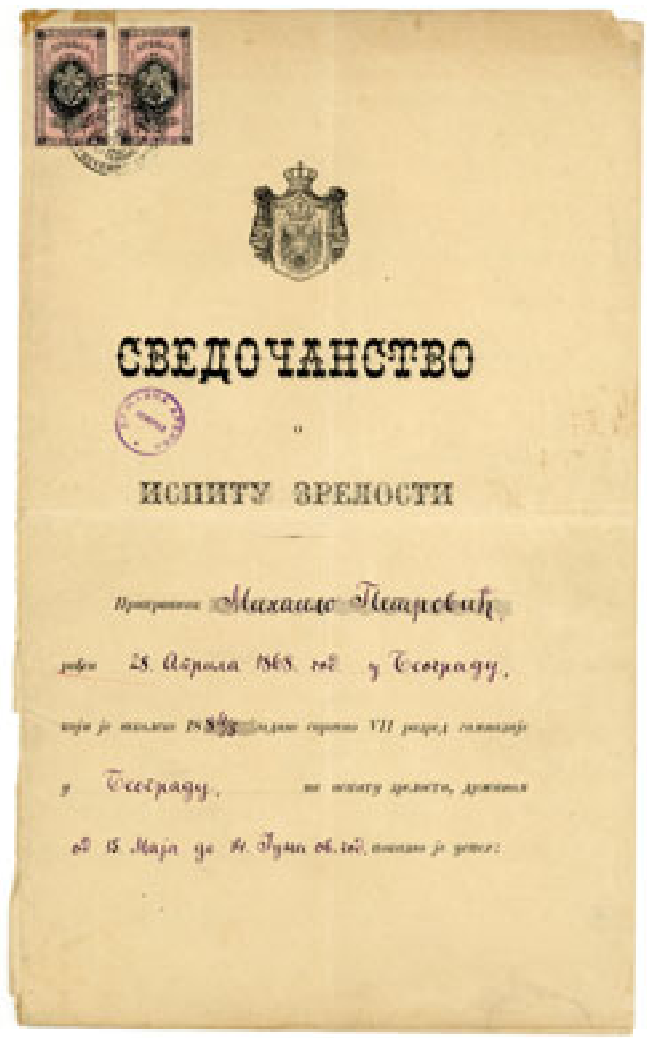
Digital legacies, Faculty of Mathematics in Belgrade:
http://alas.matf.bg.ac.rs/~websites/digitalnilegatmpalas
Virtual library of Faculty of Mathematics in Belgrade:
http://elibrary.matf.bg.ac.rs
Rade Dacić: Eight Serbian Mathematicians,
http://www.rastko.rs/istorija/delo/10296#_Toc197175291
Link to text on Mihailo Petrović's patents from magazine *Planeta* (Planet):
http://www.planeta.org.rs/56/05_Tema1.htm#.Wpa1JKjwaUl
Wikipedia, tags Mihailo Petrović (English.), Михаило Петровић Алас.
History of ICMI (International Commission on Mathematical Instruction),
http://www.icmihistory.unito.it/portrait/petrovic.php
Famous Scientiests,
https://www.famousscientists.org/mihailo-petrovic-alas
Special Issue of International Journal Non-Linear Mechanics, Vol. 73, July 2015, Dedicated to Petrović's theory: *Elements of mathematical phenomenology and Phenomenological Mappings*. Editors: K. R. (Stevanovic) Hedrih, I. Kosenko, P. Krasilnikov, P. D. Spanos
https://www.sciencedirect.com/journal/international-journal-of-nonlinear-mechanics/vol/73
Consulate General of the Republic of Serbia – New York, Serbian scientists:
http://www.serbiaconsulatenyc.com/en/greatscientists.html
Video recording *Mihailo Petrović Alas's Hydro Integrator* (7.21 min)
http://alas.mi.sanu.ac.rs/video/hidrointegrator.mp4
Video recording on Mihailo Petrović Alas's fishing activities (10.12 min)
http://alas.mi.sanu.ac.rs/video/ribarstvo.mp4

Appendices

Member of academies, foreign and international societies: RSA (Royal Serbian Academy, corresponding member from 1897, full member from 1900), YASA (Yugoslav Academy of Sciences and Arts, corresponding member from 1897), member of Czech Academy of Sciences in Prague, Academy of Sciences in Warsaw from 1929, Academy of Sciences in Bucharest from 1929, corresponding member of Polish Academy of Sciences and Arts in Krakow from 1937, Société mathématique de France, Circolo matematico di Palermo, Société Française de Physique, Societatea de sciinte din Bucuresci, Deutshe Mathematiker – Vereinigung – Leipzig, Association des docteurs des Sciences – Paris, Association française pour l'avancement des sciences (honorary president from 1928), international Committee for Unification of Terminology of Mathematical Physics – Berlin, ICMI (International Commission on Mathematical Instruction, since its founding in 1915), Shevchenko Scientific Society – Lviv, Rotary club, professeur agregé at universities in Paris and Brussels.

Member of committees, boards, associations and societies: Committee for preparation of project of the laws of the Royal Serbian Academy, Committee for reviewing of the Academy's bills, representative of the Royal Serbian Academy in International Council of Scientific Unions; Association of university professors, Main Educational Council, Permanent Professorial Examining Committee, representative of Ministry of Education at maturity exams in gymnasiums in Belgrade, Kragujevac, Niš, Jagodina; part-time supervisor of secondary schools; Board for Literature in Nikola Čupić Foundation (elected to the vacant position upon death of writer Milan Rakić in 1938); Advisory Board of Ministry of People's Economy, Committee for passing of the first Law on freshwater fishing on lakes and rivers of Serbia in 1898, Committee for negotiations about signing Convention on fishing with Romania, as well as in negotiations about protection of fishing on Sava, Danube and Drina with Austria-Hungary; Board of Oceanographic Institute in Split, Committee for artificial fishponds of the Ministry of People's Economy, Belgrade Fishing Association (founder), Managing Board of the First Serbian Royal Preferential Shipping Company, Managing board of Association of Hunting Societies of Serbia; part-time inspector of Ministry of Agriculture and Waters.

Mihailo Petrović as a high school graduate of the First Belgrade Gymnasium, 1885 (SASA Archive, 14188/14)



СВЕДОЧАНСТВО

о

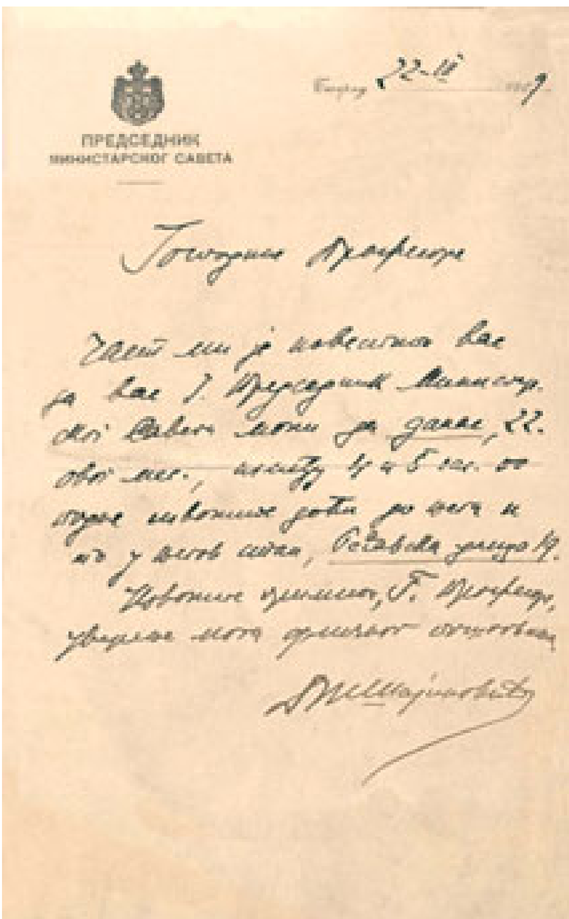
ИСПИТУ ЗРЕЛОСТИ

Михаило Петровић

Certificate of the maturity exam, 1885 (Archive of Serbia, MPS, F25, p207/894)

SASA Archive, 14197/II-22-3

SASA Archive, 14197/II-7



ПРЕДСЕДНИК
МИНИСТАРСКОГ САВЕТА

Београд 22-III 1909.

Господине Професоре

Част ми је известити вас да вас Г. Председник Министарског Савета моли да данас, 22. ов. мес., између 4 и 5 сах. по подне изволите доћи до њега и то у његов стан, Ресавска улица 19.

Изволите примити, Г. Професоре, уверење мога одличног поштовања.

An invitation to Mihailo Petrović to visit Stojan Novaković, the Prime Minister, 22 March 1909 (“Adligat” Society)

SASA Archive

SASA Archive, 14188/27

Mihailo Petrović in the presidency of the First Congress of Mathematicians from Slavic Countries, Warsaw, 23–27 September 1929 (SASA Archive, 1419717)


УНИВЕРСИТЕТ У БЕОГРАДУ

ФИЛОСОФСКИ ФАКУЛТЕТ

ПРВИ СТРУЧНИ ПРЕДМЕТ (А)

ЗАПИСНИК ДИПЛОМСКОГ ИСПИТА.

ГРУПА I Испитни предмети А Теориска Механика Б Прим. Механика В физика

КАНДИДАТ г-ђа Десанка Симић

пријавио се да полаже Теориску Механику

као први стручни предмет (раздео А).

Прегледом уписнице Испитни Одбор је утврдио да је кандидат испунио погодбе из члана 12. Уредбе Философског Факултета и да се може пустити на испит.

10. октобра 1928 год. у Београду.

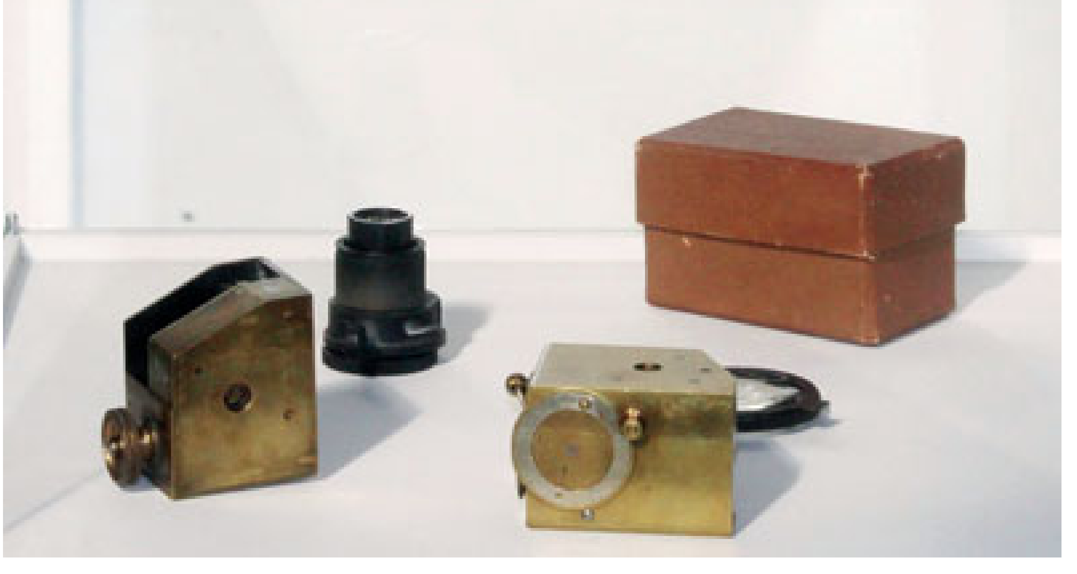
ИСПИТНИ ОДБОР:

Чланови: Н. Салтиков

Председник, Мих. Петровић.

Mihailo Petrović as the president of the exam board, 1928

Personal belongings of Mihailo Petrović in the hall of the elementary school in Belgrade that was named after him



Telemeter from 1910 –
Mihailo Petrović' patent which he constructed with general Milorad Terzić

CIP – Каталогизација у публикацији –
Народна библиотека Србије, Београд

51:929 Петровић М.(082)

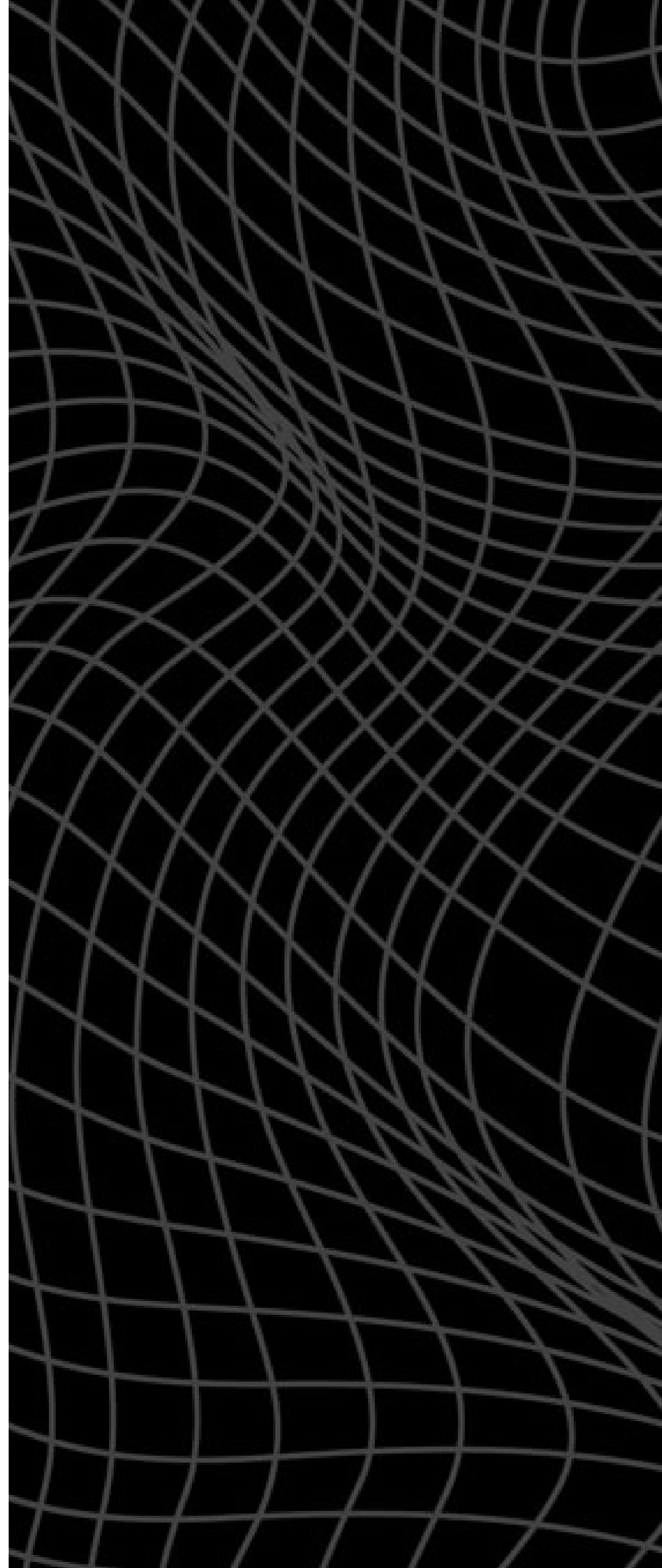
MIHAILO Petrović Alas : life, work, times : on the occasion of the 150th anniversary of his birth / [Editors Stevan Pilipović, Gradimir V. Milovanović, Žarko Mijajlović ; english translation Tatjana Ćosović ... [et al.]]. – Belgrade : SASA, 2019 (Belgrade : Planeta print). – 374 str. : ilustr. ; 24 cm

Tiraž 500. – Editor's foreword: str. 7. – Napomene i bibliografske reference uz radove. – Bibliografija uz većinu radova.

ISBN 978-86-7025-818-1

а) Петровић, Михаило (1868–1943) – Зборници

COBISS.SR-ID 276911884



9 788670 258181

Mihailo
Petrović

ALAS
Life
Work
Times